	b)	If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $r = \lvert\overline{r}\rvert$ then find div $\left(\frac{\overline{r}}{r^3}\right)$.	L3	CO3	5 M
		UNIT-V			
10	a)	Calculate $\int_S \bar{F}.\bar{n}\, ds$ where $\bar{F} = z\bar{i} + x\bar{j} - 3y^2z\bar{k}$ and 'S' is $x^2 + y^2 = 1$ in the first octant between $z = 0$ to $z = 2$.	L3	CO5	5 M
	b)	Apply the Stoke's theorem for $\int_C [(x+y)dx + (2x-z)dy + (y+z)dz]$ where 'C' is the boundary of the triangle with the vertices (2, 0, 0), (0, 3, 0) and (0, 0,6).	L3	CO5	5 M
		OR			
11		Verify the Green's theorem in the plane for $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where 'C' is a square with vertices (0, 0), (2, 0), (2, 2), (0, 2).	L4	CO5	10 M

Code: 23BS1201

I B.Tech - II Semester – Regular / Supplementary Examinations
MAY 2025

DIFFERENTIAL EQUATIONS & VECTOR CALCULUS

(Common for ALL BRANCHES)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Find the integrating factor that makes the equation $y(x^2y^2+2)dx + x(2-x^2y^2)dy = 0$ exact.	L2	CO1
1.b)	Solve $xdy - ydx = xy^2dx$.	L3	CO2
1.c)	Find the particular integral of $(D^2+4)y = \cos 2x$.	L3	CO2
1.d)	Find the solution of $(D^2+D+1)y = 0$.	L2	CO1
1.e)	Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.	L3	CO2
1.f)	Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.	L2	CO1
1.g)	Calculate the maximum value of the directional derivative of $\phi = x^2yz$ at (1, 4, 1).	L3	CO3
1.h)	Calculate the unit normal to the surface $xy = z^2$ at (4, 1, 2).	L3	CO3

1.i)	State Gauss divergence's theorem.	L1	CO1
1.j)	Given $F(t)=(5t^2-3t)i+6t^3 j-7t\,k$, then evaluate $\int_{t=2}^{4} F(t)\,dt$.	L3	CO3

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}=y^2 x\sin x$.	L3	CO2	5 M
	b)	If the air is maintained at 30^0C and the temperature of the body cools from 80^0C to 60^0C in 12 minutes, find the temperature of the body after 24 minutes.	L4	CO4	5 M
OR					
3	a)	Solve $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$.	L3	CO4	5 M
	b)	Bacteria in a culture growing exponentially so that the initial number has doubled in three hours. How many times the initial number will be present after 9 hours.	L4	CO4	5 M
UNIT-II					
4	a)	Solve $(D^2+2D+2)y=e^{-x}+\sin 2x$.	L3	CO2	5 M
	b)	Solve $(D^2-3D+2)y=x\,e^x$.	L3	CO2	5 M
OR					
5		Using method of variation of parameters, solve $(D^2+1)y=\cos x$.	L4	CO4	10 M
UNIT-III					
6	a)	Form the partial differential equation by eliminating the arbitrary function from $z=f(x^2+y^2)+x+y$.	L3	CO2	5 M
	b)	Solve $(x^2-y^2-z^2)p+2xyq=2xz$.	L3	CO2	5 M
OR					
7	a)	Solve the partial differential equation $(D^2-2DD'+D'^2)z=\sin x$.	L4	CO4	5 M
	b)	Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=e^{x+y}$.	L4	CO4	5 M
UNIT-IV					
8	a)	Find the directional derivative of $\phi=xy^2+yz^2$ at (2, -1, 1) in the direction of $\bar{i}+2\bar{j}+2\bar{k}$.	L4	CO5	5 M
	b)	Find the angle between surfaces $xy^2z=3x+z^2$ and $3x^2-y^2+2z=1$ at (1,-2, 1).	L3	CO3	5 M
OR					
9	a)	A vector field is given by $\overline{f}=2xyz^3\bar{i}+x^2z^3\bar{j}+3x^2yz^2\bar{k}$, then show that the field is irrotational and find its scalar potential function.	L4	CO5	5 M

P.V.P. Siddhartha Institute of Technology (Autonomous).
Department of Freshman Engineering.

subject Name: Differential equations & vector calculus.
subject code: 23BS1201.

scheme of evaluation

PART-A.

