PVP 23

Code: 23EE3403

II B.Tech - II Semester – Regular / Supplementary Examinations
APRIL 2026

CONTROL SYSTEMS
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

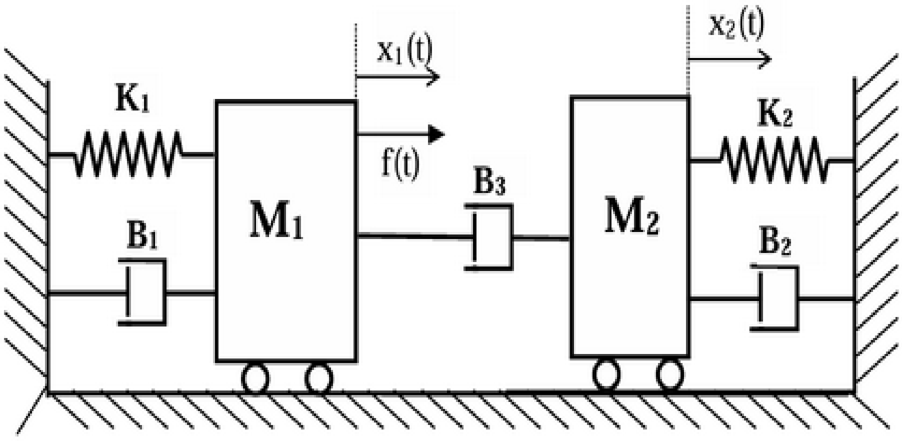
Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

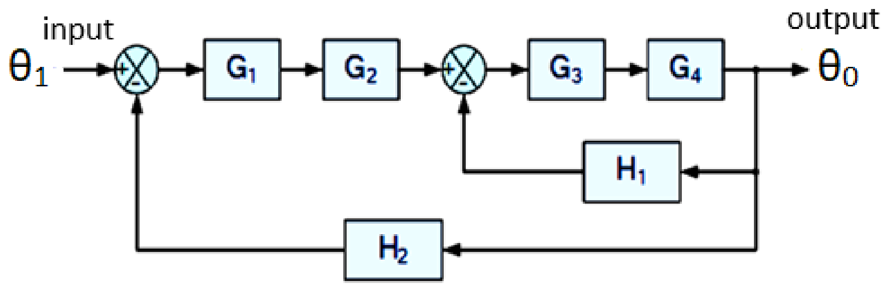
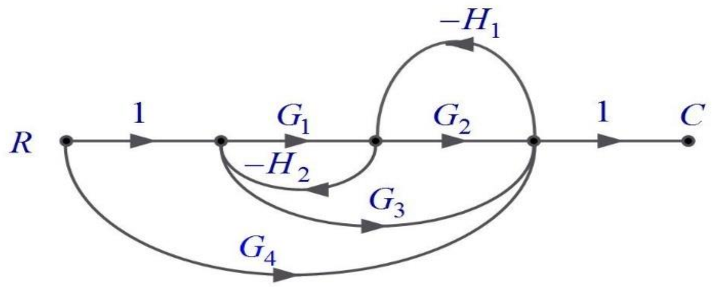
BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Define transfer function.	L2	CO1
1.b)	What are the basic elements used for modeling mechanical translational system.	L2	CO1
1.c)	Write the Masons Gain formula to determine the overall gain of the transfer function.	L2	CO1
1.d)	List the standard test signals used in control system.	L2	CO1
1.e)	What is the effect of PD controller on system performance?	L3	CO2
1.f)	What is centroid?	L2	CO1
1.g)	Draw the pole-zero plot of a lag compensator.	L3	CO3
1.h)	Define resonant peak.	L2	CO1
1.i)	List any two advantages of state space analysis over transfer function approach.	L2	CO1
1.j)	What is the need for Observability test?	L4	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Write the governing differential equations for a series RLC electrical network and determine the transfer function considering capacitor voltage as output and supply voltage as input.	L3	CO2	5 M
	b)	Distinguish between open loop and closed loop system.	L3	CO2	5 M
OR					
3	Perform the following for the mechanical translational system shown in Figure 1.  Figure 1 i. Write the differential equations governing the mechanical system with the help of free body diagram. ii. Determine the transfer function $T(S) = X_1(S)/F(S)$.		L3	CO2	10 M
UNIT-II					
4	a)	Determine the transfer function for the block diagram shown in Figure 2 using block diagram reduction technique.	L4	CO4	5 M

		 Figure 2			
	b)	Formulate an expression for the rise time of second order system considering a unit step input for the underdamped system.	L3	CO3	5 M
OR					
5	a)	Determine the transfer function for the signal flow graph shown in Figure 3 using Masons gain formula.  Figure 3	L4	CO4	5 M
	b)	Explain the pole locations of underdamped, overdamped and critically damped second-order systems.	L3	CO3	5 M
UNIT-III					
6	Sketch the root-locus plot for the given transfer function and determine the range of values of ‘K’ for the system to be stable. $$G(s)H(s) = \frac{K}{S\,(S+1)(S+3)}$$		L4	CO4	10 M
OR					
7	a)	Explain the Proportional–Integral–Derivative (PID) controller with its mathematical model and block diagram.	L3	CO2	5 M

	b)	Consider the characteristic equation of a closed loop control system is represented by the following equation: $$S^4 + 8S^3 + 18S^2 + 16S + K = 0$$ Apply the Routh Hurwitz criterion to determine the range of values of 'K' for the system to be stable.	L4	CO4	5 M
UNIT-IV					
8		Discuss the frequency-domain specifications of a control system and explain their significance.	L4	CO4	10 M
OR					
9		Sketch the Bode plot for the following transfer function to determine the phase margin and gain margin. Comment on stability. $$G(s)H(s) = \frac{20}{S\,(1+S)\,(1+0.01\,S)}$$	L4	CO4	10 M
UNIT-V					
10	a)	Explain Kalman's controllability and observability criteria with mathematical formulation.	L4	CO5	5 M
	b)	List the properties of State Transition Matrix.	L3	CO2	5 M
OR					
11		Determine the controllability and observability of a control system which is represented by the state space model given below: $$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$ $$Y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$	L4	CO5	10 M

II B.Tech II Semester- Regular Examinations- April 2026

CONTROL SYSTEMS (23EE3403)

Scheme of Valuation

Part-A

1 a) Definition of transfer function - 2m

1 b) Basic elements for modeling mechanical translational system – 2 m

1c) Mason's gain formula -2m

1 d) Any two standard test signals -2m

1 e) Any one effect of PD controller – 2m.

