UNIT-V					
10	a)	Obtain the state space representation of an n^{th} order differential equation.	L3	CO2	5 M
	b)	A second order linear system is described by $\dot{x}_1 = -2x_1 + 4x_2 + u$ $\dot{x}_2 = -x_1 - 2x_2 + u$ and $y = x_1 + x_2$. Find the transfer function.	L3	CO2	5 M
OR					
11		The state variable formulation of a system is given by $[\dot{x}] = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}[x] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $y = \begin{bmatrix} 1 & 0 \end{bmatrix}[x]$. Find the following: a) State transition matrix and b) State equation for a unit step input under zero initial condition.	L4	CO5	10 M

Code: 23EE3403

II B.Tech - II Semester – Regular Examinations - MAY 2025

CONTROL SYSTEMS
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

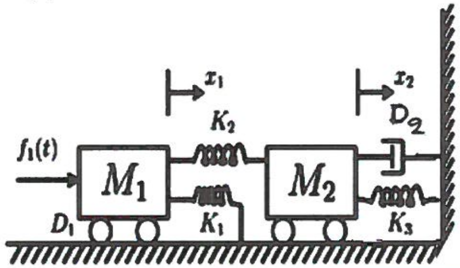
Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Differentiate open loop and closed loop control systems.	L2	CO1
1.b)	What are the effects of feedback on Sensitivity?	L3	CO2
1.c)	Define peak time and peak overshoot.	L3	CO3
1.d)	What is Steady state error?	L4	CO4
1.e)	Differentiate absolute stability and marginal stability.	L4	CO4
1.f)	What is PD controller?	L3	CO2
1.g)	State the Nyquist criterion.	L4	CO4
1.h)	Draw the circuit diagram of a lag compensator and write its transfer function.	L3	CO3
1.i)	What are the advantages of state variable techniques?	L3	CO2
1.j)	What is Kalman's test of controllability?	L4	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2		Derive the Transfer function of Armature controlled DC servo motor.	L3	CO2	10 M
		OR			
3		Develop the differential equations governing the mechanical system as shown in below figure. Also find the transfer function $X_1(s)/F(s)$ 	L4	CO4	10 M
		UNIT-II			
4		A unity feedback system has a forward path transfer function $G(s)=\dfrac{8}{s(s+2)}$. Find the value of damping ratio, undamped natural frequency of the system, percentage over shoot, peak time and settling time.	L3	CO3	10 M
		OR			
5	a)	Derive any two time domain specifications of second order system with unit step input.	L3	CO3	5 M

	b)	Explain steady state errors and error constants.	L4	CO4	5 M
		UNIT-III			
6	a)	Explain Routh's stability criterion.	L3	CO3	5 M
	b)	Write a short notes on (i) proportional (P) (ii) Proportional Integral (PI) controllers	L3	CO2	5 M
		OR			
7		Sketch the root locus plot of a unity feedback system with an open loop transfer function $G(s)=\dfrac{K}{s(s+1)(s+2)}$. Determine the range of K for stability.	L3	CO2	10 M
		UNIT-IV			
8		Sketch the Bode plot and determine the Gain margin and phase margin. For the open loop transfer function $G(s)=\dfrac{8}{s(1+0.3s)(1+0.1s)}$.	L4	CO4	10 M
		OR			
9	a)	Describe the procedure for developing the polar plot.	L4	CO4	5 M
	b)	A unity feedback control system has an open loop transfer function given by $G(s)H(s)=\dfrac{10}{s(s+3)(s+6)}$. Draw Nyquist diagram and determine stability.	L4	CO4	5 M

II B.Tech II Semester- Regular Examinations- May 2025
CONTROL SYSTEMS

Scheme of Valuation

Part-A

1 a) compare any two differences of open loop and closed loop - 2m

1 b) Sensitivity of open loop or closed loop – 2 m

1c) Peak time definition or expression -1m

Peak overshoot definition or expression- 1m

1 d) Steady state error definition or expression -2m

1 e) Absolute stability and marginal stability – 2m.

1 f) Basic concept of PD controller – 2m

1 g) Statement of Nyquist criteria – 2m

1 h) Lag circuit diagram – 1m

Transfer function – 1m

1 i) Any two advantages of state variable techniques – 2m

1 j) Definition of controllability of condition for controllability – 2m

Part-B

2. Formulation of equations-4m

Simplification – 4m

T.F derivation – 2m

3. Formulation of Differential equations – 4m

Applying Laplace transforms and Simplifacation – 4m

Finding T.F – 2m

4. Characteristic equation or closed loop formulation – 2m

Finding damping ratio -1m

Undamped natural frequency – 1m

Peak time calculation – 2m

Peak overshoot calculation – 2m

Settling time (2% or 5%) – 2m

5. a) Derivation of any two time domain specifications – 5m

b) Explanation of steady state error and error constants – 5m

6 a) Explanation of Routh stability criterion – 5m

b) Explanation of P & PI controllers – 5m

7. Formulation of all construction rules and calculation – 7m

Sketch of root locus plot – 3m

8. Magnitude calculation and plot – 4m

Phase angle calculation and plot – 4m

GM & PM from Graph – 2m

9. a) Explaining the procedure of polar plot – 5m

b) Nyquist plot and calculation – 4m

Comment on Stability – 1m

10 a) Formulation of state model – 5m

b) Finding transfer function for the given state model – 5m

11 a) Finding state transition matrix – 5m

b) State equation calculation – 5m

Part-A

1 a) Differentiate open loop and closed loop control systems

Open loop system	Closed loop system
In accurate and un reliable	Accurate and reliable
Simple and economical	Complex and costlier
The changes in output due to external disturbance are not corrected	The changes in output due to external disturbances are corrected
Stable system	Unstable system

1 b) What the effects of feedback on sensitivity?

Sensitivity of the overall gain of negative feedback closed loop control system (**T**) to the variation in open loop gain (**G**) is defined as

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\textit{Percentage change in } T}{\textit{Percentage change in } G}$$

Sensitivity of open loop system is $S_G^T = 1$

Sensitivity of closed loop system is

$$S_G^T = \frac{1}{(1+GH)}$$

It mean $\left(S_G^T\right)_{\text{closed loop}} < \left(S_G^T\right)_{\text{open loop}}$

Hence closed loop system is lesser sensitive to parameter variations; therefore closed loop system is better.

