PVP 23

Code: 23HS1403

II B.Tech - II Semester – Regular / Supplementary Examinations
APRIL 2026

OPTIMIZATION TECHNIQUES
(Common for IT, AIML, DS)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	What are the advantages of using OR techniques?	L1	CO1
1.b)	What is an infeasible solution?	L1	CO1
1.c)	Why is a dummy row or dummy column introduced in transportation problems?	L1	CO1
1.d)	What is a degenerate solution in a transportation problem?	L1	CO1
1.e)	What is meant by a sequencing problem in operations research?	L1	CO1
1.f)	What is Johnson's rule for n jobs and 2 machines?	L1	CO1
1.g)	What is the objective of inventory control?	L1	CO1
1.h)	What is meant by lead time?	L1	CO1
1.i)	Distinguish between pure strategy and mixed strategy.	L2	CO1
1.j)	Explain a saddle point in a game.	L2	CO1

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	Use Simplex method to solve the following LP problem: Max Z = $400x_1 + 400x_2$ subject to $4x_1 + 2x_2 \leq 1600$ $\frac{5}{2}x_1 + x_2 \leq 1200$ $\frac{9}{2}x_1 + \frac{3}{2}x_2 \leq 1600$ And $x_1, x_2 \geq 0$		L3	CO2	10 M
OR					
3	Apply BIG-M method to solve the following LP problem: Max Z = $8x_1 + 15x_2 + 25x_3 - x_4$ subject to $x_1 + 2x_2 + 3x_3 = 15$ $2x_1 + x_2 + 5x_3 = 20$ $x_1 + 2x_2 + x_3 + x_4 = 10$ and $x_1, x_2, x_3, x_4 \geq 0$		L3	CO2	10 M
UNIT-II					
4	Evaluate the initial basic feasible solution by VAM and optimize solution		L3	CO3	10 M
OR					

F▼ D►	D1	D2	D3	D4	D5	Capacity
F1	1	9	13	36	51	50
F2	24	12	16	20	1	100
F3	14	33	1	23	23	150
Demand	100	70	50	40	40	

5 | L3 | CO3 | 10 M

Find the optimum allotment of tasks to minimize total man hours

		SUBORDINATES			
		I	II	III	IV
TASKS	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

UNIT-III

6 | L3 | CO3 | 10 M

Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing times (in hours) are given in the following table:

Job	:	*1*	*2*	*3*	*4*	*5*
Machine *A*	:	8	10	6	7	11
Machine *B*	:	5	6	2	3	4
Machine *C*	:	4	9	8	6	5

OR

7 | L3 | CO3 | 10 M

From the following particulars of a Project
(a)Draw the network diagram.
(b) Identify the critical path
(c) Calculate the project duration and total float of each activity

Activity	1-2	1-3	2-3	2-4	3-4
Duration (In days)	3	6	9	7	3

UNIT-IV

8 | L4 | CO4 | 10 M

A company that operates for 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs 240 a meter and there is a demand for 8,000 meters a week. Each replenishment costs Rs 1,050 for administration and Rs 1,650 for delivery, while holding costs are estimated at 25 percent of value held a year. Assuming no shortages are allowed, what is the optimal

	inventory policy for the company? How would this analysis differ if the company wanted to maximize its profits rather than minimize cost? What is the gross profit if the company sells the cable for Rs 360 a meter.			
	OR			
9	A factory requires 1,500 units of an item per month, each costing Rs 27. The cost per order is Rs 150 and the inventory carrying charges work out to 20 percent of the average inventory. Find the economic order quantity and the number of orders per year. Calculate total annual inventory cost.	L4	CO4	10 M
	UNIT-V			
10	A company management and the labour union are negotiating a new three year settlement. Each of these has 4 strategies. The costs to the company are given for every pair of strategy choice. What strategy will the two sides adopt? Also determine the value of the game.	L4	CO5	10 M
	OR			
11	Mr Sethi has Rs 10,000 to invest in one of three options: A, B or C. The return on his investment depends on whether the economy experiences inflation, recession, or no change at all. The possible returns under each economic condition are given below. What should he decide, using the pessimistic criterion, optimistic criterion, equally likely criterion and regret criterion?	L4	CO5	10 M

Question 10 table:

	Company Strategies			
Union Strategies	*I*	*II*	*III*	*IV*
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	– 5	4	11	0

Question 11 table:

	State of Nature		
Strategy	*Inflation*	*Recession*	*No Change*
A	2,000	1,200	1,500
B	3,000	800	1,000
C	2,500	1,000	1,800

E1

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Scheme of Evaluation

Part-A

Q.No	Expected Points	Marks Split
1.a	Definition + 2 advantages of OR	1 + 1
1.b	Correct definition of infeasible solution	2
1.c	Reason: balancing supply & demand + zero cost concept	1 + 1
1.d	Condition m+n−1m+n-1m+n−1 + ε allocation concept	1 + 1
1.e	Definition + objective (sequence/order optimization)	1 + 1
1.f	Rule explanation (min time + placement logic)	1 + 1
1.g	Objective: cost minimization + availability	1 + 1
1.h	Definition + components (order to delivery time)	1 + 1
1.i	Pure vs Mixed (clear distinction)	1 + 1