1 f) Centroid formula or explanation – 2m

1 g) Pole zero plot of lag compensator – 2m

1 h) Resonant peak definition or expression– 2m

1 i) Any two advantages of state variable analysis over transfer function – 2m

1 j) Explaining the need of observability test – 2m

Part-B

2. a) Formulation of differential equations-2m
   Simplification – 2m
   T.F derivation – 1m

2 b) Distinguishing between open loop and closed loop with any five comparisons – 5m

3. Formulation of Differential equations – 4m
   Applying Laplace transforms and Simplification – 4m
   Finding T.F – 2m

4. a) Applying reduction rules – 4m
   Finding T.F – 1m
   b) Derivation of rise time – 5m

5. a) forward paths, loops – 2m
   Mason's gain formula and finding transfer function-3m
   b) Explaining pole locations of underdamped, overdamped and critically damped – 5m

6 Formulation of all construction rules and calculation – 7m
   Sketch of root locus plot – 3m

7. a) Explanation of PID controllers – 3m
   Mathematical model and block diagram – 2m
   b) Formulation of Routh table – 3m
   Apply routh criterion to determine the range of values of 'k' for stability – 2m

8. Explaining about frequency domain specifications and their significance– 10m

9. Magnitude calculation and plot – 4m
   Phase angle calculation and plot – 4m
   GM & PM from Graph and comment on stability– 2m

10 a) Explaining the kalman's controllability and observability with
   Mathematical formulation– 5m
   b) Listing any four properties of state transition matrix– 5m

11. Test for controllability – 5m
   Test for Observability – 5m

Part-A

1 a) The transfer function of a system is defined as the ratio of the Laplace transform of output to Laplace transform of input with zero initial conditions

1 b) Mass (M), spring (k) and dashpot (D) are the elements of mechanical translational system

1 c) Masons Gain formula states that the overall gain of the system is

$$T = \sum_{n=1}^{k} \frac{P_k \Delta_k}{\Delta}$$

k - No.of forward paths in the signal flow graph.

P_k - Gain of k^{th} forward path

Δ = 1-[sum of individual loop gains] +[sum of gain products of all possible combinations of two non touching loops]-[sum of gain products of all possible combinations of three non touching loops]+...

Δ_k - Δ for that part of the graph which is not touching k^{th} forward path
1 for all the loops touching the k^{th} forward path

1 d) Standard test signals are step, ramp, parabolic and impulse signals

1 e) effects of PD controller on system Response are:

Stability and improved time response

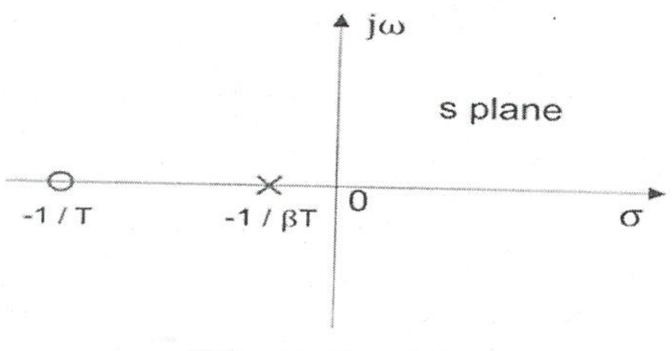
Provides more control over pole locations

1 f) Centroid

The centroid, the point of intersection of the asymptotes with real axis is given by

$$\sigma_A = \frac{\text{sum of real parts of poles} - \text{sum of real parts of zeros}}{\text{number of poles} - \text{number of zeros}}$$

1 g)



Pole-zero plot of lag compensator

1 h) Resonant Peak (M_r) $= \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ (or)

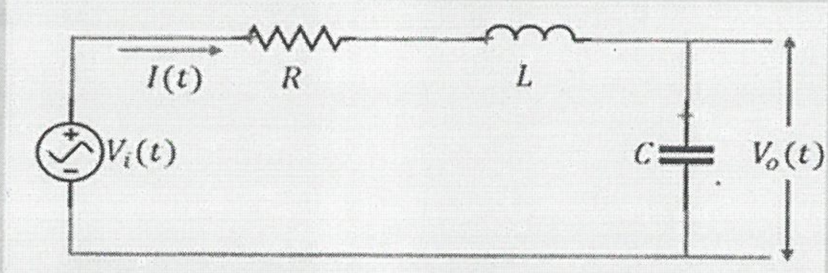
- The maximum value M_r of magnitude $|M(j\omega)|$ is known as the resonant peak. A system with large resonant peak will exhibit a large overshoot.

1 i)
- State space equation is applicable to single-input single-output as well as multi-input multi-output systems.
- State space approach can handle the systems with non-zero initial conditions.

1 j) The need for an observability test in control systems is to verify if a system's internal states can be determined from its external outputs over a finite time. It enables accurate state estimation, fault detection, and ensures system behavior

Part-B

2. a) Transfer function of series RLC electrical network



1. **Model Equations:**

$$V_i(t) = RI(t) + L\frac{dI}{dt} + \frac{1}{C}\int I dt$$

$$V_0(t) = \frac{1}{C}\int I dt$$

2. **Input and Output Variables:**
   - Input: $V_i(t)$
   - Output: $V_o(t)$

3. **Laplace Transform: (assuming initial conditions to be zero)**

$$V_i(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s)$$

$$V_0(s) = \frac{1}{sC}I(s)$$

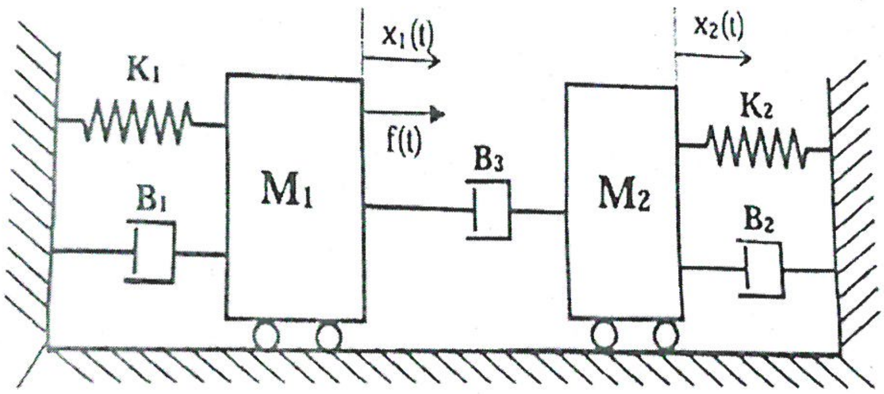
4. **Transfer Function:**

$$G(s) = \frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}I(s)}{\left(R + sL + \frac{1}{Cs}\right)I(s)} = \frac{\frac{1}{sC}}{\left(R + sL + \frac{1}{Cs}\right)} = \frac{1}{s^2LC + sRC + 1}$$

