

Code: 23CE3402

II B.Tech - II Semester – Regular Examinations - MAY 2025

STRUCTURAL ANALYSIS
(CIVIL ENGINEERING)

Duration: 3 hours

Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.

2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.

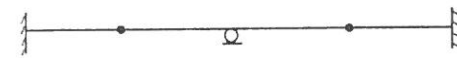
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.

4. All parts of Question paper must be answered in one place.

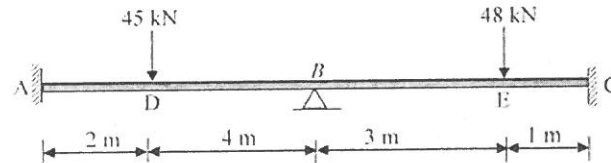
BL – Blooms Level

CO – Course Outcome

PART – A

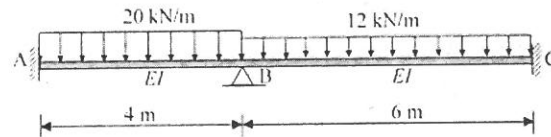
		BL	CO
1.a)	State Castigliano's first theorem.	L1	CO1
1.b)	Define Strain energy.	L1	CO1
1.c)	Determine the kinematic indeterminacy of a single bay portal frame ABCD (support A is fixed, support D is a hinge and joints B and D are pin jointed).	L2	CO2
1.d)	Define static and kinematic indeterminacies.	L1	CO2
1.e)	What is an encastre beam?	L1	CO3
1.f)	Find the degree of indeterminacy of structure given below. 	L2	CO3
1.g)	What are the assumptions made in slope-deflection method?	L1	CO4
1.h)	Write down the slope deflection equation for a beam AB fixed at A and B subjected to a settlement δ at B.	L2	CO4

9	Calculate the bending moments at A, and C for the two-span continuous beam ABC shown in figure. Use slope deflection method. EI is constant.	L4	CO4	10 M
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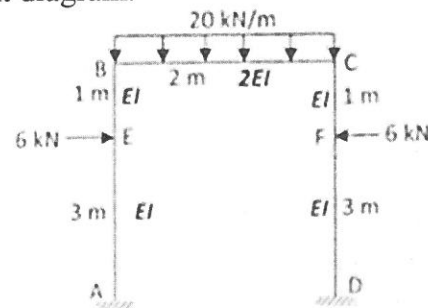
UNIT-V

10	Calculate the bending moments at A, B, and C for the two-span continuous beam ABC shown in figure. EI is constant. Use moment distribution method.	L4	CO5	10 M
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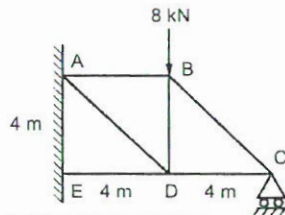
OR

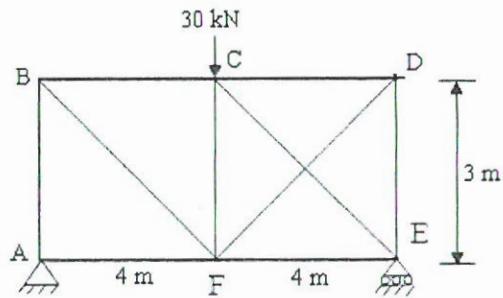
11	Analyze the frame as shown in figure by moment distribution method and draw bending moment diagram.	L4	CO5	10 M
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1.i)	Describe the term distribution factor.	L1	CO5
1.j)	Define carryover factor.	L1	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Find the strain energy stored by a prismatic rod of length l , sectional area A and modulus of elasticity E subjected to tension S .	L3	CO1	5 M
	b)	Derive the expression for strain energy stored in a beam subjected to uniform moment M .	L2	CO1	5 M
OR					
3	A cantilever beam of length l carries two concentrated loads each of magnitude P placed at distances $l/2$ and l from the fixed end. Find the strain energy stored by the cantilever beam.		L4	CO1	10 M
UNIT-II					
4	Find the forces in the members of the truss as shown in figure. The axial rigidities are same for all the members.		L4	CO2	10 M
					
OR					
5	Find the forces in the members of the truss as shown in figure. Sectional area of horizontal, vertical and diagonal members is 4000 mm^2 , 3000 mm^2 and 5000 mm^2 respectively. Take		L4	CO2	10 M

		'E' is same for all members.				
						