Part-B

Q.No	Scheme of Evaluation	Marks Split
2	Formulation + Standard form + Initial simplex table + Iterations + Final optimal solution	2 + 2 + 2 + 2 + 2
3	Artificial variables + Big-M formulation + Initial table + Iterations + Final solution	2 + 2 + 2 + 2 + 2
4	Balance check + VAM penalties + Initial allocation + Optimality test + Final cost	1 + 3 + 2 + 2 + 2
5	Cost matrix + Row reduction + Column reduction + Assignment + Total cost	1 + 2 + 2 + 3 + 2
6	Conversion to 2-machine + Johnson rule + Sequence + Time chart + Total time	2 + 3 + 2 + 2 + 1
7	Network diagram + Forward pass + Backward pass + Critical path + Float	3 + 2 + 2 + 2 + 1
8	Data identification + EOQ + Cost calculation + Profit + Interpretation	2 + 2 + 2 + 2 + 2
9	Annual demand + EOQ + Number of orders + Costs + Total cost	2 + 2 + 2 + 2 + 2
10	Payoff matrix + Saddle point + Strategy + Value of game	2 + 2 + 3 + 3
11	Payoff table + Pessimistic + Optimistic + Equal likelihood + Regret	2 + 2 + 2 + 2 + 2

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Key

1a	What are the advantages of using OR techniques ?
Sol	Operations Research techniques help in scientific decision making by providing optimal solutions to complex problems. They ensure better utilization of resources such as men, machines, and materials. OR improves planning, scheduling, and control, leading to reduced costs and increased efficiency. It also supports managers in selecting the best alternative under given constraints.
1b	What is an infeasible solution?
Sol	An infeasible solution is a solution that does not satisfy one or more of the given constraints of a problem. Such a solution cannot be accepted, as it violates the conditions imposed on the decision variables and is therefore not practically possible.
1c	Why is a dummy row or dumy column introduced in transportation problem?
Sol	A dummy row or dummy column is introduced to balance the transportation problem when total supply is not equal to total demand. It converts an unbalanced problem into a balanced one by adding artificial supply or demand with zero transportation cost. This allows the problem to be solved using standard methods.
1d	What is a degenerate solution in a transportation problem?
Sol	A degenerate solution occurs when the number of occupied cells (allocations) in a transportation table is less than $m+n-1$, where m represents rows and n represents columns. Such a situation creates difficulty in obtaining an optimal solution, and a very small quantity (ε) is assigned to some empty cell to resolve degeneracy.
1e	What is meant by a sequencing problem in operations research?
Sol	A sequencing problem involves determining the optimal order in which a set of jobs should be processed on one or more machines. Objective focuses on minimizing total processing time, idle time, or completion time while ensuring efficient utilization of resources.
1f	What is Johnson's rule for an jobs and 2 machines
Sol	Johnson's rule provides an optimal sequence of jobs to minimize total elapsed time when each job must be processed on two machines in the same order.
1g	What is the objective of inventory control?
Sol	Objective of inventory control focuses on maintaining an optimal level of inventory to meet demand without interruption. It aims to minimize total cost, including ordering and carrying costs, while ensuring continuous production and avoiding overstocking or stock outs.
1h	What is meant by lead time ?
Sol	Lead time refers to the time interval between placing an order and receiving the goods. It includes order processing time, transportation time, and delivery time, and is important for maintaining proper inventory levels.
1i	Distinguish between pure strategy and mixed strategy
Sol	**A pure strategy** involves selecting a single course of action with certainty, meaning the same decision is followed every time. In contrast, **a mixed strategy** involves choosing among two or more strategies based on probabilities, where each strategy is selected with a certain likelihood.
1j	Explain a saddle point in a game.
Sol	A saddle point refers to a position in a payoff matrix where the maximum of the row minima equals the minimum of the column maxima. At this point, both players have optimal pure strategies, and the value of the game remains stable without the need for mixed strategies.

2 Use Simplex method to solve the following LP problem

MAX Z = 400x1 + 400x2
subject to
4x1 + 2x2 <= 1600
5/2x1 + x2 <= 1200
9/2x1 + 3/2x2 <= 1600
and x1,x2 >= 0;

Sol Max $Z = 400x_1 + 400x_2$

subject to

$4x_1 + 2x_2 \leq 1600$

$2.5x_1 + x_2 \leq 1200$

$4.5x_1 + 1.5x_2 \leq 1600$

and $x_1, x_2 \geq 0$;

After introducing slack variables

Max $Z = 400x_1 + 400x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$4x_1 + 2x_2 + S_1 = 1600$

$2.5x_1 + x_2 + S_2 = 1200$

$4.5x_1 + 1.5x_2 + S_3 = 1600$

and $x_1, x_2, S_1, S_2, S_3 \geq 0$

Tableau-1	C_j	400	400	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	RHS	Ratio $= \frac{RHS}{x_1}$
R_1 0	S_1	4	2	1	0	0	1600	$\frac{1600}{4} = 400$
R_2 0	S_2	2.5	1	0	1	0	1200	$\frac{1200}{2.5} = 480$
R_3 0	S_3	(4.5)	1.5	0	0	1	1600	$\frac{1600}{4.5} = 355.5556 \rightarrow$
Z_j	$Z = 0$	0	0	0	0	0		
	$Z_j - C_j$	-400 ↑	-400	0	0	0		

Most Negative $Z_j - C_j$ is -400. So, the entering variable is x_1.

Minimum ratio is 355.5556. So, the leaving basis variable is S_3.

∴ The pivot element is 4.5.