1 c) Define Peak time and peak overshoot

- Time required for the response to reach the peak value of time response

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$$

- It is the normalised difference between the peak value of time response and the steady state value

$$M_p = 100e^{-\frac{\zeta\pi}{\sqrt{(1-\zeta^2)}}}\%$$

1d) What is steady state error?

The steady state error is defined as the value of error as time tends to infinity.

(or)

It is the error between actual output and desired output as time tends to infinity

(or)

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty}(r(t) - y(t))$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

1 e) Differentiate absolute stability and marginal stability

- **If all the roots of the characteristic equation have negative real parts, then the impulse response is bounded and eventually decreases to zero. Therefore, $\int_0^{\infty}|g(\tau)|d\tau$ is finite and the system is BIBO stable.** **Also called Absolute stability**

- **If one or more non-repeated roots of the characteristic equation are on the imaginary axis, then $g(t)$ is bounded. However, if the input signal have a common pole on the imaginary axis then the output $c(t)$ becomes unbounded. In absence of any common pole the output is bounded and has sustained oscillations. These kind of systems are called '*marginally stable*'.**

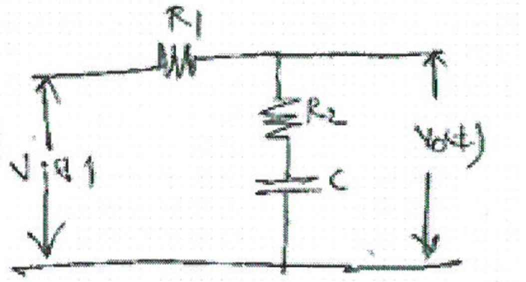
1 f) What is PD controller?

PD controller is a proportional plus derivative controller which produces an output signal consisting of two - one proportional to error signal and other proportional to the derivative of the signal.

1 g) State the Nyquist criterion.

If the Nyquist plot of the open loop transfer function G(s) corresponding to the Nyquist contour in the S-plane encircles the critical point –1+j0 in the counter clockwise direction as many times as the number of right half S-plane poles of G(s), then closed loop system is stable.

1 h) Draw the circuit diagram of a lag compensator and write its transfer function



$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{Ts+1}{T\beta s+1}$$

1 i) What are the advantages of state variable techniques?

It can be applied to non-linear as well as time varying systems. Any type of input can be considered for designing the system. It can be conveniently applied to multiple input multiple output systems. The state variables selected need not necessarily be the physical quantities of the system

1 j) What is kalman's test of controllability?

A system is said to be completely state controllable if it is possible to transfer the system state from any initial state $X(t_0)$ at any other desired state $X(t)$, in specified finite time by a control vector $U(t)$.

(or)

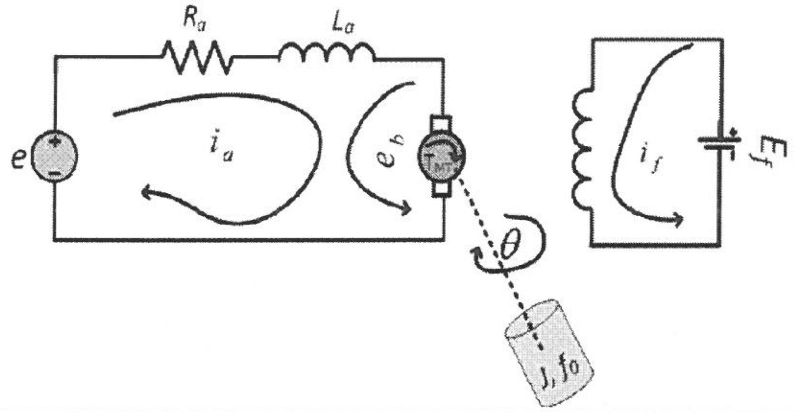
is completely state controllable if only if the rank of combined matrix

$$Q_c = \left[B, AB, \ldots, A^{n-1}B\right]$$

is n (order of the system).

Part-B

2. Derive the Transfer function of armature controlled DC Servo motor



- **Variables and Constants in the model:**
  - R_a = resistance of armature (Ω)
  - L_a = inductance of armature (H)
  - I_a = armature current (A)
  - I_f = field current (A)
  - E_a = voltage applied to armature (V)
  - E_b = back emf (V)
  - T_M = torque developed by motor (Nm)
  - θ = angular displace of motor shaft (rad)
  - J = moment of inertia of motor and load referred to motor shaft ($kg - m^2$)
  - D = friction coefficient of motor and load referred to motor shaft $\left(\frac{Nm}{rad-s}\right)$

K_T: Motor torque constant

K_b: Back emf constant

Step 1: Torque (Electrical)

Flux is developed due to the field current. This flux is proportional to field current, assuming linear range of magnetization curve.

$$\phi \; \alpha \; i_f \quad ; \qquad \phi = k_f i_f \tag{1}$$

Torque (T_{MT}) is proportional to the product of armature current and air gap flux.

$$T_{MT} \; \alpha \; i_a \phi \quad ; \quad T_{MT} = k_1 i_a \phi \tag{2}$$

Replacing eq.(1) in eq.(2), we get,

$$T_{MT} = k_1 k_f i_f i_a \quad ; \; T_{MT} = K_T i_a \tag{3}$$

Step 2: Equation of armature circuit

The differential equation of the armature circuit is determined using Kirchhoff's law which is given below.

$$e = R_a i_a + L_a \frac{di_a}{dt} + e_b \tag{4}$$

We know that the back emf is proportional to speed.

$$e_b = K_b \frac{d\theta}{dt} \tag{5}$$

Replacing eq.(5) in eq.(4), we get,

$$e = R_a i_a + L_a \frac{di_a}{dt} + K_b \frac{d\theta}{dt} \tag{6}$$

Step 3: Torque (Mechanical Effect)