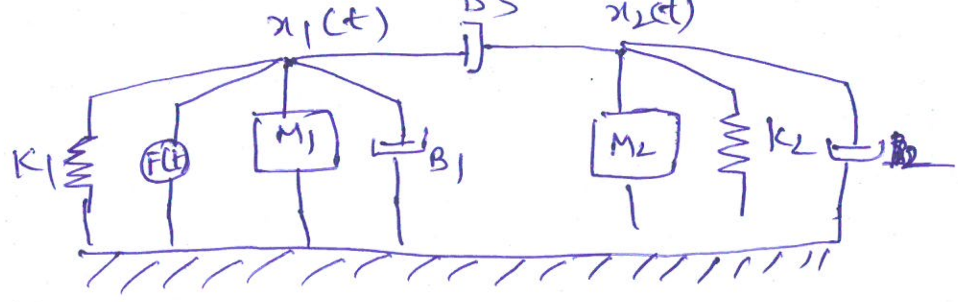
b) Comparison of open loop and closed loop systems

Open Loop	Closed Loop
Any change in output has no effect on the input i.e. feedback does not exists.	Changes in output, affects the input which is possible by use of feedback.
Output measurement is not required for operation of system.	Output measurement is necessary.
Feedback element is absent.	Feedback element is present.
Error detector is absent.	Error detector is necessary.
It is inaccurate and unreliable.	Highly accurate and reliable.
Highly sensitive to the disturbances.	Less sensitive to the disturbances.
Highly sensitive to the environmental changes.	Less sensitive to the environmental changes.
Bandwidth is small.	Bandwidth is large.
Simple to construct and cheap.	Complicated to design and hence costly.
Generally are stable in nature.	Stability is the major consideration while designing
Highly affected by nonlinearities.	Reduced effect of nonlinearities.

3. Given figure



sol: free body diagram



at node $x_1(t)$,

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_3 \frac{d(x_1 - x_2)}{dt}$$

apply L.T's

$$F(s) = M_1 s^2 x_1(s) + B_1 s x_1(s) + K_1 x_1(s) + B_3 s \left[x_1(s) - x_2(s)\right]$$

$$F(s) = x_1(s)\left[M_1 s^2 + B_1 s + K_1 + B_3 s\right] - B_3 s x_2(s) \quad \text{--- (1)}$$

at node $x_2(t)$

$$0 = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_3 \frac{d}{dt}(x_2 - x_1)$$

apply L.T.'s

$$0 = M_2 s^2 x_2(s) + B_2 s x_2(s) + K_2 x_2(s) + B_3 s\left[x_2(s) - x_1(s)\right]$$

$$0 = x_2(s)\left[M_2 s^2 + B_2 s + K_2 + B_3 s\right] - B_3 s x_1(s).$$

$$x_2(s)\left[M_2 s^2 + (B_2 + B_3)s + K_2\right] = B_3 s x_1(s)$$

$$x_2(s) = \frac{B_3 s x_1(s)}{M_2 s^2 + (B_2 + B_3)s + K_2} \quad \text{--- (2)}$$

sub eq (2) in eq (1)

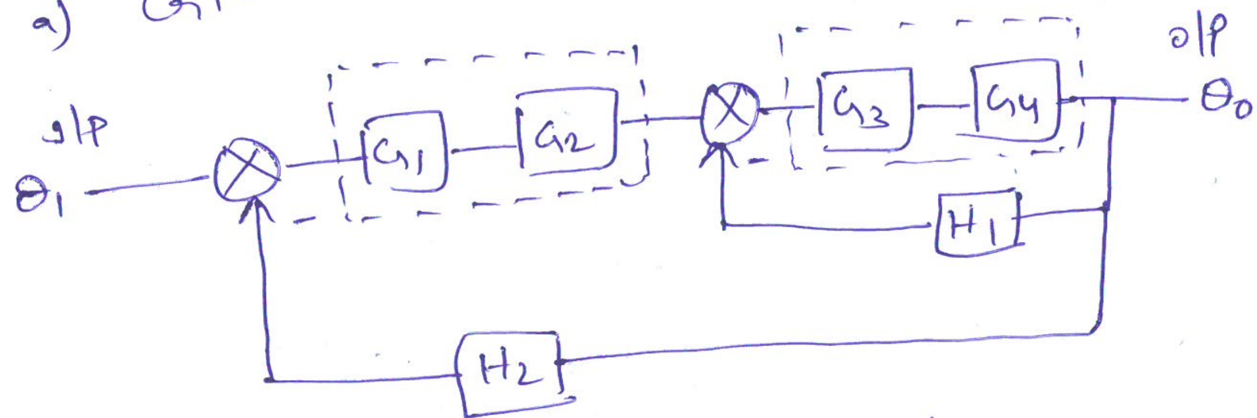
$$F(s) = x_1(s)\left[M_1 s^2 + (B_1 + B_3)s + K_1\right] - \frac{(B_3 s)^2 x_1(s)}{M_2 s^2 + (B_2 + B_3)s + K_2}$$

$$F(s) = \left\{\frac{\left[M_1 s^2 + (B_1+B_3)s + K_1\right]\left[M_2 s^2 + (B_2+B_3)s + K_2\right] - (B_3 s)^2}{M_2 s^2 + (B_2+B_3)s + K_2}\right\} x_1(s)$$