1 a) Find the I.F that makes the equation $y(x^2y^2+2)dx+x(2-x^2y^2)dy=0$ exact.

$M = x^2y^3+2y \qquad N = 2x - x^3y^2$

It is in the form of $y f(xy)dx + x g(xy) dy = 0$ } — (1M)

$\therefore I.F = \frac{1}{Mx-Ny} = \frac{1}{2x^3y^3}$ — (1M)

$Mx - Ny = 2x^3y^3$

b) solve $x\,dy - y\,dx = xy^2 dx$.

$-\frac{(y\,dx - x\,dy)}{y^2} = x\,dx$ } — (1M)

$-\int d\left(\frac{x}{y}\right) = \int x\,dx + c$

$-\frac{x}{y} = \frac{x^2}{2} + c \quad \Rightarrow \frac{x^2}{2} + \frac{x}{y} = c$ — (1M)

c) Find the P.I of $(D^2+4)y = \cos 2x$.

$y_p = \frac{1}{D^2+4}\cos 2x = \frac{x}{4}\sin 2x$ — (2M)

d) ~~Fin~~ Find the solution of $(D^2+D+1)y = 0$.

A.E $m^2+m+1=0$.

$m = -\frac{1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$ — (1M)

$y_c = e^{-x/2}\left(c_1 \cos\frac{\sqrt{3}x}{2} + c_2 \sin\frac{\sqrt{3}x}{2}\right)$. — (1M)

e) Form the P.D.E by eliminating the arbitrary function from $z = f(x^2-y^2)$

$\frac{\partial z}{\partial x} = p = f'(x^2-y^2)\,2x \rightarrow (1)$

$\frac{\partial z}{\partial y} = q = f'(x^2-y^2)(-2y) \rightarrow (2)$ } — (1M)

$(1) \div (2)$

$\frac{p}{q} = \frac{\cancel{f'}(2x)}{\cancel{f'}(-2y)} \Rightarrow -py = qx$

$\Rightarrow qx + py = 0$. — (1M)

f) Solve $2\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$.

$(2D^2+5DD'+D'^2)z=0$ where $D=\frac{\partial}{\partial x}$, $D'=\frac{\partial}{\partial y}$.

Put $D=m$, $D'=1$.

A.E is $2m^2+5m+1=0$. — (1M)

$$m=\frac{-5\pm\sqrt{25-8}}{4}=\frac{-5\pm\sqrt{17}}{4}$$

$$z=\phi_1\left[y+\left(\frac{-5+\sqrt{17}}{4}\right)x\right]+\phi_2\left[y+\left(\frac{-5-\sqrt{17}}{4}\right)x\right].$$ — (1M)

g) Calculate the maximum value of the D.D of $\phi=x^2yz$ at $(1,4,1)$

$\nabla\phi=i\frac{\partial\phi}{\partial x}+j\frac{\partial\phi}{\partial y}+k\frac{\partial\phi}{\partial z}$

$=i(2xyz)+j(x^2z)+k(x^2y)$. — (1M)

$\nabla\phi$ at $(1,4,1)=8i+j+4k$.

Max value $=|\nabla\phi|=\sqrt{64+1+16}=\sqrt{81}=9$. — (1M)

h) Calculate the unit normal to the surface $xy=z^2$ at $(4,1,2)$.

Let $f=xy-z^2$

$\nabla f=i\frac{\partial f}{\partial x}+j\frac{\partial f}{\partial y}+k\frac{\partial f}{\partial z}=yi+xj-2zk$.

Normal vector at $(4,1,2)=i+4j-4k=\bar{a}$ — (1M)

Unit normal vector $=\hat{a}=\frac{\bar{a}}{|\bar{a}|}=\frac{i+4j-4k}{\sqrt{1+16+16}}=\frac{i+4j-4k}{\sqrt{33}}$ — (1M)

i) State Gauss divergence theorem.