Entering $= x_1$, Departing $= S_3$, Key Element $= 4.5$

+ R_3(new) = R_3(old) $\div 4.5$

+ R_1(new) = R_1(old) - $4R_3$(new)

+ R_2(new) = R_2(old) - $2.5R_3$(new)

Tableau-2	C_j	400	400	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	RHS	Ratio $= \frac{RHS}{x_2}$
R_1 0	S_1	0	(0.6667)	1	0	-0.8889	177.7778	$\frac{177.7778}{0.6667} = 266.6667 \rightarrow$
R_2 0	S_2	0	0.1667	0	1	-0.5556	311.1111	$\frac{311.1111}{0.1667} = 1866.6667$
R_3 400	x_1	1	0.3333	0	0	0.2222	355.5556	$\frac{355.5556}{0.3333} = 1066.6667$
Z_j	$Z = 142222.2222$	400	133.3333	0	0	88.8889		
	$Z_j - C_j$	0	-266.6667 ↑	0	0	88.8889		

Most Negative $Z_j - C_j$ is -266.6667. So, the entering variable is x_2.

Minimum ratio is 266.6667. So, the leaving basis variable is S_1.

∴ The pivot element is 0.6667.

Entering $= x_2$, Departing $= S_1$, Key Element $= 0.6667$

+ R_1(new) = R_1(old) ÷ 0.6667

+ R_2(new) = R_2(old) - 0.1667R_1(new)

+ R_3(new) = R_3(old) - 0.3333R_1(new)

Tableau-3	C_j	400	400	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	RHS	Ratio $= \frac{RHS}{S_3}$
R_1 400	x_2	0	1	1.5	0	-1.3333	266.6667	$\frac{266.6667}{-1.3333}$ (ignore, denominator is -ve)
R_2 0	S_2	0	0	-0.25	1	-0.3333	266.6667	$\frac{266.6667}{-0.3333}$ (ignore, denominator is -ve)
R_3 400	x_1	1	0	-0.5	0	(0.6667)	266.6667	$\frac{266.6667}{0.6667} = 400 \rightarrow$
Z_j	$Z = 213333.3333$	400	400	400	0	-266.6667		
	$Z_j - C_j$	0	0	400	0	-266.6667 ↑		

Most Negative $Z_j - C_j$ is -266.6667. So, the entering variable is S_3.

Minimum ratio is 400. So, the leaving basis variable is x_1.

∴ The pivot element is 0.6667.

Entering $= S_3$, Departing $= x_1$, Key Element $= 0.6667$

+ R_3(new) = R_3(old) ÷ 0.6667

+ R_1(new) = R_1(old) + 1.3333R_3(new)

+ R_2(new) = R_2(old) + 0.3333R_3(new)

Tableau-4	C_j	400	400	0	0	0		
C_B	**Basis**	x_1	x_2	S_1	S_2	S_3	RHS	**Ratio**
R_1 400	x_2	2	1	0.5	0	0	800	
R_2 0	S_2	0.5	0	-0.5	1	0	400	
R_3 0	S_3	1.5	0	-0.75	0	1	400	
Z_j	$Z = 320000$	800	400	200	0	0		
	$Z_j - C_j$	400	0	200	0	0		

Since all $Z_j - C_j \geq 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 800$

Max $Z = 320000$

3 Apply BIG-M method to solve the following LP problem:
MAX Z = 8x1 + 15x2 + 25x3 - x4
subject to
x1 + 2x2 + 3x3 = 15
2x1 + x2 + 5x3 = 20
x1 + 2x2 + x3 + x4 = 10
and x1,x2,x3,x4 >= 0;

Sol

subject to

$$x_1 + 2x_2 + 3x_3 + A_1 = 15$$
$$2x_1 + x_2 + 5x_3 + A_2 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$$

and $x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$

Tableau-1	C_j	8	15	25	-1	$-M$	$-M$	$-M$		
C_B	**Basis**	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS	**Ratio** $= \frac{RHS}{x_3}$
R_1 $-M$	A_1	1	2	3	0	1	0	0	15	$\frac{15}{3} = 5$
R_2 $-M$	A_2	2	1	(5)	0	0	1	0	20	$\frac{20}{5} = 4 \rightarrow$
R_3 $-M$	A_3	1	2	1	1	0	0	1	10	$\frac{10}{1} = 10$
Z_j	$Z = -45M$	$-4M$	$-5M$	$-9M$	$-M$	$-M$	$-M$	$-M$		
	$C_j - Z_j$	$4M + 8$	$5M + 15$	$9M + 25 \uparrow$	$M - 1$	0	0	0		

Most Positive $C_j - Z_j$ is $9M + 25$. So, the entering variable is x_3.

Minimum ratio is 4. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is 5.

Entering $= x_3$, Departing $= A_2$, Key Element $= 5$

+ R_2(new) = R_2(old) $\div 5$

+ R_1(new) = R_1(old) - $3R_2$(new)

+ R_3(new) = R_3(old) - R_2(new)

Tableau-2	C_j	8	15	25	-1	$-M$	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS	Ratio = $\frac{RHS}{x_2}$
R_1 $-M$	A_1	-0.2	(1.4)	0	0	1	-0.6	0	3	$\frac{3}{1.4} = 2.1429 \rightarrow$
R_2 25	x_3	0.4	0.2	1	0	0	0.2	0	4	$\frac{4}{0.2} = 20$
R_3 $-M$	A_3	0.6	1.8	0	1	0	-0.2	1	6	$\frac{6}{1.8} = 3.3333$
Z_j	$Z = -9M + 100$	$-0.4M + 10$	$-3.2M + 5$	25	$-M$	$-M$	$0.8M + 5$	$-M$		
	$C_j - Z_j$	$0.4M - 2$	$3.2M + 10 \uparrow$	0	$M - 1$	0	$-1.8M - 5$	0		

Most Positive $C_j - Z_j$ is $3.2M + 10$. So, the entering variable is x_2.

Minimum ratio is 2.1429. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is 1.4.