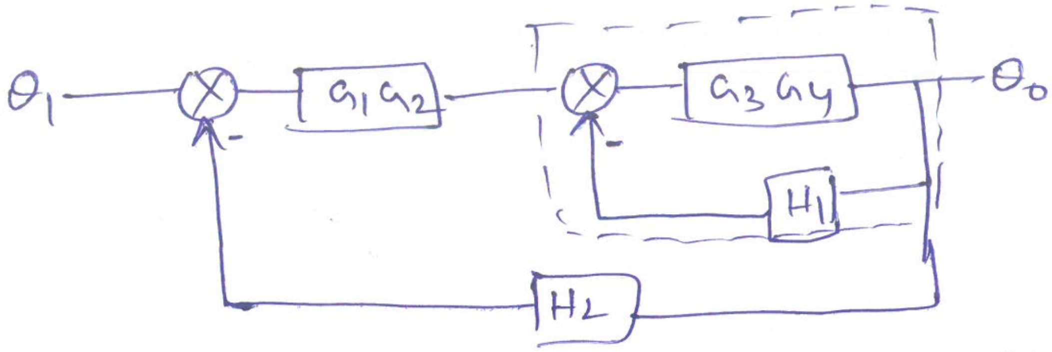
$\therefore$ required transfer function,

$$\boxed{\frac{x_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2+B_3)s + K_2}{\left[M_1 s^2 + (B_1+B_3)s + K_1\right]\left[M_2 s^2 + (B_2+B_3)s + K_2\right] - (B_3 s)^2}}$$

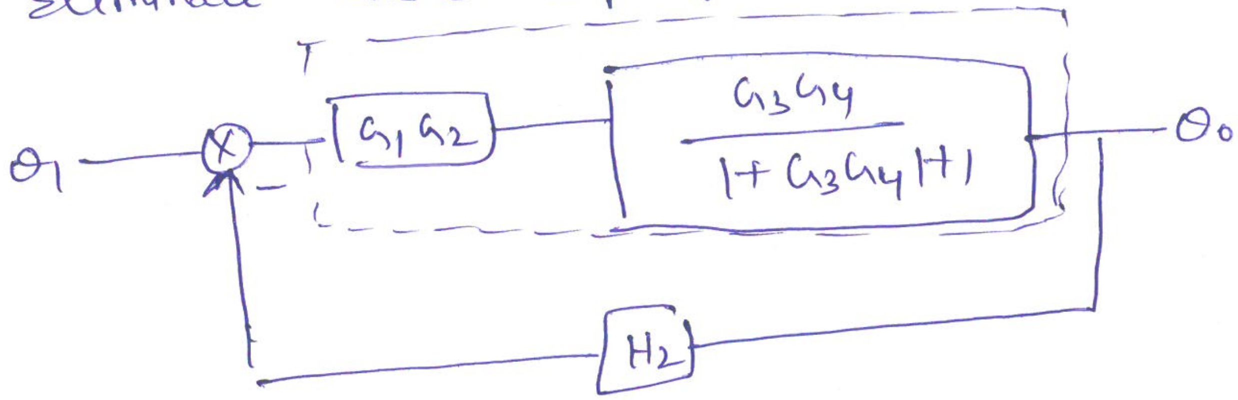
4. a) Given Block diagram



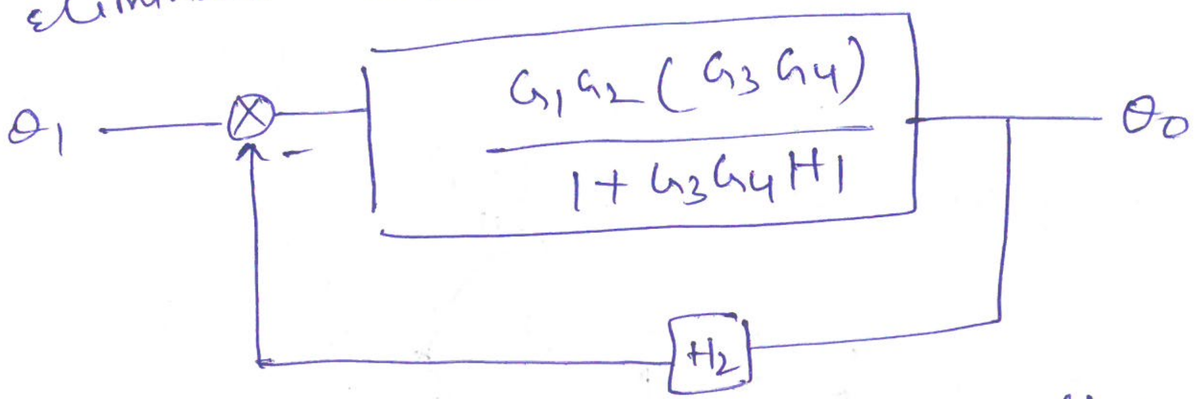
- Reduce cascade block of G_1 & G_2
- similarly reduce cascade block of G_3 & G_4



• eliminate closed loop of G_3G_4 with H_1 feedback



• eliminate cascade block



∴ Required Transfer function after eliminating closed loop,

$$\frac{\theta_0}{\theta_1} = \frac{\dfrac{G_1G_2G_3G_4}{1+G_3G_4H_1}}{1+\dfrac{G_1G_2G_3G_4 \cdot H_2}{1+G_3G_4H_1}}$$

$$\boxed{T.F\ \frac{\theta_0}{\theta_1} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_1G_2G_3G_4H_2}}$$

4. b)

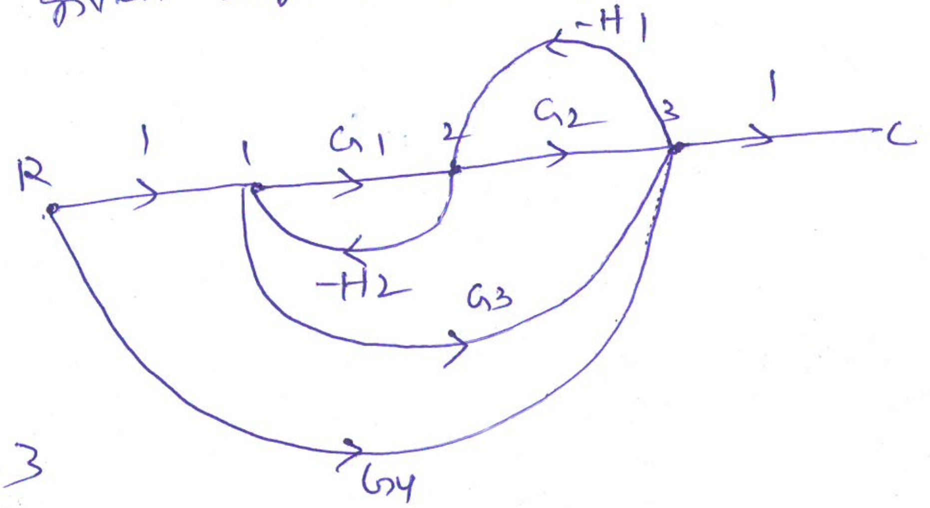
Expression for Rise Time

- Consider a 2nd order underdamped system
- Rise time t_r is the time to taken by the step response to go from 0 to 100% of the final value i.e., one
- $y(t_r) = 1 = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{(1-\zeta^2)}}\sin(\omega_d t_r + \theta)$ where $(\theta = \cos^{-1}\zeta)$