Let S be a closed surface enclosing a volume V. If $\bar{F}$ is a continuously differentiable vector point function then

$\int_V \operatorname{div}\bar{F}\,dV=\int_S \bar{F}\cdot\bar{n}\,dS$, where $\bar{n}$ is the outward normal vector at every pt of S. — (2M)

j) Given $F(t)=(5t^2-3t)i+6t^3j-7tk$, then evaluate $\int_{t=2}^{4}F(t)\,dt$

$\left(\frac{5t^3}{3}-\frac{3t^2}{2}\right)_2^4 i+6\left(\frac{t^4}{4}\right)_2^4 j-7\left[\frac{t^2}{2}\right]_2^4 k$ — (1M)

$\frac{226}{3}i+360j-42k$. — (1M)

PART - B

UNIT - I

2a) Solve the D.E $\frac{dy}{dx} + \frac{y}{x} = y^2 x \sin x$.

This is Bernoulli's D.E.

$\frac{1}{y^2}\frac{dy}{dx} + \frac{y}{x}\cdot\frac{1}{y^2} = x\sin x$.

$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{x}\cdot\frac{1}{y} = x\sin x$. —— (1M)

Let $\frac{1}{y} = t$.

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$.

$-\frac{dt}{dx} + t\cdot\frac{1}{x} = x\sin x$

$\frac{dt}{dx} - t\cdot\frac{1}{x} = -x\sin x$ —— (1M)

$P = -1/x$ $\quad Q = -x\sin x$

$I.F\ e^{\int P dx} = e^{-\log x} = e^{\log x^{-1}} = 1/x$. —— (1M)

G.S $t\cdot I.F = \int Q\cdot I.F\,dx + C$. —— (1M)

$\frac{1}{y}\cdot\frac{1}{x} = -\int x\sin x\cdot\frac{1}{x}\,dx + C$

$\frac{1}{xy} = \cos x + C$. —— (1M)

2b). By Newton's law of cooling,

$\theta = \theta_0 + c e^{-kt}$ —— (1M)

Given $\theta = 80 \quad t = 0 \quad \theta_0 = 30$.

$\theta = 60 \quad t = 12$.

$\theta = ? \quad t = 24$. —— (1M)

i). Put $\theta = 80$, $t = 0$. then $c = 50$ —— (1M)

ii) Put $\theta = 60$, $t = 12$ then $k = \frac{1}{12}\log\frac{5}{3}$. —— (1M)

iii) Put $t = 24$, $\theta = ?$

$\theta = \theta_0 + c e^{-kt}$

$= 30 + 50\, e^{-\frac{1}{12}\log\frac{5}{3}\times 24}$.

$= 30 + 50\, e^{\log(\frac{5}{3})^{-2}}$. —— (1M)

$= 30 + 50\left(\frac{9}{25}\right) = 48°$

3 a). Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

$M = x^2y - 2xy^2$ $\quad N = -x^3 + 3x^2y$

$\frac{\partial M}{\partial y} = x^2 - 4xy$ $\quad \frac{\partial N}{\partial x} = -3x^2 + 6xy$. — (2M)

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \Rightarrow$ not exact.

The given equation is in Homogeneous form.

$\therefore I.F = \frac{1}{Mx + Ny}$.

$Mx + Ny = (x^2y - 2xy^2)x + (3x^2y - x^3)y$.

$= \cancel{x^3y} - 2x^2y^2 + 3x^2y^2 - \cancel{x^3y} = x^2y^2$.

— (1M)

$I.F = \frac{1}{x^2y^2}$.

$\frac{1}{x^2y^2}(x^2y - 2xy^2)dx + \frac{1}{x^2y^2}(-x^3 + 3x^2y)dy = 0$.

$M_1 = \frac{1}{y} - \frac{2}{x}$ $\quad N_1 = -\frac{x}{y^2} + \frac{3}{y}$.

$\frac{\partial M_1}{\partial y} = -\frac{1}{y^2}$ $\quad \frac{\partial N_1}{\partial x} = -\frac{1}{y^2} \Rightarrow \frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$ exact.

G.S $\int M_1 dx + \int N_1 dy = c$.
(y const) (which do not contain x term). — (1M)

$\frac{1}{y}\int dx - 2\int \frac{1}{x}dx + 3\int \frac{1}{y}dy = c$.