UNIT-III						
6	A fixed beam of 5 m span carries a gradually varying load of 12 kN/m at one end to 32 kN/m at the other end. Find the fixed end moments and reactions at fixed supports.	L4	CO3	10 M		
OR						
7	A continuous beam of ABC of 4 m span each, A is fixed and B, C are pinned is subjected to a clockwise couple of 12 kNm at 1 m from the left support A and UDL of 10 kN/m on BC span. Draw shear force and bending moment diagrams. Take $EI = 1000 \text{ kNm}^2$. Use theorem of three moments.	L4	CO3	10 M		
UNIT-IV						
8	A continuous beam ABC consists of span $AB = 3 \text{ m}$ and $BC = 4 \text{ m}$, the ends A and C are fixed. AB and BC carry uniformly distributed loads of intensity 4 kN/m and 5 kN/m respectively. The beam is of uniform section throughout. Draw the bending moment diagram for the beam. Use slope deflection method. Take $EI = 7000 \text{ kNm}^2$.	L4	CO4	10 M		
OR						

23CE3402

II-II B.Tech Regular Exams - May-2025

STRUCTURAL ANALYSIS

PVP-23

Civil Engineering

Short Answers:- Part-A.

- a) Definition of Castigliano's theorem - 1M.
Formula - 1M.
- b) Strain energy definition - 2M.
- c) D_k formula - 1M.
Calculation - 1M.
- d) Static Indeterminacy definition - 1M.
Kinematic Indeterminacy definition - 1M.
- e) Encastre beam meaning - 2M.
- f) D_s formula - 1M.
Calculation - 1M.
- g) Any two assumptions made in slope-deflection - 2M.
- h) Slope deflection equation - 2M. $- 2 \times 1M = 2M.$
- i) Definition of distribution factor - 2M.
- j) Definition of carryover factor - 2M.

Part-B

— o —

- 2 a) Defining section properties - 2M.
Tension relation calculation - 2M.
Final strain energy formula - 1M.
- 2 b) Diagram - 1M.
Calculation - 2M.
Final strain energy formula - 2M.

3.

diagram - 1M.

Strain energy formula - 2M.

Calculation - 6M.

Final formula - 2M.

4.

Consider correct procedure 70% marks will be awarded
for all questions - ie, 7M.

Forces resisting for external loads - 3M.

Due to unit load, forces in members at redundant at all joints - 3M.

Table for calculation - 3M. (Including member forces)
Calculation of X - 1M.

5.

External load application, forces at all joints - 3M.

Due to unit load at redundant, forces in members at all joints - 3M.

Table for calculation - 3M (Including member forces)

Calculation of X - 1M.

6.

Considering UDL and calc of $M_A + M_B$ - 2M.Triangular load, Calc of $M_A' + M_B'$ - 2M.~~Final~~ Calculation - 2M.

Final moment - 2M.

Calc of reactions - 2M.

7.

Clapeyron's formula - 2M.

Calculation - 2M.

Zero beam calculation - 2M.

Calc final moments - 2M.

Answer - 1M.

BMD - 1M.

8. calc of fixed end moments - 3M.
 calc of slope deflection equations - 3M.
 calc of equilibrium condition - 2M.
 calc of fixed moments - 2M.
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9. calc of fixed end moments - 3M.
 calc of slope deflection equations - 3M.
 calc of equilibrium conditions - 2M.
 calc of fixed moments - 2M.
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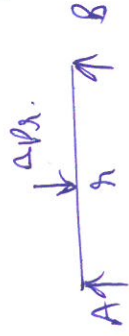
10. calc of fixed end moments - 3M.
 calc of distribution factor - 3M.
 Moment distribution method - 3M.
 calc of fixed moments - 2M.
-

11. calc of fixed end moments - 3M.
 calc of distribution factor - 2M.
 Moment distribution method - 3M.
 calc of fixed moments - 2M.
-