$$\Rightarrow \sin(\omega_d t_r + \theta) = 0 \Rightarrow \omega_d t_r + \theta = \pi$$

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{(1-\zeta^2)}}$$

5. a) given signal flow graph



n = 3

forward paths are

R 1 2 3 C → $G_1 G_2 = P_1$
R 1 3 C → $G_3 = P_2$
R 3 C → $G_4 = P_3$

forward path gaines

loops

$l_1 = 232 = -G_2 H_1$

$l_2 = 121 = -G_1 H_2$

$l_3 = 1321 = G_3 H_1 H_2$

Mason's gain formula to find T.F

$$T.F = \frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$\Delta_1 = \Delta_2 = 1$, $\Delta_3 = 1 + G_1 H_2$

$\Delta = 1 + G_2 H_1 + G_1 H_2 - G_3 H_1 H_2$

$$\frac{C}{R} = \frac{G_1 G_2 + G_3 + G_4 (1 + G_1 H_2)}{1 + G_2 H_1 + G_1 H_2 - G_3 H_1 H_2}$$

5 b) The characteristic equation of second order system is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The roots of this equation is given by,

$$s_1, s_2 \Rightarrow s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2 - 1}}{2}$$

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

(when $\xi = 0$, $s_1, s_2 = \pm j\omega_n$ – roots are purely imaginary)

when $\xi = 1$, $s_1, s_2 = -\omega_n$ – roots are real, negative & equal

when $\xi > 1$, $s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$ – roots are real & unequal, distinct.

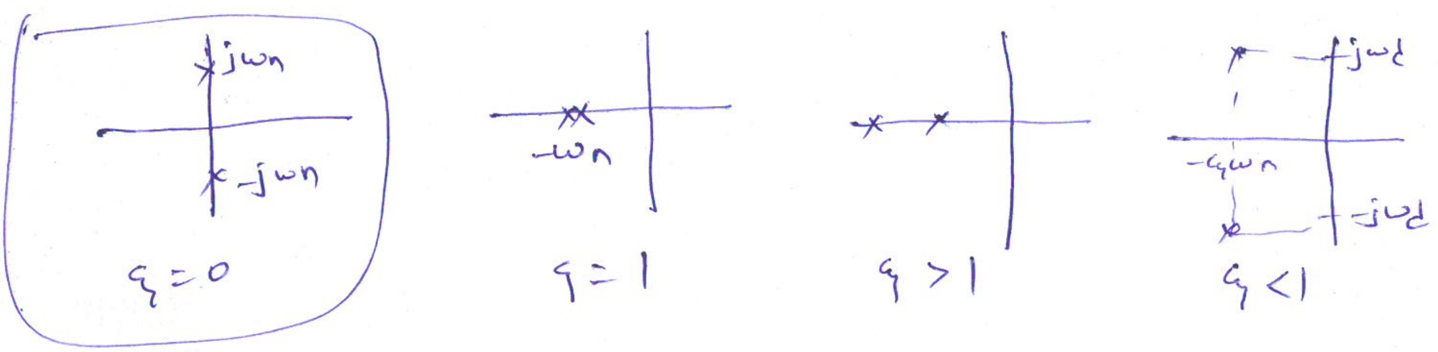
when $\xi < 1$, $s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$$= -\xi\omega_n \pm \omega_n\sqrt{(-1)(1-\xi^2)}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$= -\xi\omega_n \pm j\omega_d$$ – roots are complex conjugate

ω_d – damped frequency of oscillation of the system, rad/sec



6. Given, $T.F = \dfrac{K}{s(s+1)(s+3)}$

soln:- 1) Root locus is symmetrical about real x-axis

2) Asymptodes, $P = 0, -1, -3$, $Z = nil$

$A = n - m = 3 - 0 = 3$

3) Asymptodic angle, $\theta = \pm \dfrac{(2q+1)}{n-m} \times 180^\circ$

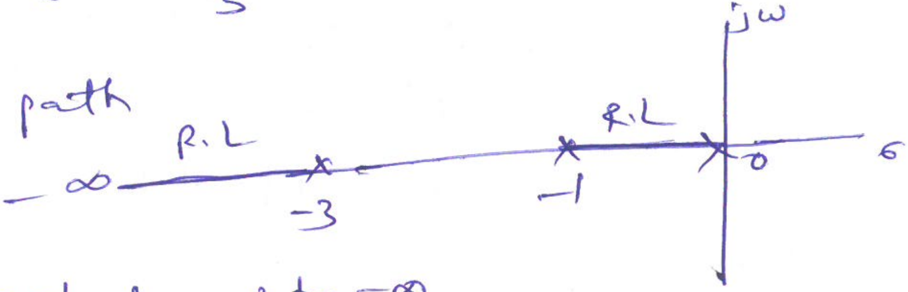
$q = 0, 1, 2$, $q = 0 \rightarrow \theta_1 = \pm 60^\circ$