— (1M)

$\frac{x}{y} - 2\log x + 3\log y = c$.

b). By law of Natural growth $N = ce^{kt}$ — (2M)

let N_0 be inthe initial no at $t = 0$.

i) $c = N_0$. — (1M)

Given that initial no. has doubled in 3hrs.

ii) $N = 2N_0$, $t = 3$; $N = ce^{kt} \Rightarrow 2\cancel{N_0} = \cancel{N_0}e^{3k}$.

$\Rightarrow e^{3k} = 2 \Rightarrow k = \frac{1}{3}\log 2$.

— (1M)

iii) $N = ?$, $t = 9$

$N = ce^{kt}$

$= N_0 e^{(\frac{1}{3}\log 2)9^3} = N_0 e^{\log 8} = 8N_0$. — (1M)

i.e after 9 hrs bacterial in aculture becomes 8 times the initial no

UNIT - II.

4a) Solve $(D^2+2D+2)y = e^{-x} + \sin 2x$.

A.E $m^2+2m+2=0$

$m = -1 \pm i$ — (2M)

$y_c = e^{-x}(c_1 \cos x + c_2 \sin x)$.

$y_p = \frac{1}{D^2+2D+2} e^{-x} + \frac{1}{D^2+2D+2} \sin 2x = y_{p_1} + y_{p_2}$

$y_{p_1} = \frac{e^{-x}}{D^2+2D+2} = \frac{e^{-x}}{1-2+2} = e^{-x}$. — (1M)

$y_{p_2} = \frac{1}{D^2+2D+2} \sin 2x$.

Put $D^2 = -a^2 = -4$

$= \frac{1}{-4+2D+2} \sin 2x = \frac{\sin 2x}{2(D-1)}$.

$= \frac{1}{2} \frac{D+1}{D^2-1} \sin 2x = -\frac{1}{10}(D+1)\sin 2x = -\frac{1}{10}(2\cos 2x + \sin 2x)$. — (1M)

$y_p = e^{-x} - \frac{1}{10}(2\cos 2x + \sin 2x)$

$y = y_c + y_p = e^{-x}(c_1 \cos x + c_2 \sin x) + e^{-x} - \frac{1}{10}(2\cos 2x + \sin 2x)$ — (1M)

4b). Solve $(D^2-3D+2)y = xe^x$.

A.E $m^2 - 3m + 2 = 0$.

$m = 1, 2$

$y_c = c_1 e^x + c_2 e^{2x}$. — (2M)

$y_p = \frac{1}{D^2-3D+2} xe^x$.

$= e^x \frac{1}{(D+1)^2 - 3(D+1) + 2} x$.

$= e^x \frac{1}{D^2 - D} x$.

$= e^x \frac{1}{D} \cdot (1-D)^{-1} x$

$= -e^x \frac{1}{D} [1 + D + D^2 + \cdots] x$. — (2M)

$= -e^x \frac{1}{D} [x+1]$

$= -e^x \left[\frac{x^2}{2} + x\right]$.

$y = y_c + y_p = c_1 e^x + c_2 e^{2x} - e^x \left[\frac{x^2}{2} + x\right]$. — (1M)

5) using method of variation of parameters, solve
$(D^2+1)y = \cos x$.

A.E $m^2+1=0 \Rightarrow m^2=-1$, $m = \pm i$

$y_c = c_1\cos x + c_2 \sin x = c_1 u(x) + c_2 v(x)$. — (2M)

where $u = \cos x$ $\quad v = \sin x$

$u' = -\sin x$ $\quad v' = \cos x$

$w = uv' - vu' = \cos^2 x + \sin^2 x = 1$. — (2M)

$A = -\int \frac{vR}{w}\, dx$.

$= -\frac{1}{2}\int 2\sin x \cdot \cos x \, dx = -\frac{1}{2}\int \sin 2x\, dx$. — (2M)

$= \frac{1}{2} \cdot \frac{\cos 2x}{2} = \frac{\cos 2x}{4}$

$B = \int \frac{uR}{w}\, dx$.