The electrical torque rotates the load at a speed $\dot{\theta}$ against the moment of inertia J and the viscous friction coefficient B . This is given as,

$$J\frac{d^2\theta}{dt^2}+B\frac{d\theta}{dt}=T_{MT}=K_T i_a \qquad (7)$$

Modelling using transfer function approach

Applying Laplace transform to eq.(5), (6) and (7), we get,

$$E_b(s)=K_b s\theta(s) \qquad (8)$$

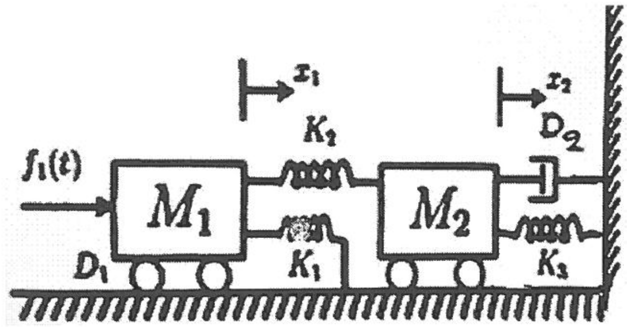
$$E(s)=R_a I_a(s)+L_a sI_a(s)+E_b(s) \qquad (9)$$

$$Js^2\theta(s)+Bs\theta(s)=T_{MT}(s)=K_T I_a(s) \qquad (10)$$

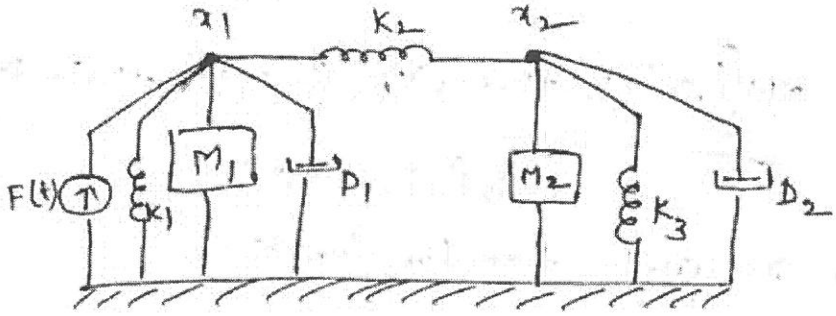
Simplifying above equations, we get,

$$G(s)=\frac{\theta(s)}{E(s)}=\frac{K_T}{s\left((Js+B)(R_a+sL_a)+K_T K_b\right)}$$

3. Given figure



Formulate differential equations, from free body diagram,



at node x_1

$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + D_1 \frac{dx_1}{dt} + K_1 x_1 + K_2(x_1 - x_2)$$

apply Laplace trasforms,

$$F(s) = s^2 M_1 x_1(s) + s D_1 x_1(s) + K_1 x_1(s) + K_2[x_1(s) - x_2(s)]$$

$$F(s) = x_1(s)[M_1 s^2 + s D_1 + K_1 + K_2] - K_2 x_2(s) \quad \text{—(1)}$$

at node, x_2

$$0 = M_2 \frac{d^2 x_2}{dt^2} + D_2 \frac{dx_2}{dt} + K_3 x_2 + K_2(x_2 - x_1)$$

apply L.T's

$$0 = s^2 M_2 x_2(s) + D_2 s\, x_2(s) + K_3 x_2(s) + K_2[x_2(s) - x_1(s)]$$

$$0 = x_2(s)[M_2 s^2 + D_2 s + K_3 + K_2] - K_2 x_1(s)$$

$$x_2(s)[M_2 s^2 + D_2 s + K_3 + K_2] = K_2 x_1(s)$$

$$x_2(s) = \frac{K_2 x_1(s)}{M_2 s^2 + D_2 s + K_3 + K_2} \quad \text{—(2)}$$

Sub. eq(2) in eq(1)

$$F(s) = x_1(s)[M_1 s^2 + D_1 s + K_1 + K_2] - \frac{K_2^2 x_1(s)}{M_2 s^2 + D_2 s + K_3 + K_2}$$

$$F(s) = \frac{x_1(s)\left[(M_1 s^2 + D_1 s + K_1 + K_2)(M_2 s^2 + D_2 s + K_3 + K_2) - K_2^2\right]}{M_2 s^2 + D_2 s + K_3 + K_2}$$

$\therefore$ required transfer function,

$$\boxed{\frac{x_1(s)}{F(s)} = \frac{M_2 s^2 + D_2 s + K_3 + K_2}{(M_1 s^2 + D_1 s + K_1 + K_2)(M_2 s^2 + D_2 s + K_3 + K_2) - K_2^2}}$$

4.

Sol:- Given OLTF, $G(s) = \frac{8}{s(s+2)}$

charac. eqn $\rightarrow 1 + G(s)H(s) = 0$

$$1 + \frac{8}{s(s+2)} \cdot 1 = 0$$

$$s(s+2) + 8 = 0$$

$$s^2 + 2s + 8 = 0 \quad \text{—} \quad (1)$$

Compare eq (1) with standard charac. eqn

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2s + 8$$

$$\omega_n^2 = 8 \; ; \; \omega_n = \sqrt{8} = 2.82 \text{ r/s}$$

$$2\zeta\omega_n = 2$$

$$\zeta = \frac{2}{2\omega_n} = \frac{2}{2 \times 2.82} = 0.35$$

$\therefore$ damping ratio $\zeta = 0.5$

undamped natural freq, $\omega_n = 2.82$ r/s

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{2.82\sqrt{1-(0.35)^2}}$$

$$= 1.19 \text{ sec.}$$

$$t_s(2\%) = \frac{4}{\zeta\omega_n} = \frac{4}{0.35 \times 2.82} = 4 \text{ sec.}$$

$$t_s(5\%) = \frac{3}{\zeta\omega_n} = \frac{3}{0.35 \times 2.82} = 3 \text{ sec.}$$

$$\% M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = 30.7\%$$

Expression for Rise Time

- Consider a 2nd order underdamped system
- Rise time t_r is the time to taken by the step response to go from 0 to 100% of the final value i.e., one
- $y(t_r) = 1 = 1 - \frac{e^{-\zeta\omega_n t_r}}{\sqrt{(1-\zeta^2)}} \sin(\omega_d t_r + \theta)$ where $(\theta = \cos^{-1}\zeta)$

$$\Rightarrow \sin(\omega_d t_r + \theta) = 0 \Rightarrow \omega_d t_r + \theta = \pi$$

$$\boxed{t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \cos^{-1}\zeta}{\omega_n\sqrt{(1-\zeta^2)}}}$$