$q = 1 \rightarrow \theta_2 = \pm 180^\circ$

$q = 2 \rightarrow \theta_3 = \pm 300^\circ$

4) centroid $= \dfrac{(0 - 1 - 3) - 0}{3} = \dfrac{-4}{3} = -1.33$

5) Root locus path



R.L is 0 to −1 & −3 to −∞

6) BWP : $\dfrac{dK}{ds} = 0$

$1 + \dfrac{K}{s(s+1)(s+3)} = 0$

$s(s+1)(s+3) + K = 0$

$s(s^2 + 4s + 3) + K = 0$

$s^3 + 4s^2 + 3s + K = 0$

$K = -(s^3 + 4s^2 + 3s)$

$\dfrac{dK}{ds} = 3s^2 + 8s + 3 = 0$

$s = -2.215, -0.45$

7. Intersection with imaginary axis

$s^3 + 4s^2 + 3s + K = 0$ → By R-H criteria

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	
s^0	K	

→ $K > 0$

→ $\frac{12-K}{4} > 0$

$12 - K > 0$

$12 > K$

∴ $K_{max} = 12$ → for stability

$4s^2 + K = 0$

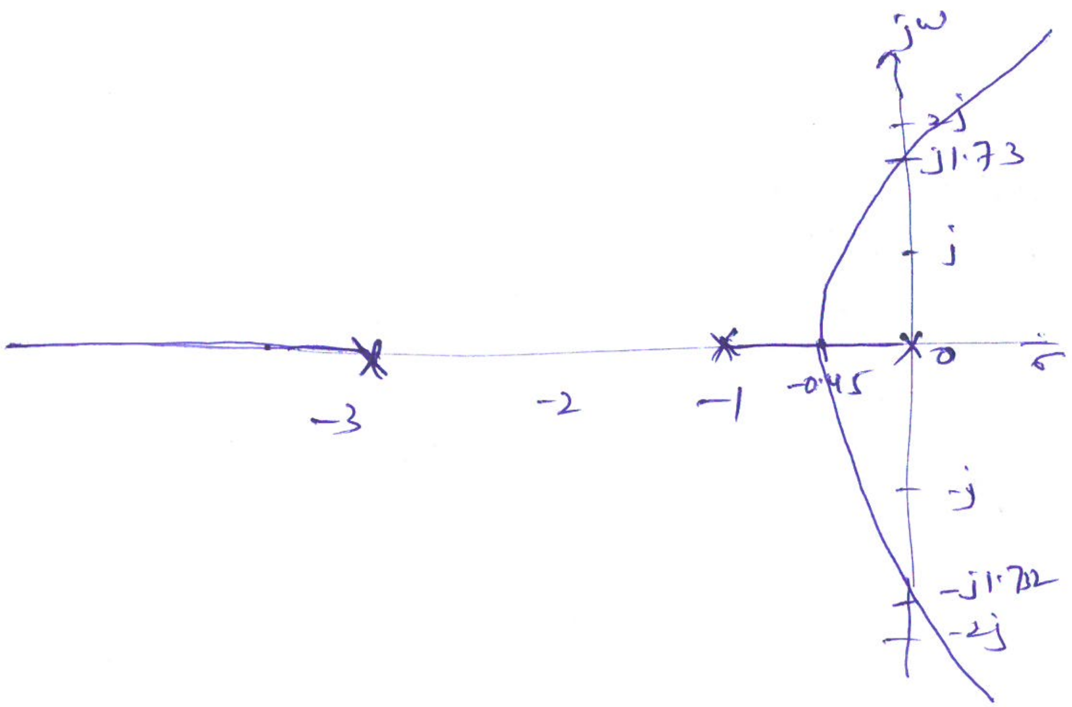
$4s^2 + 12 = 0$

$4s^2 = -12$

$s^2 = -3$

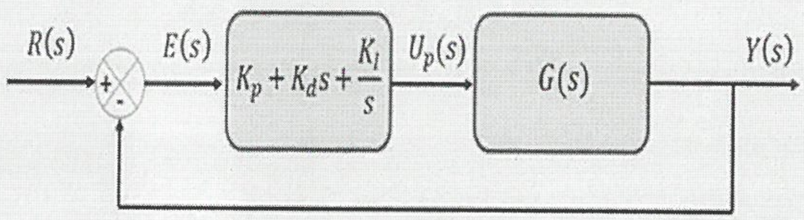
$s = \pm j1.732$

8. Root locus plot



7 a)

Proportional, Integral and Derivative actions



$$U_p(s) = (K_p + K_d s + \frac{K_I}{s})E(s) \quad \overset{\mathcal{L}}{\leftrightarrow} \quad u_p(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int_0^t e(\tau)\, d\tau$$

Proportional control

- Simplest to implement
- May not be sufficient to meet the design requirements

Proportional + Derivative Control

- Stability and improved time response
- Provides more control over pole locations

Proportional + Integral Control

- Perfect tracking of constant reference inputs
- Rejection of constant disturbance inputs

Proportional + Derivative + Integral Control
For second order systems ⟶

- Stability
- Control over pole locations
- Perfect tracking of constant reference inputs
- Constant disturbance rejection

b) $s^4 + 8s^3 + 18s^2 + 16s + K = 0$

By R-H criteria,

s^4	1	18	K
s^3	8	16	
s^2	16	K	
s	$\frac{256-8K}{16}$		
s^0	K		

for stability,

$K > 0$

$\frac{256-8K}{16} > 0$

$256 - 8K > 0$

$256 > 8K$

$32 > K$

∴, for stability the range of 'K' should be

$\boxed{0 < K < 32}$

8.

Frequency Domain Specifications

Resonant Peak (M_r)

- The maximum value M_r of magnitude $|M(j\omega)|$ is known as the resonant peak. A system with large resonant peak will exhibit a large overshoot.
- In general, the magnitude of M_r gives indication on the relative stability of a stable closed-loop system.

Resonant Frequency (ω_r)

- The frequency at which the output of the system has maximum magnitude is resonant frequency.

Bandwidth (ω_b)