Entering $= x_2$, Departing $= A_1$, Key Element $= 1.4$

+ R_1(new) = R_1(old) $\div 1.4$

+ R_2(new) = R_2(old) - $0.2R_1$(new)

+ R_3(new) = R_3(old) - $1.8R_1$(new)

Tableau-3	C_j	8	15	25	-1	$-M$	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS	Ratio = $\frac{RHS}{x_4}$
R_1 15	x_2	-0.1429	1	0	0	0.7143	-0.4286	0	2.1429	$\frac{2.1429}{0}$ (ignore, denominator is 0)
R_2 25	x_3	0.4286	0	1	0	-0.1429	0.2857	0	3.5714	$\frac{3.5714}{0}$ (ignore, denominator is 0)
R_3 $-M$	A_3	0.8571	0	0	(1)	-1.2857	0.5714	1	2.1429	$\frac{2.1429}{1} = 2.1429 \rightarrow$
Z_j	$Z = -2.1429M + 121.4286$	$-0.8571M + 8.5714$	15	25	$-M$	$1.2857M + 7.1429$	$-0.5714M + 0.7143$	$-M$		
	$C_j - Z_j$	$0.8571M - 0.5714$	0	0	$M - 1$ $\uparrow$	$-2.2857M - 7.1429$	$-0.4286M - 0.7143$	0		

Most Positive C_j - Z_j is M - 1. So, the entering variable is x_4.

Minimum ratio is 2.1429. So, the leaving basis variable is A_3.

∴ The pivot element is 1.

Entering $= x_4$, Departing $= A_3$, Key Element $= 1$

+ R_3(new) = R_3(old)

+ R_1(new) = R_1(old)

+ R_2(new) = R_2(old)

Tableau-4	C_j	8	15	25	-1	$-M$	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS	Ratio = $\frac{RHS}{x_1}$
R_1 15	x_2	-0.1429	1	0	0	0.7143	-0.4286	0	2.1429	$\frac{2.1429}{-0.1429}$ (ignore, denominator is -ve)
R_2 25	x_3	0.4286	0	1	0	-0.1429	0.2857	0	3.5714	$\frac{3.5714}{0.4286} = 8.3333$
R_3 -1	x_4	**(0.8571)**	0	0	1	-1.2857	0.5714	1	2.1429	$\frac{2.1429}{0.8571} = 2.5 \rightarrow$
Z_j	$Z = 119.2857$	**7.7143**	**15**	**25**	**-1**	**8.4286**	**0.1429**	**-1**		
	$C_j - Z_j$	**0.2857** ↑	0	0	0	$-M - 8.4286$	$-M - 0.1429$	$-M + 1$		

Most Positive C_j - Z_j is 0.2857. So, the entering variable is x_1.

Minimum ratio is 2.5. So, the leaving basis variable is x_4.

∴ The pivot element is 0.8571.

Entering $= x_1$, Departing $= x_4$, Key Element $= 0.8571$

+ R_3(new) = R_3(old) $\div 0.8571$

+ R_1(new) = R_1(old) + $0.1429 R_3$(new)

+ R_2(new) = R_2(old) - $0.4286 R_3$(new)

Tableau-5	C_j	8	15	25	-1	$-M$	$-M$	$-M$		
C_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	RHS	Ratio
R_1 15	x_2	0	1	0	0.1667	0.5	-0.3333	0.1667	2.5	
R_2 25	x_3	0	0	1	-0.5	0.5	0	-0.5	2.5	
R_3 8	x_1	1	0	0	1.1667	-1.5	0.6667	1.1667	2.5	
Z_j	$Z = 120$	**8**	**15**	**25**	**-0.6667**	**8**	**0.3333**	**-0.6667**		
	$C_j - Z_j$	0	0	0	-0.3333	$-M - 8$	$-M - 0.3333$	$-M + 0.6667$		

Since all $C_j - Z_j \leq 0$

Hence, optimal solution is arrived with value of variables as
$x_1 = 2.5, x_2 = 2.5, x_3 = 2.5, x_4 = 0$

Max $Z = 120$

4 Evaluate the initial basic feasible solution by VAM and optimize solution

	D1	D2	D3	D4	D5	Capacity
F1	1	9	13	36	51	50
F2	24	12	16	20	1	100
F3	14	33	1	23	23	150
Demand	100	70	50	40	40	

Sol

Initial feasible solution is

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	1(50)	9	13	36	51	50	8 | 8 | 8 | -- | -- | -- | -- |
S_2	24	12(60)	16	20	1(40)	100	11 | 4 | 8 | 8 | -- | -- | -- |
S_3	14(50)	33(10)	1(50)	23(40)	23	150	13 | 13 | 9 | 9 | 9 | 9 | 14 |
Demand	100	70	50	40	40		
Column Penalty	13 13 13 10 14 14 14	3 3 3 21 33 -- --	12 12 -- -- -- -- --	3 3 3 3 23 23 --	22 -- -- -- -- -- --		

Here, the number of allocated cells = 7 is equal to m + n - 1 = 3 + 5 - 1 = 7

The minimum total transportation cost = 1*50+12*60+1*40+14*50+33*10+1*50+23*40=Rs.2810/-

Iteration-1 of optimality test

1. Find u_i and v_j for all occupied cells(i,j), where $c_{ij} = u_i + v_j$

1. Substituting, $u_3 = 0$, we get

2. $c_{31} = u_3 + v_1 \Rightarrow v_1 = c_{31} - u_3 \Rightarrow v_1 = 14 - 0 \Rightarrow v_1 = 14$

3. $c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 1 - 14 \Rightarrow u_1 = -13$

4. $c_{32} = u_3 + v_2 \Rightarrow v_2 = c_{32} - u_3 \Rightarrow v_2 = 33 - 0 \Rightarrow v_2 = 33$