Civil Engg

1a) Castigliano's first theorem:-

The deflection at any load point is equal to the partial derivation or differentiation of strain energy with respect to load at that point.



$$\delta_h = \frac{\partial w_i}{\partial R_h}$$

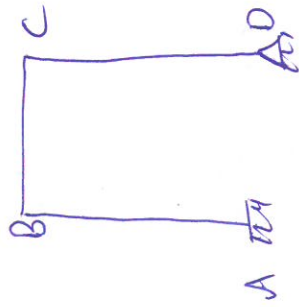
— (2M)

1b) Strain Energy:-

When an elastic member is deformed under the action of an external loading the member is said to have possessed or stored energy which is called "strain energy".

— (2M)

1c)



$D_k = \text{kinematic indeterminacy}$

$$D_k = 3j - R$$

$$R = 3 + 2 = 5$$

$$= 3 \times 4 - 5$$

$$= 7$$

— (2M)

1d) Static Indeterminacy:-

No. of additional unknown reactions to solve a structure.

$$D_s = \text{no. of reactions} - \text{no. of equilibrium} - \text{no. of internal hinges}$$

Kinematic Indeterminacy:-

— (1M)

It is defined as no. of independent displacement components in a structure.

$$D_k = 3j - R \quad \text{for frames.}$$

$$D_k = 2j - R \quad \text{for beams}$$

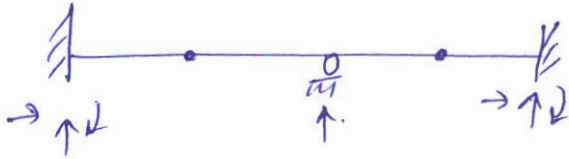
— (1M)

1e) Encastre beam:-

Fixed beam is also called as encastre beam, whose ends are fixed.

— (2M)

1f)



$$\text{Unknowns} = 3 + 3 + 1 = 7.$$

$$D_s = 7 - 3 - 2 = 2.$$

— (2M)

1g) Assumptions made in slope-deflection method:-

- All joints are rigid
- Rotations of joints are treated as unknowns
- Between each pair of supports beam section is constant.
- Shear deformations are neglected.

Any two - 2M

1h) Slope deflection method:-

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l} \left[2\theta_A + \theta_B - \frac{3\delta}{l} \right]$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l} \left[\theta_A + 2\theta_B - \frac{3\delta}{l} \right]$$

$$\theta_A = 0.$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l} \left[2\theta_B - \frac{3\delta}{l} \right]$$

— (2M).

1i) Distribution factor:-

The ratio of stiffness of member to the total stiffness of all the members meeting at the joint.

— (2M)

1j) carryover factor:- A factor that determines the amount of moment transferred from one end to its another end.

Half of the moment from one end to another end.

— (2M)

(2)

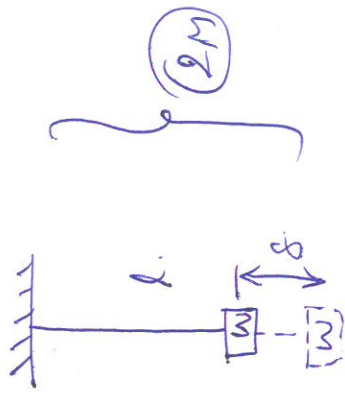
2a) Strain energy stored in a bar

Elastic member of length 'l'

Cross-sectional area 'A'

External axial load 'W'

Extension of member 's'.



load is applied gradually, magnitude of load is increased gradually from zero to the value 'W' the member also has gradually extended.

The work done by load on member in stretching it equals the product of average load and displacement 's'

$$\text{External work done} = W_e = \frac{1}{2} Ws$$

$$\text{Energy stored by member} = W_i$$

$$W_e = W_i$$

Tension in member be 'S'

$$\text{Intensity of tensile stress } \sigma = \frac{S}{A}$$

$$\text{Tensile strain } \epsilon = \frac{\sigma}{A} = \frac{S}{AE}$$

$$\text{change in length of member} = \delta = \text{strain} \times \text{original length}$$

$$\delta = \epsilon \cdot l = \frac{S}{AE} \cdot l$$

$$\text{Strain energy stored} = \text{work done}$$

$$= \frac{1}{2} Ws = \frac{1}{2} \cdot S \cdot \frac{Sl}{AE} = \frac{S^2 l}{2AE}$$