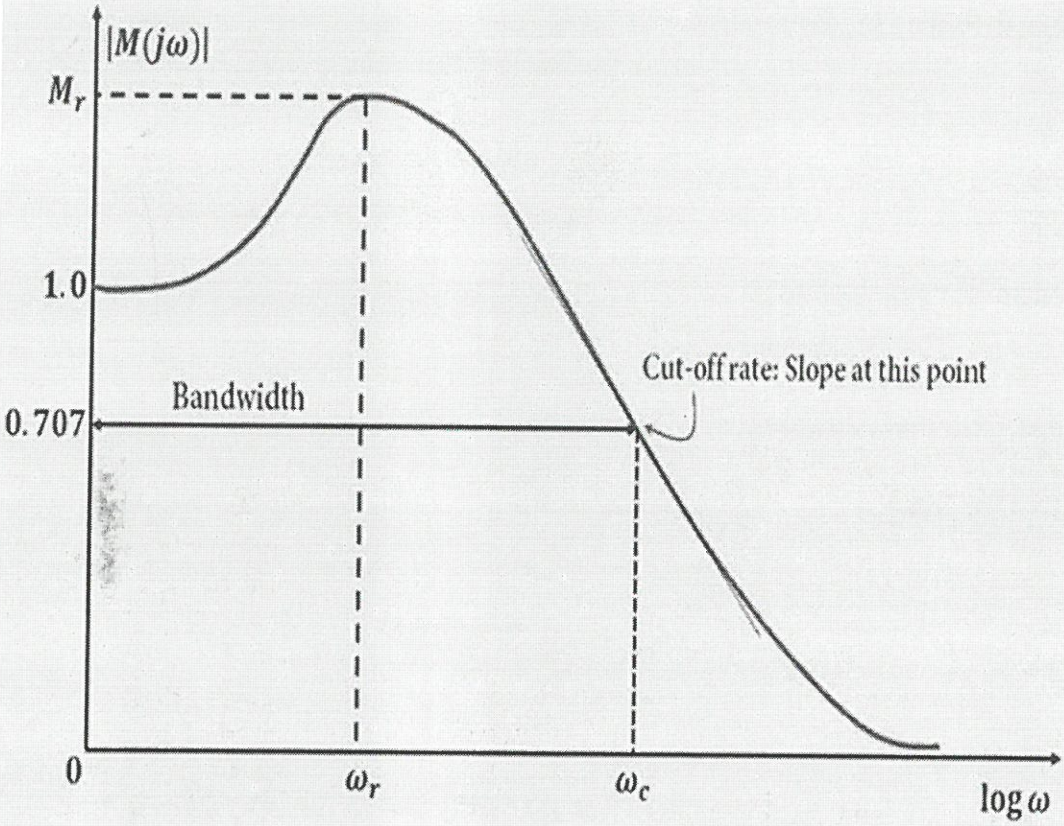
- The bandwidth BW is the frequency at which $|M(j\omega)|$ drops to 70.7% of, or 3dB down from, its zero-frequency value.

Cut-off Frequency (ω_c)

- The frequency at which the magnitude $|M(j\omega)|$ is $\frac{1}{\sqrt{2}}$ times of its maximum value is known as cut-off frequency.
- In other words, the frequency at which the magnitude of the closed loop system is 3dB less than its maximum value is called cut-off frequency.

Cut-off Rate

- The cut-off rate is the rate of change of slope of the magnitude at cut-off frequency.
- In other words, the cut-off rate is the slope of the frequency response near the cut-off frequency.



- Consider the standard second order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$$

- The closed loop transfer function is

$$M(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n and ζ are natural frequency and damping ratio respectively.

- The closed loop frequency response is

$$M(j\omega) = \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2}$$

Resonant Peak (M_r)

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

Resonant Frequency (ω_r)

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

Bandwidth (ω_r)

$$\omega_b = \omega_n\sqrt{1-2\zeta^2+\sqrt{4\zeta^4-4\zeta^2+2}}$$

9. given OLTF = $\dfrac{20}{s(1+s)(1+0.01s)}$

sol: $G(j\omega) = \dfrac{20}{j\omega(1+j\omega)(1+j0.01\omega)}$

Mag. plot

factor	corner frequency	slope in db/dec.	change in slope db/dec.
$\frac{20}{j\omega}$	–	−20	−20
$\frac{1}{1+j\omega}$	$\omega c_1 = 1$	−20	−40
$\frac{1}{1+j0.01\omega}$	$\omega c_2 = 100$	−20	−60

choose, $\omega_l = 0.1$; $M = 20\log \frac{20}{|j\omega|}$

$= 20\log \frac{20}{0.1} = 46\,db$

$\omega_{c_1} = 1$; $M = 20\log \frac{20}{1} = 26\ db$

$\omega c_2 = 100$; $M = -40\log 100 + 26$

$= -54\,db$

chose, $\omega_h = 500$; $M = -60\log \frac{500}{100} - 54$

$= -96\ db$

Phase angle plot :

$\phi = -90 - \tan^{-1}(\omega) - \tan^{-1}(0.01\omega)$

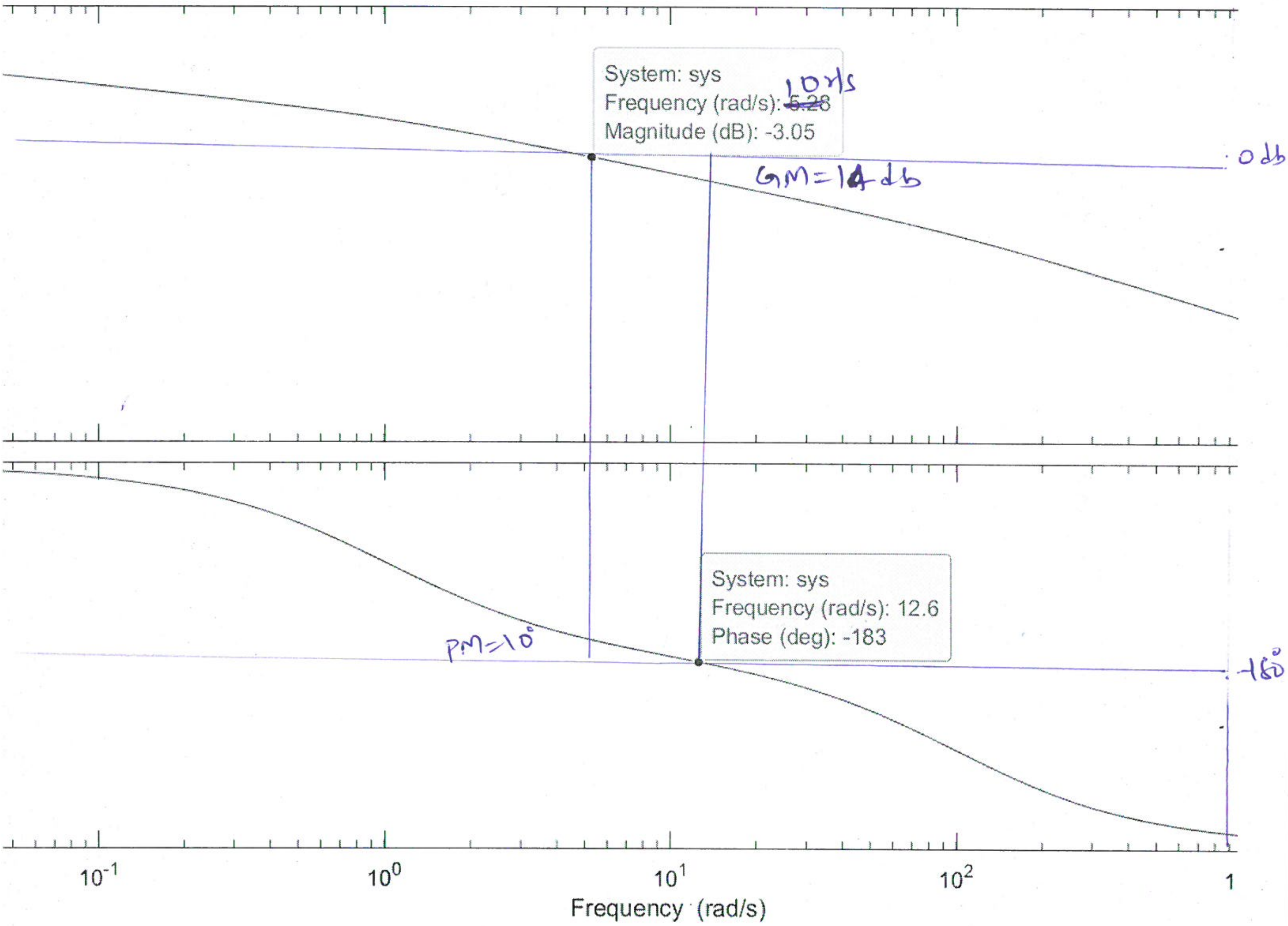
ω	ϕ
0.1	$-96°$
0.5	$-117°$
1	$-135°$
10	-180
100	$-224°$