5. $c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 12 - 33 \Rightarrow u_2 = -21$

6. $c_{25} = u_2 + v_5 \Rightarrow v_5 = c_{25} - u_2 \Rightarrow v_5 = 1 + 21 \Rightarrow v_5 = 22$

7. $c_{33} = u_3 + v_3 \Rightarrow v_3 = c_{33} - u_3 \Rightarrow v_3 = 1 - 0 \Rightarrow v_3 = 1$

8. $c_{34} = u_3 + v_4 \Rightarrow v_4 = c_{34} - u_3 \Rightarrow v_4 = 23 - 0 \Rightarrow v_4 = 23$

Optimality test

	D_1	D_2	D_3	D_4	D_5	Supply	u_i
S_1	1 (50)	9	13	36	51	50	$u_1 = -13$
S_2	24	12 (60)	16	20	1 (40)	100	$u_2 = -21$
S_3	14 (50)	33 (10)	1 (50)	23 (40)	23	150	$u_3 = 0$
Demand	100	70	50	40	40		
v_j	$v_1 = 14$	$v_2 = 33$	$v_3 = 1$	$v_4 = 23$	$v_5 = 22$		

2. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij} = c_{ij} - \left(u_i + v_j\right)$

1. $d_{12} = c_{12} - \left(u_1 + v_2\right) = 9 - (-13 + 33) = -11$

2. $d_{13} = c_{13} - \left(u_1 + v_3\right) = 13 - (-13 + 1) = 25$

3. $d_{14} = c_{14} - \left(u_1 + v_4\right) = 36 - (-13 + 23) = 26$

4. $d_{15} = c_{15} - \left(u_1 + v_5\right) = 51 - (-13 + 22) = 42$

5. $d_{21} = c_{21} - \left(u_2 + v_1\right) = 24 - (-21 + 14) = 31$

6. $d_{23} = c_{23} - \left(u_2 + v_3\right) = 16 - (-21 + 1) = 36$

7. $d_{24} = c_{24} - \left(u_2 + v_4\right) = 20 - (-21 + 23) = 18$

8. $d_{35} = c_{35} - \left(u_3 + v_5\right) = 23 - (0 + 22) = 1$

	D_1	D_2	D_3	D_4	D_5	Supply	u_i
S_1	1 (50)	9 [-11]	13 [25]	36 [26]	51 [42]	50	$u_1 = -13$
S_2	24 [31]	12 (60)	16 [36]	20 [18]	1 (40)	100	$u_2 = -21$
S_3	14 (50)	33 (10)	1 (50)	23 (40)	23 [1]	150	$u_3 = 0$
Demand	100	70	50	40	40		
v_j	$v_1 = 14$	$v_2 = 33$	$v_3 = 1$	$v_4 = 23$	$v_5 = 22$		

3. Now choose the most negative value from all d_{ij} (opportunity cost) = d_{12} = [-11]

and draw a closed path from S_1D_2.

Closed path is $S_1D_2 \rightarrow S_1D_1 \rightarrow S_3D_1 \rightarrow S_3D_2$

Closed path and plus/minus sign allocation...

	D_1	D_2	D_3	D_4	D_5	Supply	u_i
S_1	1 (50) (-)	9 [-11] (+)	13 [25]	36 [26]	51 [42]	50	$u_1 = -13$
S_2	24 [31]	12 (60)	16 [36]	20 [18]	1 (40)	100	$u_2 = -21$
S_3	14 (50) (+)	33 (10) (-)	1 (50)	23 (40)	23 [1]	150	$u_3 = 0$
Demand	100	70	50	40	40		
v_j	$v_1 = 14$	$v_2 = 33$	$v_3 = 1$	$v_4 = 23$	$v_5 = 22$		

4. Minimum allocated value among all negative position (-) on closed path = 10
Substract 10 from all (-) and Add it to all (+)

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	1 (40)	9 (10)	13	36	51	50
S_2	24	12 (60)	16	20	1 (40)	100
S_3	14 (60)	33	1 (50)	23 (40)	23	150
Demand	100	70	50	40	40	

5. Repeat the step 1 to 4, until an optimal solution is obtained.

Iteration-2 of optimality test

1. Find u_i and v_j for all occupied cells(i,j), where $c_{ij} = u_i + v_j$

1. Substituting, $u_3 = 0$, we get

2. $c_{31} = u_3 + v_1 \Rightarrow v_1 = c_{31} - u_3 \Rightarrow v_1 = 14 - 0 \Rightarrow v_1 = 14$

3. $c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 1 - 14 \Rightarrow u_1 = -13$

4. $c_{12} = u_1 + v_2 \Rightarrow v_2 = c_{12} - u_1 \Rightarrow v_2 = 9 + 13 \Rightarrow v_2 = 22$

5. $c_{22} = u_2 + v_2 \Rightarrow u_2 = c_{22} - v_2 \Rightarrow u_2 = 12 - 22 \Rightarrow u_2 = -10$

6. $c_{25} = u_2 + v_5 \Rightarrow v_5 = c_{25} - u_2 \Rightarrow v_5 = 1 + 10 \Rightarrow v_5 = 11$

7. $c_{33} = u_3 + v_3 \Rightarrow v_3 = c_{33} - u_3 \Rightarrow v_3 = 1 - 0 \Rightarrow v_3 = 1$

8. $c_{34} = u_3 + v_4 \Rightarrow v_4 = c_{34} - u_3 \Rightarrow v_4 = 23 - 0 \Rightarrow v_4 = 23$