Strain energy stored per unit volume of member.

$$= \frac{S^2 l}{2AE} / Al = \frac{S^2}{2AE} = \frac{\sigma^2}{2E}$$

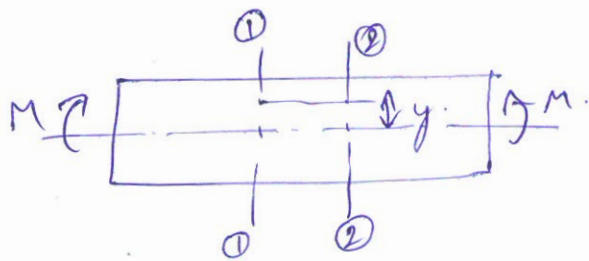
Strain energy stored in member due to axial loading

$$= \frac{\sigma^2}{2E}$$

(2M)

(1M)

2b) Expression of strain energy stored due to bending:-



— (1M)

Consider a beam subjected to a uniform moment 'M'. Consider an elemental length 'ds' of beam between two sections 1-1 and 2-2 'ds' apart, area 'da'.

$$\sigma = \frac{M}{I} \cdot y.$$

Energy stored by elemental cylinder = energy stored $\times$ vol of cylinder per unit volume

$$= \frac{\sigma^2}{2E} \cdot da \cdot ds = \frac{1}{2E} \left(\frac{M}{I} \cdot y \right)^2 \cdot da \cdot ds$$

$$= \frac{M^2}{2EI^2} \cdot ds \cdot da \cdot y^2$$

$$= \frac{M^2}{2EI^2} \cdot ds \cdot da \cdot y^2 = \frac{M^2 ds}{2EI^2} \sum da y^2$$

$\sum da y^2$ = moment of inertia of beam section about the neutral axis = I

$$\text{Energy stored by 'ds' length of beam} = \frac{M^2 ds}{2EI}$$

$$\therefore \text{Total energy stored by whole beam} = \int \frac{M^2 ds}{2EI}$$

$$= \frac{M^2}{2EI} \int ds = \frac{M^2 l}{2EI}$$

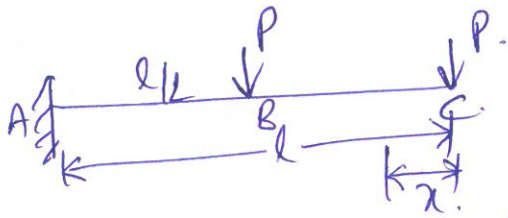
$$I = \frac{bd^3}{12}$$

$$\frac{M}{I} = \frac{\sigma}{y} = M = \frac{bd^3}{12} \cdot \frac{\sigma}{(d/2)} = \frac{\sigma \cdot bd^2}{6}$$

$$W_i = \frac{1}{2E} \left(\frac{1}{6} \cdot \sigma \cdot bd^2 \right) \frac{1}{bd^3/12} = \frac{\sigma^2 b d l}{6E} = \frac{\sigma^2}{6E} \times \text{vol of beam}$$

$$\therefore \text{Strain energy stored per unit volume} = \frac{\sigma^2}{6E}$$

(3)



(3)

(1M)

Strain energy $U = \int_0^l \frac{M^2 dx}{2EI}$

(2M)

from 0 to $l/2$.

$$M_1(x) = P(l-x)$$

from $l/2$ to l .

$$M_2(x) = P(l-x) + P\left(\frac{l}{2} - x\right)$$

$$U = \int_0^{l/2} \frac{M_1^2 dx}{2EI} + \int_{l/2}^l \frac{M_2^2 dx}{2EI}$$

(2M)

$$= \int_0^{l/2} \frac{P^2(l-x)^2}{2EI} dx + \int_{l/2}^l \frac{P^2 \left[(l-x) + \left(\frac{l}{2} - x\right) \right]^2}{2EI} dx$$

$$= \frac{P^2}{2EI} \left[\int_0^{l/2} (l-x)^2 dx + \int_{l/2}^l \left(\frac{3l}{2} - 2x \right)^2 dx \right]$$

$$= \frac{P^2}{2EI} \left[\left. \frac{(l-x)^3}{3} \cdot (-1) \right|_0^{l/2} + \left. \left(\frac{\frac{3l}{2} - 2x}{3} \right)^3 \cdot \left(-\frac{1}{2}\right) \right|_{l/2}^l \right]$$