$= \int \cos^2 x\, dx = \int \left(\frac{1+\cos 2x}{2}\right) dx$ — (2M)

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]$

$y_p = Au + Bv = \frac{1}{4}\cos 2x \cdot \cos x + \frac{\sin x}{2}\left(x + \frac{\sin 2x}{2}\right)$

$y = y_c + y_p$ — (1M)

$= c_1\cos x + c_2 \sin x + \frac{1}{4}\cos 2x \cos x + \frac{\sin x}{2}\left(x + \frac{\sin 2x}{2}\right)$. — (1M)

UNIT – III

6a) Form the P.D.E by eliminating the arbitrary function from
$z = f(x^2+y^2) + x + y$.

$\frac{\partial z}{\partial x} = f'(x^2+y^2)(2x) + 1$

$p = f'(x^2+y^2)(2x) + 1 \Rightarrow f'(x^2+y^2)(2x) = p - 1 \rightarrow (1)$ — (2M)

$\frac{\partial z}{\partial y} = f'(x^2+y^2)(2y) + 1$

$q = f'(x^2+y^2)(2y) + 1$. — (2M)

$2f'(x^2+y^2)y = q - 1 \rightarrow (2)$

(1) ÷ (2)

$\frac{\cancel{f'(x^2+y^2)}(2x)}{\cancel{f'(x^2+y^2)}(2y)} = \frac{p-1}{q-1}$

$\Rightarrow qx - x = py - y \quad \Rightarrow qx - py = x - y$. — (1M)

6b) solve $(x^2-y^2-z^2)p + 2xyq = 2xz$.

A.E $\cancel{\frac{dx}{x^2-y^2-z^2}}$ $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. — (1M)

$$\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}.$$

$$\frac{dy}{\cancel{2x}y} = \frac{dz}{\cancel{2x}z}.$$

$$\int \frac{dy}{y} = \int \frac{dz}{z} + \log c_1$$

$$\log y - \log z = \log c_1$$

$$\Rightarrow c_1 = \frac{y}{z}.$$ — (2M)

using multipliers x, y, z we have

each fraction $= \frac{x\,dx + y\,dy + z\,dz}{x(x^2-y^2-z^2) + 2xy^2 + 2xz^2}$.

$$= \frac{x\,dx + y\,dy + z\,dz}{x(x^2+y^2+z^2)}.$$

$$\therefore \frac{x\,dx + y\,dy + z\,dz}{\cancel{x}(x^2+y^2+z^2)} = \frac{dy}{\cancel{2x}y}.$$

$$\int \frac{2(x\,dx + y\,dy + z\,dz)}{x^2+y^2+z^2} = \int \frac{dy}{y} + \log c_2$$

$$\log(x^2+y^2+z^2) = \log y + \log c_2$$

$$c_2 = \frac{x^2+y^2+z^2}{y}.$$ — (1M)

G.S $f\left(\frac{y}{z}, \frac{x^2+y^2+z^2}{y}\right) = 0$. — (1M)

7a) solve $(D^2 - 2DD' + D'^2)z = \sin x$.

Replace $D = m$, $D' = 1$.

A.E $m^2 - 2m + 1 = 0$.

$(m-1)^2 = 0$.

$m = 1, 1$

C.F $= f_1(y+x) + x f_2(y+x)$. — (2M)

$P.I = \frac{1}{D^2-2DD'+D'^2} e^{x+y} \sin x$.

Replace $D^2 = -a^2 = -1$, $DD' = -ab = 0$, $D'^2 = -b^2 = 0$. — (2M)

$= \frac{1}{-1} \sin x = -\sin x$.

$z = C.F + P.I$

$= f_1(y+x) + x f_2(y+x) - \sin x$. — (1M)

7b) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+y}$.

$(D^2-2DD'+D'^2)z = e^{x+y}$ where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$.

Replace $D = m$, $D' = 1$

$m^2 - 2m + 1 = 0$

$\Rightarrow (m-1)^2 = 0$, $m = 1, 1$ — (2M)

$C.F = f_1(y+x) + x f_2(y+x)$

$P.I = \frac{1}{D^2-2DD'+D'^2} e^{x+y}$.

Put $D = a = 1$, $D' = b = 1$.