	D_1	D_2	D_3	D_4	D_5	Supply	u_i
S_1	1 (40)	9 (10)	13	36	51	50	$u_1 = -13$
S_2	24	12 (60)	16	20	1 (40)	100	$u_2 = -10$
S_3	14 (60)	33	1 (50)	23 (40)	23	150	$u_3 = 0$
Demand	100	70	50	40	40		
v_j	$v_1 = 14$	$v_2 = 22$	$v_3 = 1$	$v_4 = 23$	$v_5 = 11$		

2. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij} = c_{ij} - (u_i + v_j)$

1. $d_{13} = c_{13} - (u_1 + v_3) = 13 - (-13 + 1) = 25$

2. $d_{14} = c_{14} - (u_1 + v_4) = 36 - (-13 + 23) = 26$

3. $d_{15} = c_{15} - (u_1 + v_5) = 51 - (-13 + 11) = 53$

4. $d_{21} = c_{21} - (u_2 + v_1) = 24 - (-10 + 14) = 20$

5. $d_{23} = c_{23} - (u_2 + v_3) = 16 - (-10 + 1) = 25$

6. $d_{24} = c_{24} - (u_2 + v_4) = 20 - (-10 + 23) = 7$

7. $d_{32} = c_{32} - (u_3 + v_2) = 33 - (0 + 22) = 11$

8. $d_{35} = c_{35} - (u_3 + v_5) = 23 - (0 + 11) = 12$

	D_1	D_2	D_3	D_4	D_5	Supply	u_i
S_1	1 (40)	9 (10)	13 [25]	36 [26]	51 [53]	50	$u_1 = -13$
S_2	24 [20]	12 (60)	16 [25]	20 [7]	1 (40)	100	$u_2 = -10$
S_3	14 (60)	33 [11]	1 (50)	23 (40)	23 [12]	150	$u_3 = 0$
Demand	100	70	50	40	40		
v_j	$v_1 = 14$	$v_2 = 22$	$v_3 = 1$	$v_4 = 23$	$v_5 = 11$		

Since all $d_{ij} \geq 0$.

So final optimal solution is arrived.

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	1 (40)	9 (10)	13	36	51	50
S_2	24	12 (60)	16	20	1 (40)	100
S_3	14 (60)	33	1 (50)	23 (40)	23	150
Demand	100	70	50	40	40	

The minimum total transportation cost $= 1 \times 40 + 9 \times 10 + 12 \times 60 + 1 \times 40 + 14 \times 60 + 1 \times 50 + 23 \times 40 = 2700$

5 Find the optimum allotment of tasks to minimize total man hours (10 Marks)

Sol

		SUBORDINATAES			
		I	II	III	IV
TASKS	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

Step-1: Find out each row minimum element and subtract it from that row

Step-2: Find out each column minimum element and subtract it from that column.

Step-3: Cover all zeros with a minimum number of lines

Determine the minimum number of lines, required to cover all zeros in the matrix.

There are 4 lines required to cover all zeros, which is equal to size of matrix (4), so an optimal assignment exists and the algorithm stops.

	1	2	3	4	
A	0	18	9	3	(-8)
B	9	24	0	22	(-4)
C	23	4	3	0	(-15)
D	9	16	14	0	(-10)

Step-1

	1	2	3	4	
A	0	14	9	3	
B	9	20	0	22	
C	23	0	3	0	
D	9	12	14	0	
	(-0)	(-4)	(-0)	(-0)	

Step-2

	1	2	3	4	
A	0	14	9	3	
B	9	20	0	22	
C	23	0	3	0	
D	9	12	14	0	
	✓	✓	✓	✓	

Step-3

	1	2	3	4	
A	[0]	14	9	3	
B	9	20	[0]	22	
C	23	[0]	3	0	
D	9	12	14	[0]	

Optimal assignment

	SUBORDINATAES				
		I	II	III	IV
TASKS	A	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

		Sub	Cost
TASKS	A	I	8
	B	III	4
	C	II	19
	D	IV	10
	Total		41

6 Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC processing times (in hours) are given in the following table

Job	1	2	3	4	5
Machine-1	8	10	6	7	11
Machine-2	5	6	2	3	4
Machine-3	4	9	8	6	5

Sol

Since any of condition $\min\{T_{1j}\} \geq \max\{T_{ij}\}$ and/or $\min\{T_{mj}\} \geq \max\{T_{ij}\}$, for j=2,3,...,m-1 is satisfied.

So given problem can be converted to 2-machine problem.

Machine-G

13	16	8	10	15

Machine-H

9	15	10	9	9

1. The smallest processing time is 8 hour for job 3 on Machine-G. So job 3 will be processed first.

3				

2. The next smallest processing time is 9 hour for job 1,4,5 on Machine-H and for this jobs 10 is smallest on Machine-G. So job 4 will be processed last.

3				4

3. The next smallest processing time is 9 hour for job 1,5 on Machine-H and for this jobs 13 is smallest on Machine-G. So job 1 will be processed before job 4.