5 a)

Expression for Peak Time

- Peak time t_p is the time taken by the step response to reach the peak value
- At peak, the time derivative of response is zero
- $\frac{dy}{dt}\Big|_{t_p} = 0 = \frac{\zeta\omega_n e^{-\zeta\omega_n t_p}}{\sqrt{(1-\zeta^2)}} \sin(\omega_d t_p + \theta) - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{(1-\zeta^2)}} \omega_d \cos(\omega_d t_p + \theta)$

$$\Rightarrow \zeta \sin(\omega_d t_p + \theta) - \sqrt{(1-\zeta^2)} \cos(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_p + \theta)\cos\theta - \cos(\omega_d t_p + \theta)\sin\theta = 0$$

$$\Rightarrow \sin(\omega_d t_p) = 0 \Rightarrow \omega_d t_p = 0, \pi, 2\pi, \ldots$$

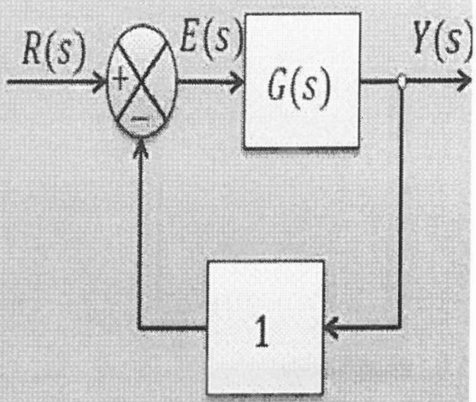
$$\boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}}$$

(corresponding to first peak)

5b)

Steady State Error

- It is the error between the actual output and the desired output as $t \to \infty$



Unity feedback system

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{t\to\infty}(r(t) - y(t))$$

By final value theorem,

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$E = R - Y = R - \frac{GR}{1+G} = \frac{R}{1+G}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

Steady State Error for Standard Inputs

- **Unit step input:** $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{1}{1+G(s)} = \frac{1}{1+K_p}$$

where $K_p = \lim_{s\to 0} G(s)$ is called position error constant

- **Unit ramp (velocity) input:**

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s\to 0}\frac{1}{s+sG(s)} = \lim_{s\to 0}\frac{1}{sG(s)} = \frac{1}{K_v}$$

where $K_v = \lim_{s\to 0} sG(s)$ is called velocity error constant

Note: Velocity error is not error in the velocity but it is error in position due to ramp input

Steady State Error for Standard Inputs

- **Unit parabolic (acceleration) input:**

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s\to 0}\frac{1}{s^2 + s^2G(s)} = \lim_{s\to 0}\frac{1}{s^2G(s)} = \frac{1}{K_a}$$

where $K_a = \lim_{s\to 0} s^2G(s)$ is called acceleration error constant

- The error constants K_p, K_v and K_a describe the ability of a system to reduce or eliminate steady state errors
- These values mostly depend on the type of the system
- As the type of the system becomes higher, more steady-state errors are eliminated

6 a) Routh-Hurwitz Criterion

- **Consider a system with general form of transfer function**

$$T(s) = \frac{p(s)}{q(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \qquad 1$$

- **The characteristic equation of the system is given by**

$$q(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0 \qquad 2$$

- **For stability it is necessary to determine whether any roots of the system lies in the RHP of the *s*-plane.**
- **The characteristic equation is represented in factored form as**

$$q(s) = a_n(s - p_1)(s - p_2)\cdots(s - p_n) = 0 \qquad .3$$

$$\Rightarrow q(s) = a_n\prod_{i=1}^{n}(s - p_i) = 0 \qquad .4$$

- The Routh-Hurwitz criterion is a *necessary* and *sufficient* condition for the stability of linear time invariant systems.
- The method requires two step
  i. Generating Routh array
  ii. Interpreting the Routh array for location of poles in the *s*-plane.
- The Routh-Hurwitz criterion states that the number of roots of the characteristic equation with positive real parts is equal to the number of changes in sign of the first column of the Routh array

Routh Array

- Consider the characteristic equation as in equation (2)

$$q(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

- The coefficients of the characteristic equation are arranged as rows in an array as follows

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \end{array}$$

- The remaining rows are formed by using the following procedure

$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\ s^{n-2} & b_{n-1} & b_{n-2} & b_{n-3} & \cdots \end{array}$$

$$b_{n-1} = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}, b_{n-2} = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}, \cdots$$

- Similarly

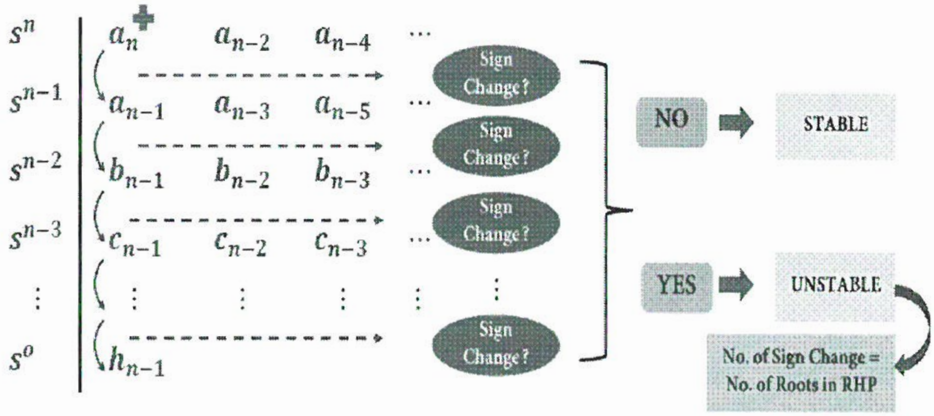
$$\begin{array}{c|cccc} s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\ s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\ s^{n-2} & b_{n-1} & b_{n-2} & b_{n-3} & \cdots \\ s^{n-3} & c_{n-1} & c_{n-2} & c_{n-3} & \cdots \end{array}$$

$$c_{n-1} = \frac{b_{n-1}a_{n-3} - a_{n-1}b_{n-2}}{b_{n-1}}, c_{n-2} = \frac{b_{n-1}a_{n-5} - a_{n-1}b_{n-3}}{b_{n-1}}, \cdots$$