(2M)

$$= \frac{P^2}{2EI} \left[\frac{(l/2)^3}{3} + \left(\frac{3l}{2} - 2x \right)^3 \right]_{l/2}^l = \frac{P^2}{2EI} \left[\frac{l^3}{24} - \left(-\frac{l}{2} \right)^3 - \frac{(l/2)^3}{6} \right]$$

$$= \frac{P^2}{2EI} \left[\frac{l^3}{24} + \frac{2l^3}{8 \times 6} \right] = \frac{P^2}{2EI} \left[\frac{l^3}{24} + \frac{l^3}{24} \right] = \frac{P^2 l^3}{24EI}$$

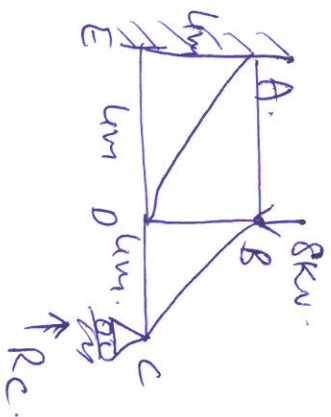
(1M)

$$\therefore U = \frac{P^2 l^3}{24EI}$$

(2M)

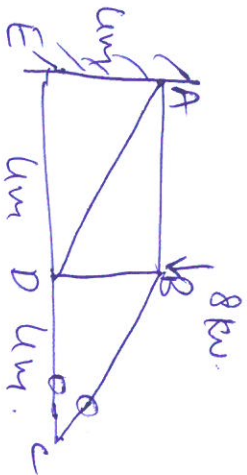
Correct procedure 70% of marks will be awarded
— 7M.

14



Consider correct procedure
10% marks will be awarded
if 71.

Take R_c as redundant.



$$BC = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$$

$$\sin \theta = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Taking moments at A.

$$H_E \times 4 = 8 \times 4 \Rightarrow H_E = 8 \text{ kN} \rightarrow$$

$$H_A = 8 \text{ kN} \leftarrow$$

$$V_A = 8 \text{ kN} \uparrow$$

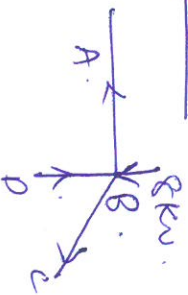
At joint E:-



$$P_{bc} \cos \theta = P_{bc} \Rightarrow P_{bc} = 0$$

$$P_{bc} \sin \theta = 0 \Rightarrow P_{bc} = 0$$

At joint B:-



$$P_{bc} \sin \theta + 8 = P_{bd}$$

$$0 + 8 = P_{bd}$$

$$P_{ab} = P_{bc} \cos \theta$$

$$P_{ab} = 0$$

At joint D:-



$$P_{bd} = P_{ad} \sin \theta$$

$$8 = P_{ad} \frac{1}{\sqrt{2}} \Rightarrow P_{ad} = 8\sqrt{2} \text{ kN}$$

$$P_{ad} \cos \theta = P_{ed}$$

$$8\sqrt{2} \frac{1}{\sqrt{2}} = P_{ed} = 8 \text{ kN}$$

At joint E:-

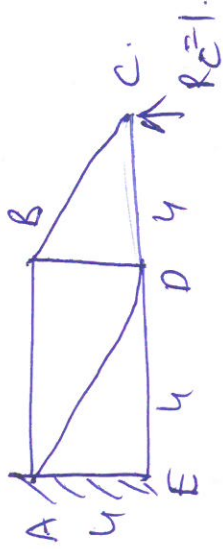
$$P_{ed} = H_E$$

$$8 = 8 \text{ kN} \text{ (satisfied)}$$

3/1/19

Apply unit load at redundant as R_e .

(4)



Take moments at $A = 0$.