3			1	4

4. The next smallest processing time is 9 hour for job 5 on Machine-H. So job 5 will be processed before job 1.

3		5	1	4

5. The next smallest processing time is 15 hour for job 2 on Machine-H. So job 2 will be processed before job 5.

3	2	5	1	4

Job	M_1 In time	M_1 Out time	M_2 In time	M_2 Out time	M_3 In time	M_3 Out time	Idle time M_2	Idle time M_3
3	0	0 + 6 = 6	6	6 + 2 = 8	8	8 + 8 = 16	6	8
2	6	6 + 10 = 16	16	16 + 6 = 22	22	22 + 9 = 31	8	6
5	16	16 + 11 = 27	27	27 + 4 = 31	31	31 + 5 = 36	5	-
1	27	27 + 8 = 35	35	35 + 5 = 40	40	40 + 4 = 44	4	4
4	35	35 + 7 = 42	42	42 + 3 = 45	45	45 + 6 = 51	2	1

The total minimum elapsed time = 51

Idle time for Machine-1

= 51 - 42

= 9

Idle time for Machine-2

= (6) + (16 - 8) + (27 - 22) + (35 - 31) + (42 - 40) + (51 - 45)

= 6 + 8 + 5 + 4 + 2 + 6

= 31

Idle time for Machine-3

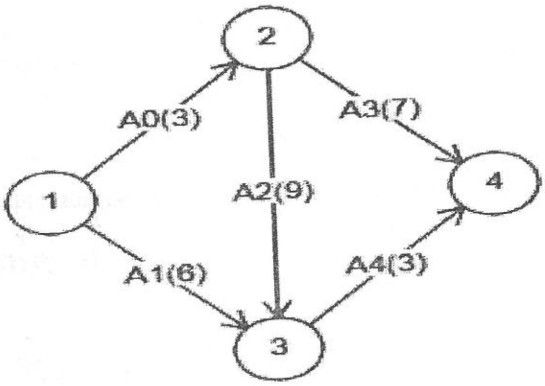
= (8) + (22 - 16) + (40 - 36) + (45 - 44) + (51 - 51)

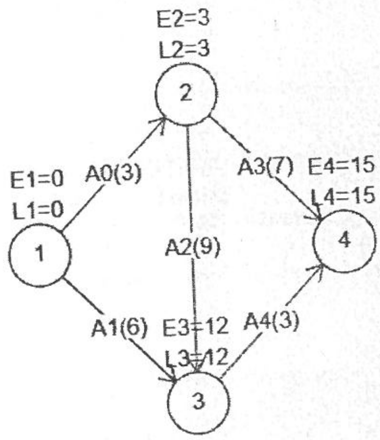
= 8 + 6 + 4 + 1 + 0

= 19

7 From the following particulars of a project
a) Draw the network diagram b) Identify the critical Path c) Calculate the project duration and total float of each activity

Sol





Forward Pass Method

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 3 = 3$

$E_3 = Max\left\{E_i + t_{i,3}\right\} [i = 1, 2]$

$= Max\left\{E_1 + t_{1,3}; E_2 + t_{2,3}\right\}$

$= Max\{0 + 6; 3 + 9\}$

$= Max\{6; 12\}$

$= 12$

$E_4 = Max\left\{E_i + t_{i,4}\right\} [i = 2, 3]$

$= Max\left\{E_2 + t_{2,4}; E_3 + t_{3,4}\right\}$

$= Max\{3 + 7; 12 + 3\}$

$= Max\{10; 15\}$

$= 15$

Backward Pass Method

$L_4 = E_4 = 15$

$L_3 = L_4 - t_{3,4} = 15 - 3 = 12$

$L_2 = \text{Min}\left\{L_j - t_{2,j}\right\} [j = 3, 4]$

$= \text{Min}\left\{L_3 - t_{2,3}; L_4 - t_{2,4}\right\}$

$= \text{Min}\{12 - 9; 15 - 7\}$

$= \text{Min}\{3; 8\}$

$= 3$

$L_1 = \text{Min}\left\{L_j - t_{1,j}\right\} [j = 2, 3]$

$= \text{Min}\left\{L_2 - t_{1,2}; L_3 - t_{1,3}\right\}$

$= \text{Min}\{3 - 3; 12 - 6\}$

$= \text{Min}\{0; 6\}$

$= 0$

For each activity, the total float, free float and independent float calculations are shown in Table

Activity (i,j) (1)	Duration t_{ij} (2)	Earliest Start E_i (3)	E_j (4)	L_i (5)	Latest Finish L_j (6)	Earliest Finish $= E_i + t_{ij}$ (7) = (3) + (2)	Latest Start $= L_j - t_{ij}$ (8) = (6) - (2)	Total Float $= L_j - t_{ij} - E_i$ (9) = (8) - (3)	Free Float $= E_j - E_i - t_{ij}$ (10) = (4) - (3) - (2)	Independent Float $= E_j - L_i - t_{ij}$ (11) = (4) - (5) - (2)	Critical Activity? (12)
1-2	3	0	3	0	3	$0 + 3 = 3$	$3 - 3 = 0$	$0 - 0 = 0$	$3 - 0 - 3 = 0$	$3 - 0 - 3 = 0$	Yes
1-3	6	0	12	0	12	$0 + 6 = 6$	$12 - 6 = 6$	$6 - 0 = 6$	$12 - 0 - 6 = 6$	$12 - 0 - 6 = 6$	No
2-3	9	3	12	3	12	$3 + 9 = 12$	$12 - 9 = 3$	$3 - 3 = 0$	$12 - 3 - 9 = 0$	$12 - 3 - 9 = 0$	Yes
2-4	7	3	15	3	15	$3 + 7 = 10$	$15 - 7 = 8$	$8 - 3 = 5$	$15 - 3 - 7 = 5$	$15 - 3 - 7 = 5$	No
3-4	3	12	15	12	15	$12 + 3 = 15$	$15 - 3 = 12$	$12 - 12 = 0$	$15 - 12 - 3 = 0$	$15 - 12 - 3 = 0$	Yes

8 A company that operates for 50 weeks in a year is concerned about its stocks of copper cable. this costs Rs 240 a meter and there is a demand for 8,000 meters a week. each replenshment costs Rs 1,050 for administration and Rs. 1,650 for delivery, while holding costs are estimated at 25 percent of value held a year assuming no shortages are allowed, what is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize its profits rather tha minimize cost? What is the gross profit if the company sells the cable for Rs 360 a meter.