$= \frac{1}{+1-2+1} e^{x+y} = \frac{1}{0} e^{x+y}$

$= \frac{x}{\frac{\partial}{\partial D}(D^2-2DD'+D'^2)} e^{x+y}$

$= \frac{x}{2D-2D'} e^{x+y} = \frac{x}{0} e^{x+y}$.

$= \frac{x^2}{\frac{\partial^2}{\partial D^2}(D^2-2DD'+D'^2)} e^{x+y}$.

$= \frac{x^2}{2} e^{x+y}$. — (2M)

$G.S = C.F + P.I$

$z = f_1(y+x) + x f_2(y+x) + \frac{x^2}{2} e^{x+y}$. — (1M)

UNIT- V.

8 a) Find the D.D of $\phi = xy^2 + yz^2$ at $(2,-1,1)$ in the direction of $i+2j+2k$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$= y^2 i + j(2xy + z^2) + k(2yz).$$

$\nabla\phi$ at $(2,-1,1)$.

$$\nabla\phi = i - 3j - 2k$$ — 2M

unit vector in the direction of the vector $i+2j+2k$.

$$\hat{a} = \frac{i+2j+2k}{\sqrt{1+4+4}} = \frac{1}{3}(i+2j+2k).$$ — 2M

D.D along the given direction $= \nabla\phi \cdot \hat{a}$

$$= (i-3j-2k)\cdot\frac{1}{3}(i+2j+2k).$$

$$= \frac{1}{3}(1-6-4) = -\frac{9}{3} = -3.$$ — 1M

---

8 b) Find the angle between surfaces $xy^2z = 3x + z^2$ & $3x^2 - y^2 + 2z = 1$ at $(1,-2,1)$.

Let $f = 3x + z^2 - xy^2z$, $g = 3x^2 - y^2 + 2z - 1$

$$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$= (3-y^2z)i - 2xyz\,j + (2z - xy^2)k.$$ — 1M

∇f at $(1,-2,1)$ — 1M

$$\bar{n}_1 = -i + 4j - 2k.$$

$$\nabla g = i\frac{\partial g}{\partial x} + j\frac{\partial g}{\partial y} + k\frac{\partial g}{\partial z}.$$

$$= 6xi - 2yj + 2k.$$ — 1M

∇g at $(1,-2,1)$

$$\bar{n}_2 = 6i + 4j + 2k.$$ — 1M

$$\cos\theta = \frac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}_1||\bar{n}_2|} = \frac{(-i+4j-2k)\cdot(6i+4j+2k)}{\sqrt{1+16+4}\;\sqrt{36+16+4}}$$ — 1M

$$= \frac{-6+16-4}{\sqrt{21}\,\sqrt{56}} = \frac{6}{\sqrt{21}\cdot 2\sqrt{14}} = \frac{3}{\sqrt{21}\sqrt{14}} = \frac{3}{7\sqrt{6}}$$ $\quad\theta = \cos^{-1}\left(\frac{3}{7\sqrt{6}}\right)$

9 a). A vector field is given by $\bar{F} = 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$ then S.T the field is irrotational & find its scalar potential function.

$$\text{curl } \bar{F} = \nabla \times \bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz^3 & x^2z^3 & 3x^2yz^2 \end{vmatrix}$$

$$= i\left[\cancel{3x^2z^2} - \cancel{3x^2z^2}\right] - j\left[6xyz^2 - 6xyz^2\right] + k\left[2xz^3 - 2xz^3\right]$$

$$= \bar{0}$$ — (3M)

$\therefore$ $\bar{F}$ is irrotational.

then $\exists$ a scalar $\phi \ni \bar{F} = \nabla\phi$ — (1M)

$$2xyz^3 i + x^2z^3 j + 3x^2yz^2 k = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}.$$

By comparing the components. on both sides,

$$\frac{\partial\phi}{\partial x} = 2xyz^3, \quad \frac{\partial\phi}{\partial y} = x^2z^3, \quad \frac{\partial\phi}{\partial z} = 3x^2yz^2.$$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz.$$

By integrating, we get

$$\int d\phi = \int(2xyz^3\,dx + x^2z^3\,dy + 3x^2yz^2\,dz)$$ — (1M)

$$\phi = \int d\,(x^2z^3y)$$

$$\phi = x^2z^3y + c.$$

9 b) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ & $r = |\bar{r}|$ then find $\text{div}\left(\frac{\bar{r}}{r^3}\right)$.