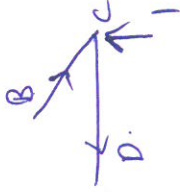
$$R_e(8) = H_e(4)$$

$$H_e = \frac{8}{4} = 2 \text{ kN} \leftarrow$$

$$H_a = 2 \text{ kN} \rightarrow$$

$$V_a = 1 \text{ kN} \downarrow$$

At joint C



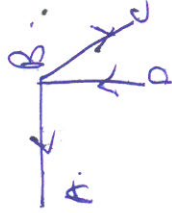
$$R_{bc} \sin \theta = 1$$

$$R_{bc} = \sqrt{2} \text{ kN (comp)}$$

$$R_{bc} \cos \theta = R_{dc}$$

$$\sqrt{2} \cdot \frac{1}{\sqrt{2}} = R_{dc} = 1 \text{ kN (Tensile)}$$

At joint B



$$R_{bc} \sin \theta = R_{bd}$$

$$\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1 \text{ kN (Comp)}$$

$$R_{bc} \cos \theta = R_{db}$$

$$\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1 \text{ kN (Tensile)}$$

At joint D



$$R_{bd} = R_{ad} \sin \theta$$

$$1 = R_{ad} \cdot \frac{1}{\sqrt{2}} \Rightarrow R_{ad} = \sqrt{2} \text{ kN (Comp)}$$

$$R_{ad} \cos \theta = R_{ed} + R_{dc}$$

$$\sqrt{2} \cdot \frac{1}{\sqrt{2}} = R_{ed} + 1 \Rightarrow R_{ed} = 0$$

(4M)

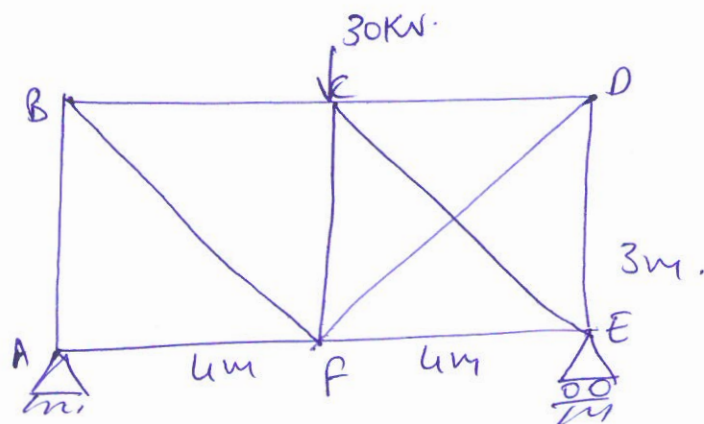
Member	P	K	L	PkL	KL	forces in members. $S = P + XK$
AB.	0	-1	4	0	4	3.537 (Comp)
BC	0	$+\sqrt{2}$	$4\sqrt{2}$	0	$8\sqrt{2}$	-5.00 (Tensile)
CD.	0	+1	4	0	4	-3.537 (Tensile)
BD.	+8	+1	4	32	4	-3.537 (Tensile)
AD.	$+8\sqrt{2}$	$+\sqrt{2}$	$4\sqrt{2}$	90.509	$8\sqrt{2}$	-5.00 (Tensile)
ED.	-8	0	4	-32	0	0

L = 3M

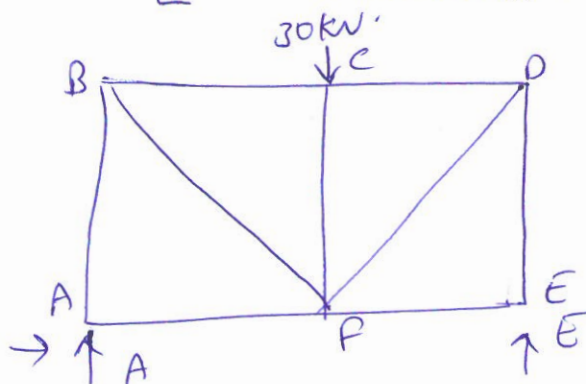
$$X = - \frac{\sum PkL}{\sum KL} = - \frac{122.509}{34.627} = -3.537$$

(1M)

(5)



Let us take ~~CDE~~ as redundant and remove.