Sol

Given Data

Annual demand:

$D = 8000 \times 50 = 400{,}000$ meters/year

Cost per meter: Rs 240

Ordering cost per order:

$S = 1050 + 1650 = 2700$

Holding cost (25% of value):

$H = 0.25 \times 240 = 60$ per meter/year

Selling price: Rs 360 per meter

2. Inventory Policy

Optimal order size = 6000 meters

Number of orders per year

$$\frac{400000}{6000} \approx 66.67 \approx 67 \text{ orders/year}$$

Time between orders

$$\frac{50}{67} \approx 0.75 \text{ weeks } (\approx 5\text{–}6 \text{ days})$$

Economical Optimal Quantity

$$EOQ = \sqrt{\frac{2DS}{H}}$$

$$EOQ = \sqrt{\frac{2 \times 400000 \times 2700}{60}}$$

$$EOQ = \sqrt{36{,}000{,}000} = 6000 \text{ meters}$$

$$\frac{D}{Q} \times S = \frac{400000}{6000} \times 2700 = 180000$$

$$\frac{Q}{2} \times H = \frac{6000}{2} \times 60 = 180000$$

$$= 180000 + 180000 = Rs\ 360000$$

Revenue and purchase cost remain unchanged
Only controllable cost = inventory cost
Profit maximization gives the same EOQ as cost minimization
(because minimizing cost automatically maximizes profit under these conditions)
Revenue: 400000×360=144000000
Purchase Cost: 400000×240=96000000
Gross Profit (before inventory cost): =144000000−96000000=48000000
Net Profit (after inventory cost): =48000000−360000=Rs 47,640,

9 A factory requires 1500 units of an item per month each costing Rs 27 the cost per order is Rs 150 and the inventory carrying charges work out to 20 percent of the average inventory find the economic order quantity and the number of orders per year calculated total annual inventory cost

Sol

Monthly demand = 1500 units

Annual demand $D = 1500 \times 12 = 18000$ units

Unit cost = Rs 27

Ordering cost $S = Rs\ 150$ per order

Carrying cost rate = 20% of unit cost

Carrying cost per unit per year

$H = 0.20 \times 27 = Rs\ 5.4$

$$EOQ = \sqrt{\frac{2DS}{H}}$$

$$EOQ = \sqrt{\frac{2 \times 18000 \times 150}{5.4}}$$

$$EOQ = \sqrt{\frac{5400000}{5.4}} = \sqrt{1000000}$$

$$EOQ = 1000 \text{ units}$$

$$\text{Number of orders} = \frac{D}{EOQ} = \frac{18000}{1000} = 18 \text{ orders}$$

Carrying cost per year

$$= \frac{EOQ}{2} \times H = \frac{1000}{2} \times 5.4 = 500 \times 5.4 = Rs\ 2700$$

Total annual inventory cost

$$= 2700 + 2700 = Rs\ 5400$$

10 A company management and the labour union are negotiating a new three-year settlement. Each of

these has 4 strategies: The costs to the company are given for every pair of strategy choice. What strategy will the two sides adopt? Also determine the value of the game.

Union Strategies	Company Strategies I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	– 5	4	11	0

Sol Applying the rule of finding out the saddle point, we obtain the saddle point that is enclosed both in a circle and a rectangle, as shown in Table

Union Strategies	Company Strategies I	II	III	IV	Row minimum
I	20	15	(12)	35	12 ← *Maximin*
II	25	14	8	10	8
III	40	2	10	5	2
IV	– 5	4	11	0	– 5
Column maximum	40	15	(12) ↑ *Minimax*	35	

since Maximin = Minimax = Value of game = 12, therefore the company will always adopt strategy III – Legalistic strategy and union will always adopt strategy I – Hard and aggressive bargaining

11 Mr. Sethi has Rs. 10,000 to invest in one of threee options A, B or C the return on his investment depends on whether the economy experiences inflation, recession, or no change at all the possinle returns under eache economic conditions are given below what should he decide, using the pessimistic criterion, optimistic criterion, eqally likely criterion and regret criterion?

Strategy	Inflation	Recession	No Change
A	2000	1200	1500
B	3000	800	1000
C	2500	1000	1800

Sol **Solution:**

Given payoffs (in Rs.):

Strategy	Inflation	Recession	No Change
A	2000	1200	1500
B	3000	800	1000
C	2500	1000	1800

(i) Pessimistic Criterion (Maximin):

Strategy	Minimum Payoff
A	1200
B	800
C	1000

Maximin = 1200, hence choose **Strategy A.**

(ii) Optimistic Criterion (Maximax):

Strategy	Maximum Payoff
A	2000
B	3000
C	2500

Maximax $= 3000$, hence choose **Strategy B.**

(iii) Equally Likely Criterion (Laplace):

Strategy	Average Payoff
A	$\frac{2000+1200+1500}{3} = 1567$
B	$\frac{3000+800+1000}{3} = 1600$
C	$\frac{2500+1000+1800}{3} = 1767$

Maximum average $= 1767$, hence choose **Strategy C.**

(iv) Regret Criterion (Minimax Regret):

Column maximums: Inflation = 3000, Recession = 1200, No Change = 1800

Strategy	Inflation	Recession	No Change	Max Regret
A	1000	0	300	1000
B	0	400	800	800
C	500	200	0	500

Minimum regret $= 500$, hence choose **Strategy C.**

Final Answer:

Pessimistic → A

Optimistic → B

Equally Likely → C

Regret Criterion → C