- **The process is continued till s^0 and the complete table of array is obtained as shown below**

Interpretation of Routh Array

- For a system to be stable it is sufficient that all elements of the first column in the Routh array is positive.
- If the condition is not met, then the system is unstable and the number of roots with positive real part is equal to the number of changes in the sign of the elements of the first column of the array.



6 b)

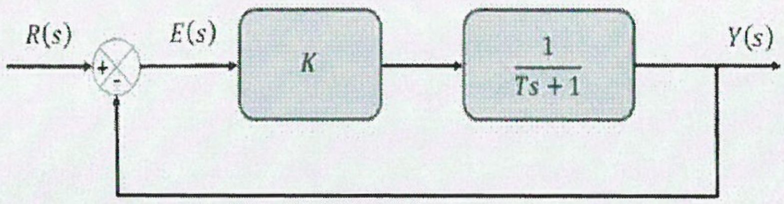
Proportional control action

Let us take a closer look at the proportional control action. Consider a first order plant

$$G(s) = \frac{K}{Ts+1}$$

Then the closed-loop transfer function is

$$C(s) = \frac{K}{Ts+1+K}$$

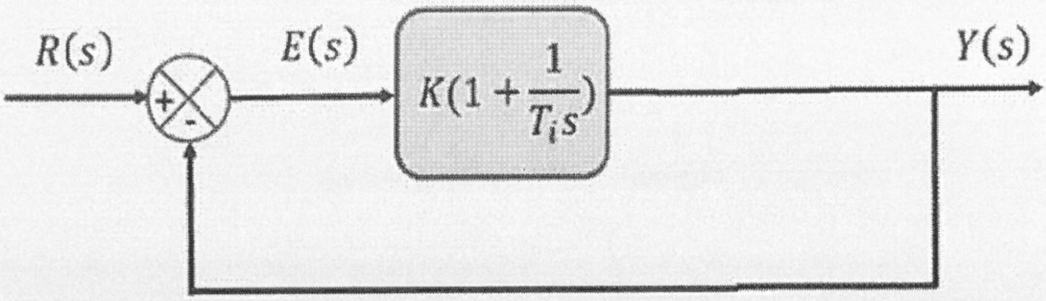


$$E(s) = \frac{1}{1+G(s)}R(s) = \frac{1}{1+\frac{K}{Ts+1}}\frac{1}{s} = \frac{Ts+1}{Ts+1+K}\frac{1}{s} \qquad e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{Ts+1}{Ts+1+K} = \frac{1}{K+1}$$

- Proportional controller has improved the time constant from T to $\frac{T}{(1+k)}$. However, there is steady state error.
- The steady state error can be reduced by choosing a large K, but high gain has the tendency to destabilize the higher order plants.

Proportional + Integral Control

Let us look at how Proportional + Integral control fares in this situation.



The proportional + integral control action eliminated the steady state error.

. The proportional term ensures stability while the Integral terms eliminates steady state error.

7.

Sol:- Given OLTF, $G(s) = \dfrac{K}{s(s+1)(s+2)}$

1) Root locus is symmetrical about real x-axis

2) Asymptodes, $A = n - m$

open loop poles $\rightarrow 0, -1, -2$, $n = 3$

open loop zeros $\rightarrow 0$, $m = 0$

$A = 3 - 0 = 3$

3) Asymptodic angle, $\theta = \pm \dfrac{(2q+1)}{n-m} \times 180^\circ$

$q = 0, 1, 2 \ldots (n-m)-1$

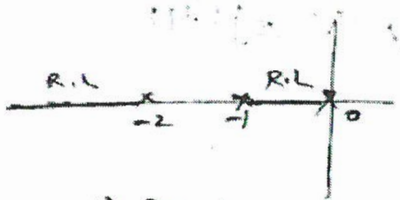
$q = 0$, $\theta_1 = \pm \dfrac{(2(0)+1)}{3} \times 180 = \pm 60^\circ$