Bode plot is shown in next page

From graph ~~GCF~~ GCF= 10 r/s , PCF = 12 r/s

∴ GM= 14 db, PM= 10° & both are positive . Hence the system is stable

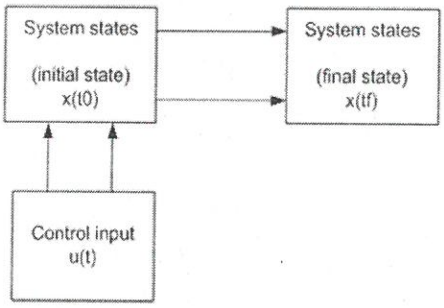
Bode plot for the transfer function = 20/s(1+s)(1+0.01s)



10 a)

Controllability

If the system state transfer from any initial state $x(t_0)$ to any other desired state $x(t_f)$ in specified finite time by a control vector $u(t)$, then the system is called as state controllable.



Testing of Controllability

Kalman test: The system is represented as

$$\dot{x} = Ax + Bu$$

is completely state controllable if only if the rank of combined matrix

$$Q_c = \left[B, AB, \ldots, A^{n-1}B\right]$$

is n (order of the system).

Observability

- A system is said to be observable If every state $x(t_0)$ can be completely identified by measurements of the output $y(t)$ over a finite time interval.
- A system is called observable if every state $x(t_0)$ can be determined from the observation of $y(t)$ over a finite time interval.
- Every state affects every element of the output vector.

Kalman test of Observability

Kalman test: The system represented as

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

is completely observable if only if the rank of combined matrix

$$Q_{observable} = \left[C^T, A^T C^T, \dots, \left(A^T\right)^{n-1} C^T \right]$$

is n (order of the system).

b)

Properties of the state transition matrix (STM):

The STM is calculated as

$$\phi(t) = e^{At} = L^{-1}\{[sI - A]^{-1}\} \quad (1)$$

Property 1: $\phi(0) = I$ (2)

Proof: From (1), at $t=0$ we get

$$\phi(0) = e^{A \cdot 0} = I \quad (3)$$

Property 2: $\phi(-t) = e^{-At}$ (4)

Proof: From (1), $\phi(t) = e^{At}$ (5)

Multiplying both sides with e^{-At} in the above equation (5), we get,

$$e^{-At}\phi(t) = e^{-At}e^{At} = I \quad (6)$$

Post-multiplying both sides with $\phi^{-1}(t)$ in eq. (6), we get,

$$e^{-At}\phi(t)\phi^{-1}(t) = \phi^{-1}(t) \quad (7)$$

$$e^{-At} = \phi^{-1}(t) \quad (8)$$

$$e^{-At} = \phi(-t) \quad (9)$$

From (8) and (9), we get,

$$\phi^{-1}(t) = \phi(-t) \quad (10)$$

Property 3: $$\phi(t_2-t_1)\phi(t_1-t_0)=\phi(t_2-t_0) \tag{11}$$

Proof: From (1), we get,

$$\phi(t_2-t_1)\phi(t_1-t_0)=e^{A(t_2-t_1)}e^{A(t_1-t_0)} \tag{12}$$

Simplifying (12), we get,

$$\phi(t_2-t_1)\phi(t_1-t_0)=e^{At_2}e^{-At_1}e^{At_1}e^{-At_0} \tag{13}$$

$$\phi(t_2-t_1)\phi(t_1-t_0)=e^{At_2}e^{-At_0} \tag{14}$$

$$\phi(t_2-t_1)\phi(t_1-t_0)=e^{A(t_2-t_0)}=\phi(t_2-t_0) \tag{15}$$

Property 4: $$\left[\phi(t)\right]^m=\phi(mt) \tag{16}$$

Proof: From (1), we get,

$$\left[\phi(t)\right]^m=e^{At}\times e^{At}\times e^{At}\times e^{At}\times\ldots\ (m\ times) \tag{17}$$

$$\left[\phi(t)\right]^m=e^{mAt} \tag{18}$$

$$\left[\phi(t)\right]^m=\phi(mt) \tag{19}$$

11. Given state model, $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$

Test for controllability,

$Q_c = \begin{bmatrix} B & AB \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

$Q_c = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = -2 \neq 0$

$\therefore$, the given system is <u>controllable</u>

Test for observability

$Q_o = \begin{bmatrix} C^T & A^T C^T \end{bmatrix}$

$A^T C^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$Q_o = \begin{vmatrix} 0 & -2 \\ 1 & -3 \end{vmatrix} = 2 \neq 0$

$\therefore$, the given system is observable