$$\bar{r} = xi + yj + zk.$$

$$|\bar{r}| = \sqrt{x^2+y^2+z^2}.$$

$$r^2 = x^2 + y^2 + z^2.$$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}.$$ — (2M)

$$\text{div}\left(\frac{\bar{r}}{r^3}\right) = \nabla\cdot\frac{\bar{r}}{r^3}$$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\frac{1}{r^3}(xi + yj + zk).$$

$$= \sum\frac{\partial}{\partial x}\cdot\left(\frac{x}{r^3}\right).$$ — (1M)

$$= \sum \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) \quad = \sum \frac{\partial}{\partial x}\left(x r^{-3}\right).$$

$$= \sum \left[-3 r^{-4} \frac{\partial r}{\partial x} \cdot x + r^{-3}\right]$$

$$= \sum \left[-3 x^2 r^{-5} + r^{-3}\right].$$

$$= \cancel{-3r^{-5}(x^2+} \; -3r^{-5} \sum x^2 + r^{-3} \sum 1.$$ — (2M)

$$= -3r^{-5}(x^2+y^2+z^2) + 3r^{-3}$$

$$= -3r^{-3} + 3r^{-3} = 0.$$

UNIT-V.

10a) Given $\bar{F} = zi + xj - 3y^2 z k$.

Let $\phi = x^2 + y^2 - 1$.

$\nabla\phi = 2xi + 2yj$

unit normal vector $\bar{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{\cancel{2}(xi+yj)}{\cancel{2}\sqrt{x^2+y^2}}.$ — (2M)

$= xi + yj \quad [\because x^2+y^2=1]$

Let R be the projection on yz plane.

$$\iint_S \bar{F}\cdot\bar{n}\, dS = \iint_R \bar{F}\cdot\bar{n} \frac{dy\,dz}{|n\cdot i|}$$ — (2M)

$$= \iint (zx + xy) \frac{dy\,dz}{x}.$$

$$= \int_{y=0}^{1}\int_{z=0}^{2} (z+y)\, dy\, dz$$

$$= \int_{y=0}^{1} dy \int_0^2 z\, dz + \int_{y=0}^{1} y\, dy \int_{z=0}^{2} dz.$$ — (1M)

$$= (1)(2) + \left(\tfrac{1}{2}\right)(2) = 2+1 = 3$$

10b) Given $\int (x+y)dx + (2x-z)dy + (y+z)dz$.

$\int [(x+y)i + (2x-z)j + (y+z)k]\cdot[dx\,i + dy\,j + dz\,k]$.

$\int \bar{F}\cdot d\bar{r}$ — (1M)

i.e $\bar{F} = (x+y)i + (2x-z)j + (y+z)k$.

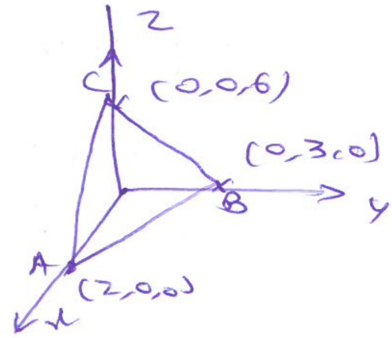
By Stokes theorem.

$$\oint_C \bar{F}\cdot d\bar{r} = \iint_S \text{curl}\,\bar{F}\cdot\bar{n}\, dS$$ — (1M)

$$\text{curl}\,\bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & 2x-z & y+z \end{vmatrix}$$ —— (1M)

$$= 2i + k$$

Equation of the plane through A, B, C is



$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$$

$$3x + 2y + z = 6.$$

Let $\phi = 3x + 2y + z - 6$.