$q = 1$, $\theta_1 = \pm \dfrac{(2(1)+1)}{3} \times 180 = \pm 180^\circ$

$q = 2$, $\theta_2 = \pm \dfrac{(2(2)+1)}{3} \times 180^\circ = \pm 300$

5) Root locus path :- R.L $\longrightarrow$ 0 to -1 &

-2 to ∞



6) Break away point (BWP) $= \frac{dk}{ds} = 0$

charac. eqn $\quad 1 + GH = 0$

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2) + k = 0$$

$$s^3 + 3s^2 + 2s + k = 0$$

$$k = -(s^3 + 3s^2 + 2s)$$

$$\frac{dk}{ds} = 3s^2 + 6s + 2 = 0$$

$$s = \frac{-6 \pm \sqrt{36 - 24}}{6} = -1.577, -0.422$$

-0.422 is the valid BWP

7) Intersection with imaginary axis by R-H criteria,

charac. eqn $\quad s^3 + 3s^2 + 2s + K = 0$

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	
s^0	K	

$$\frac{6-K}{3} > 0$$

$$6 - K > 0$$

$$6 > K$$

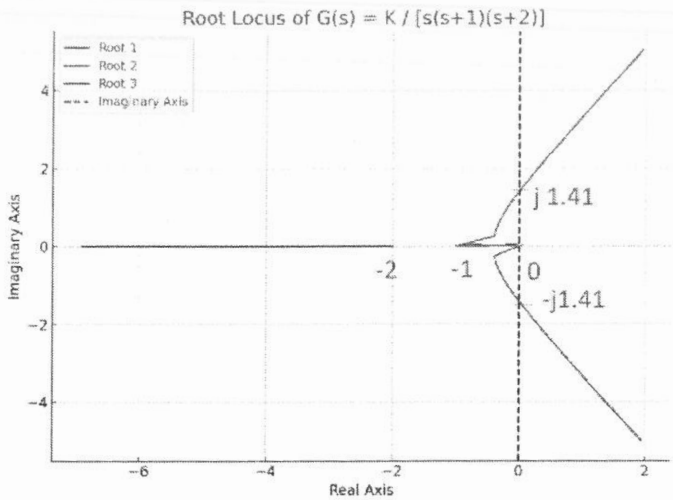
$$\therefore K < 6$$

$\therefore$ for stability the range of 'K' should lies $\boxed{0 < K < 6}$

consider $\quad 3s^2 + K = 0$

$$3s^2 = -6$$

$$s^2 = \frac{-6}{3} ; \quad s = \pm j1.41$$



8. Sol:— Given OLTF, $G(s) = \dfrac{8}{s(1+0.3s)(1+0.1s)}$

Sinusoidal T.F,

$$G(j\omega) = \frac{8}{j\omega(1+j0.3\omega)(1+j0.1\omega)}$$

Magnitude plot:

Factor	C.F	Slope db/dec.	change in slope db/dec.
$\frac{8}{j\omega}$	—	−20	−20
$\frac{1}{1+j0.3\omega}$	$\omega_{c1} = 3.3$	−20	−40
$\frac{1}{1+j0.1\omega}$	$\omega_{c2} = 10$	−20	−60

choose $\omega_l = 0.1$; $M = 20\log\dfrac{8}{|j\omega|}$

$= 20\log\dfrac{8}{0.1} = 38$ db

$\omega_{c1} = 3.3$, $M = 20\log\dfrac{8}{3.3} = 7.6$ db

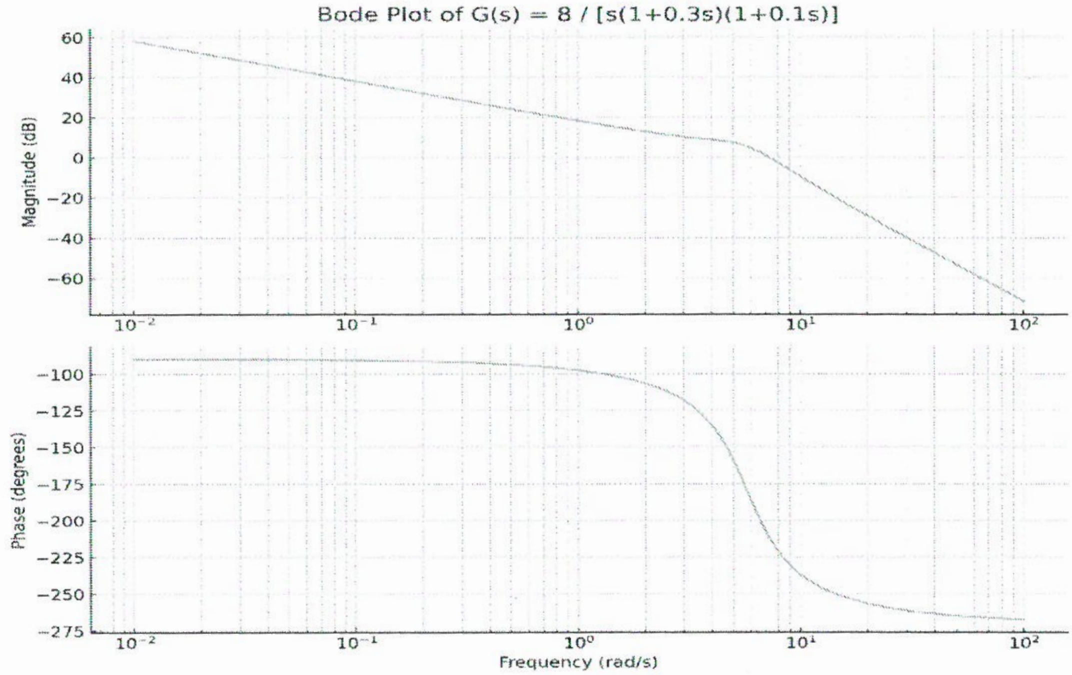
$\omega_{c2} = 10$; $M = -40 \times \log\dfrac{10}{3.33} + 7.6 = -11$ db

Choose $\omega_h = 100$; $M = -60 \times \log\dfrac{100}{10} - 11 = -71$ db

Phase plot

$$\phi = -90 - \tan^{-1}(0.3\omega) - \tan^{-1}(0.1\omega)$$

ω	0.1	0.5	1	3	5	6	7
ϕ	−92°	−101°	−112°	−149°	−173°	−182°	−189°



From graph,

- Gain Margin (GM): ≈ 5.4 dB
- Phase Margin (PM): ≈ 20°

9a)

- **The polar plot of a sinusoidal transfer function $M(j\omega)$ is a plot of its magnitude of $M(j\omega)$ versus the phase angle of $M(j\omega)$.**
- **Thus the polar plots is the locus of the vectors $|M(j\omega)|$ and $\angle M(j\omega)$ as ω is varied from 0 to ∞.**
- **The polar plot is also known as *Nyquist Plot.***
- **One advantage of using polar plot is that it depicts the frequency-response characteristics of a system over the entire frequency range in a single plot.**
- **One disadvantage is that the plot does not clearly indicate the contributions of each individual factor of the open loop transfer function.**

9 b)

Sol: Given OLTF, $G(s) = \dfrac{10}{s(s+3)(s+6)}$

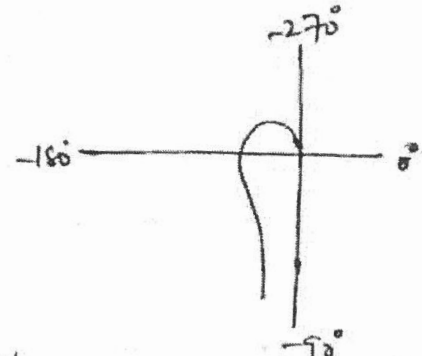
$$G(j\omega) = \frac{10}{j\omega(3+j\omega)(6+j\omega)}$$

Magnitude, $|G(j\omega)| = \dfrac{10}{\omega\sqrt{\omega^2+9}\sqrt{\omega^2+36}}$

Phase angle, $\angle G(j\omega) = -90 - \tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{6}\right)$

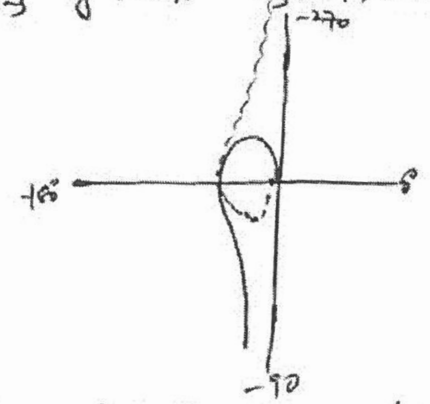
1. Mapping of Section 'C_1' — polar plot

ω	M	ϕ
0	∞	$-90°$
⋮		
∞	0	-270



2. Mapping of section 'C_2' is of zero radius, so not require

3. Mapping of section 'C_3' — inverse polar plot



4. Mapping of section 'C_4' as there is a pole at origin & letting $s = \lim_{R \to 0} Re^{j\theta}$ $(1+sT \approx sT)$

$$G(s) = \frac{10}{18s(1+0.33s)(1+0.166s)}$$

$$= \frac{0.55}{s(1+0.33s)(1+0.166s)} = \frac{0.55}{s\times1\times1} = \frac{0.55}{s}$$

$\theta = -\pi/2$; $G(s) = \frac{0.55}{\underset{R\to0}{Lt}\ Re^{j\theta}} = \frac{0.55}{0} e^{-j\theta}$

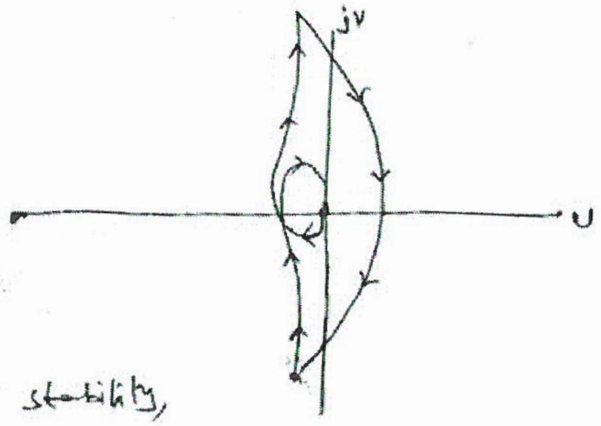
$= \infty e^{-j\theta}$

$= \infty e^{j\pi/2}$

$\theta = \pi/2$, ; $G(s) = \infty e^{-j\pi/2}$

i.e G_4 section ∞ radius $[-90$ to -90

Complete Nyquist plot is



For stability,

$N = P - Z$

$P = 0$, to cal. encirclements equate $\phi = -180°$

$$-180 = -90 - \tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{6}\right)$$

$$\tan[90] = \tan\left[\tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\left(\frac{\omega}{6}\right)\right]$$

$$\frac{1}{0} = \frac{\frac{\omega}{2} + \frac{\omega}{6}}{1 - \frac{\omega^2}{18}}$$

$$1 - \frac{\omega^2}{18} = 0 \Rightarrow$$

$18 - \omega^2 = 0$

$18 = \omega^2$

$\omega = 4.24$ r/s

Sub $\omega = 4.24$ in Mag.

$$M = \frac{10}{\omega\sqrt{\omega^2+9}\sqrt{\omega^2+36}} = 0.011 < 1$$

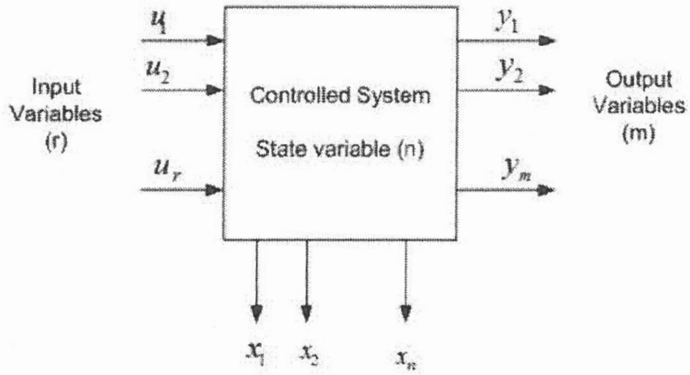
$\therefore$ $N = 0$, $Z = 0$

$\therefore$ the given system is open loop & closed loop stable.

10 a)

State space representation (Mathematical Analysis)

- Consider MIMO System as



The state representation can be arranged in the form of *n* first order differential equations :

- State equation

$$\frac{dx_1(t)}{dt} = \dot{x}_1(t) = f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$
$$\frac{dx_2(t)}{dt} = \dot{x}_2(t) = f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$
$$\vdots$$
$$\frac{dx_n(t)}{dt} = \dot{x}_n(t) = f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$

$$\dot{x}(t) = f(x, u, t)$$

Output equation

$$y_1(t) = g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$
$$y_2(t) = g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$
$$\vdots$$
$$y_m(t) = g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)$$

$$y(t) = g(x, u, t)$$

State model of a linear time invariant system is a special case of the general time invariants models :

In this case, each state variable now becomes linear combination of system states and inputs, i.e.,

$$
\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + \ldots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + \ldots + b_{2r}u_r \\
&\vdots \qquad\qquad\qquad \vdots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + \ldots + b_{nr}u_r
\end{aligned}
$$