$$\nabla\phi = 3i + 2j + k.$$

$$\bar{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{3i+2j+k}{\sqrt{14}}.$$

$$\iint \text{curl}\,\bar{F}\cdot\bar{n}\,ds = \iint (2i+k)\cdot\frac{1}{\sqrt{14}}(3i+2j+k)\,ds.$$

$$= \frac{1}{\sqrt{14}}\iint (6+1)\,ds$$ —— (1M)

$$= \frac{7}{\sqrt{14}}\iint ds.$$

Let R be projected on xy plane, then

$$= \frac{7}{\sqrt{14}}\int_{x=0}^{2}\int_{y=0}^{\frac{6-3x}{2}} \frac{dx\,dy}{|\bar{n}\cdot k|}.$$

$$= \frac{7}{\sqrt{14}}\int_0^2\int_0^{\frac{6-3x}{2}} \frac{dx\,dy}{1/\sqrt{14}}.$$

$$= 7\int_0^2 [y]_0^{\frac{6-3x}{2}}\,dx.$$

$$= \frac{7}{2}\int_0^2 [6-3x]\,dx.$$

$$= \frac{7}{2}\left[6x - \frac{3x^2}{2}\right]_0^2$$

$$= \frac{7}{2}(6) = 21$$ —— (1M)

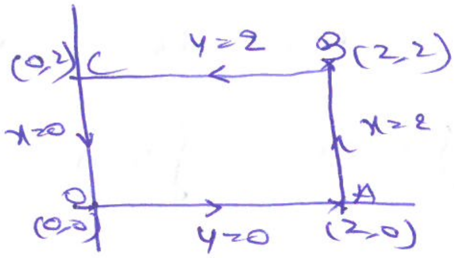
11: verify the Green's theorem in the plane for $\int_C (x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy$ where 'c' is a square with vertices (0, 0), (2, 0), (2, 2), (0, 2).

By Green's Theorem.

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \qquad \text{— (2M)}$$

$M = x^2 - xy^3 \qquad N = y^2 - 2xy.$

$\frac{\partial M}{\partial y} = -3xy^2 \qquad \frac{\partial N}{\partial x} = -2y.$



$$R.H.S = \iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy.$$

$$= \int_{x=0}^{2}\int_{y=0}^{2} (-2y + 3xy^2)\,dx\,dy.$$

$$= \int_0^2 \left[-\cancel{2}\frac{y^2}{\cancel{2}} + \cancel{3}x\frac{y^3}{\cancel{3}}\right]_0^2 dx.$$

$$= \int_0^2 (-4 + 8x)\,dx = \left[-4x + 8\frac{x^2}{2}\right]_0^2 \qquad \text{— (3M)}$$

$$= 8$$

$$L.H.S \int_C x^2 - xy^3)\,dx + (y^2 - 2xy)\,dy = \int_{OA} \bar{F}\cdot d\bar{r} + \int_{AB} \bar{F}\cdot d\bar{r} + \int_{BC} \bar{F}\cdot d\bar{r} + \int_{CO} \bar{F}\cdot d\bar{r}.$$

i) along the line OA.

$y = 0,\ dy = 0.\ ,\ x \to 0$ to 2.

$$\therefore \int_{OA} \bar{F}\cdot d\bar{r} = \int_0^2 x^2\,dx = \frac{8}{3} \qquad \text{— (1M)}$$

ii) along the line AB.

$x = 2,\ dx = 0,\ y \to 0$ to 2.

$$\therefore \int_{AB} \bar{F}\cdot d\bar{r} = \int_0^2 (y^2 - 4y)\,dy. \qquad \text{— (1M)}$$

$$= -\frac{16}{3}.$$

iii) along the line BC.

$y=2,\ dy=0,\ x\to 2$ to 0

$$\int_{BC}\bar{F}\cdot d\bar{r} = \int_2^0 (x^2-8x)\,dx$$

$$= \frac{40}{3}$$ — (1M)

iv) along the line CO

$x=0,\ dx=0,\ y\to 2$ to 0.

$$\int_{CO}\bar{F}\cdot d\bar{r} = \int_2^0 y^2\,dy = -\frac{8}{3}.$$ — (1M)

$$\int \bar{F}\cdot d\bar{r} = \cancel{\frac{8}{3}} - \frac{16}{3} + \frac{40}{3} - \cancel{\frac{8}{3}} = \frac{24}{3} = 8$$

L.H.S = R.H.S.

Hence the theorem is verified. — (1M)

Please accord marks for any alternate method also.

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