In vector matrix form,

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Output variables at time 't' are linear combination of the values of the input and state variables at time 't', i.e.,

$$
\begin{aligned}
&y_1(t) = c_{11}x_1(t) + \ldots + c_{1n}x_n(t) + d_{11}u_1(t) + \ldots + d_{1r}u_r(t) \\
&\mathrm{M} \\
&y_m(t) = c_{m1}x_1(t) + \ldots + c_{mn}x_n(t) + d_{m1}u_1(t) + \ldots + d_{mr}u_r(t)
\end{aligned}
$$

$$y(t) = Cx(t) + Du(t)$$

$$[C]_{(m\times n)} = \begin{bmatrix} c_{11} & c_{12} \ldots & c_{1n} \\ c_{21} & c_{22} \ldots & c_{2n} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \\ c_{m1} & c_{m2} \ldots & c_{mn} \end{bmatrix}, [D]_{(m\times r)} = \begin{bmatrix} d_{11} & d_{12} \ldots & d_{1r} \\ d_{21} & d_{22} \ldots & d_{2r} \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \\ d_{m1} & d_{m2} \ldots & d_{mr} \end{bmatrix}$$

The state model of linear time invariant system is given as

$$\dot{x}(t) = Ax(t) + Bu(t) \quad ; \textit{ State equation}$$

$$y(t) = Cx(t) + Du(t) ; \textit{ Output equation}$$

where, A : System matrix, B : Input matrix, C : Output matrix, D : Coupling matrix (Transmission matrix)

10 b)

Sol: given,

$\dot{x}_1 = -2x_1 + 4x_2 + 4$

$\dot{x}_2 = -x_1 - 2x_2 + 4$ and

$y = x_1 + x_2$

Stat model is represented as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} 4$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T.F = \frac{Y(s)}{U(s)} = C[sI-A]^{-1}B$$

$$[sI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} s+2 & -4 \\ 1 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{(s+2)^2+4} \begin{bmatrix} s+2 & 4 \\ -1 & s+2 \end{bmatrix}$$

$$[sI-A]^{-1}B = \frac{1}{(s+2)^2+4} \begin{bmatrix} s+2 & 4 \\ -1 & s+2 \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times1}$$

$$= \frac{1}{(s+2)^2+4} \begin{bmatrix} s+6 \\ s+1 \end{bmatrix}$$

$$C[sI-A]^{-1}B = \frac{1}{(s+2)^2+4} \begin{bmatrix} 1 & 1 \end{bmatrix}_{1\times2} \begin{bmatrix} s+6 \\ s+1 \end{bmatrix}_{2\times1}$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{2s+7}{(s+2)^2+4}}$$

Sol: Given, $\dot{x} = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$; $y = [1 \quad 0] x$

a) state transition matrix, $\phi(t) = L^{-1}[sI - A]^{-1}$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s(s+3)+2} \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \\ \frac{-1}{(s+2)(s+1)} & \frac{s+3}{(s+2)(s+1)} \end{bmatrix}$$

Consider,

$$\frac{s}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = \frac{2}{s+2} \quad \frac{-1}{s+1}$$

$s = A(s+1) + B(s+2)$

Let $s = -1$ | $s = -2$

$-1 = B$ | $-2 = -A$

$$\frac{2}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = \frac{-2}{s+2} + \frac{2}{s+1}$$

$2 = A(s+1) + B(s+2)$

$s = -1$ | $s = -2$

$2 = B$ | $-A = 2$

$$\frac{-1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = \frac{1}{s+2} \quad \frac{-1}{s+1}$$

$-1 = A(s+1) + B(s+2)$

$s = -1$ | $s = -2$

$B = -1$ | $-A = -1$

$$\frac{s+3}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = \frac{-1}{s+2} + \frac{2}{s+1}$$

$$s+3 = A(s+1) + B(s+2)$$

$s=-1$	$s=-2$
$B=2$	$-A=1$

$\therefore$ STM $= L^{-1}[sI-A]^{-1}$

$$e^{At} = \phi(t) = L^{-1}\begin{bmatrix} \frac{2}{s+2} - \frac{1}{s+1} & \frac{-2}{s+2} + \frac{2}{s+1} \\ \frac{1}{s+2} - \frac{1}{s+1} & \frac{-1}{s+2} + \frac{2}{s+1} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -2e^{-2t} + 2e^{-t} \\ e^{-2t} - e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix}$$

b) state equation for unit step i/p under zero Initial condition,

$$x(t) = L^{-1}\{[sI-A]^{-1} B\, U(s)\}$$

$$L^{-1}\left\{\begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{2}{(s+2)(s+1)} \\ \frac{-1}{(s+2)(s+1)} & \frac{s+3}{(s+2)(s+1)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix}\right\}$$

$$L^{-1}\left\{\begin{bmatrix} \frac{2}{(s+2)(s+1)} \\ \frac{s+3}{(s+2)(s+1)} \end{bmatrix}\begin{bmatrix} \frac{1}{s} \end{bmatrix}\right\} = L^{-1}\left\{\begin{matrix} \frac{2}{s(s+2)(s+1)} \\ \frac{s+3}{s(s+2)(s+1)} \end{matrix}\right\}$$

consider, $\frac{2}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$

$\therefore 2 = A(s+2)(s+1) + Bs(s+1) + Cs(s+2)$

$s=0$	$s=-1$	$s=-2$
$2A=2$	$-C=2$	$2B=2$
$A=1$		$B=1$

$\frac{s+3}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$

$s+3 = A(s+2)(s+1) + Bs(s+1) + Cs(s+2)$

$s=0$	$s=-1$	$s=-2$
$3=2A$	$-C=2$	$2B=1$
$A=3/2$		$B=1/2$

$$\therefore X(t) = L^{-1}\begin{bmatrix} \frac{1}{s} + \frac{1}{s+2}, \frac{-2}{s+1} \\ \frac{3/2}{s} + \frac{1/2}{s+2} - \frac{2}{s+1} \end{bmatrix} = \begin{bmatrix} 1+e^{-2t}-2e^{-t} \\ \frac{3}{2}+\frac{1}{2}e^{-2t}-2e^{-t} \end{bmatrix}$$