Consider Correct procedure 70% marks will be awarded ie, 7M.

(5)

Take moments at A

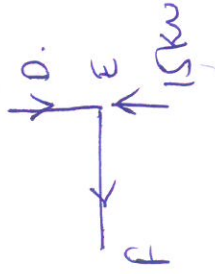
$$R_E (8) - 30(4) = 0$$

$$R_E = 15 \text{ kN} \uparrow$$

$$R_A = 15 \text{ kN} \uparrow$$

$$\tan \theta = 3/4, \quad \sin \theta = 3/5, \quad \cos \theta = 4/5$$

At joint E:-



$$P_{ED} = 15 \text{ kN (comp)}$$

$$P_{EF} = 0$$

At joint A:-

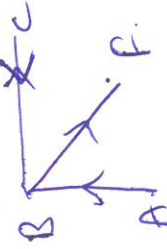


$$P_{AB} = 15 \text{ kN (comp)}$$

$$P_{AF} = 0 \text{ kN}$$

(34)

At joint B:-



$$P_{AB} = P_{BF} \sin \theta$$

$$15 = P_{BF} \cdot \frac{3}{5}$$

$$P_{BF} = 25 \text{ kN (Tensile)}$$

$$P_{BF} \cos \theta = P_{BC}$$

$$25 \times \frac{4}{5} = 20 = P_{BC} \text{ (comp)}$$

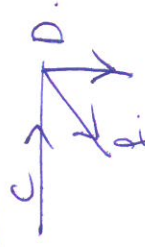
At joint C:-



$$P_{BC} = P_{CD} \Rightarrow P_{CD} = 20 \text{ kN (comp)}$$

$$P_{CF} = 30 \text{ kN (comp)}$$

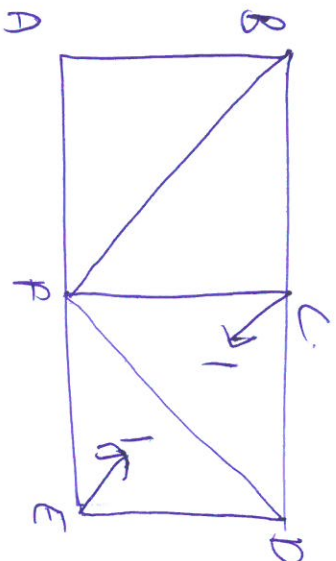
At joint D:-



$$P_{CD} = P_{DF} \cos \theta$$

$$20 = P_{DF} \times \frac{4}{5} \Rightarrow P_{DF} = 25 \text{ kN (Tensile)}$$

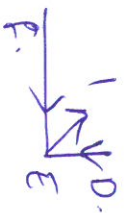
Apply unit force at redundant. $\delta^C(E)$.



There will be no reactions.

So, $R_{ab} = R_{af} = 0$.

At joint E:-



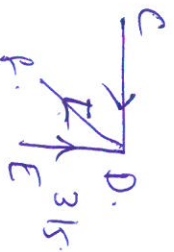
$R_{ef} \cos \theta = 1 \cos \theta$

$R_{ef} = 4/5$ (Comp).

$R_{ed} \sin \theta = 1 \sin \theta$

$R_{ed} = 3/5$ (Comp).

At joint D:-



$R_{df} \sin \theta = R_{ed}$

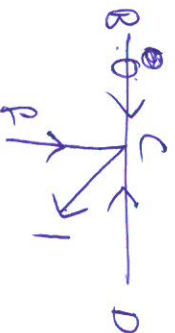
$R_{df} \cdot \frac{3}{5} = \frac{3}{5}$

$R_{df} = 1$ kN (Tensile)

$R_{df} \cos \theta = R_{ed}$

$4/5 = R_{ed}$ (Comp).

At joint C:-

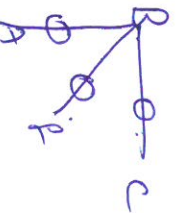


1 case $\neq R_{ed} = R_{ed}$

$4/5 + R_{ed} = 4/5$

$R_{ed} = 0$.

At joint B:-



1 case $\neq R_{ef} = 3/5$ (Comp).

$R_{bf} = 0$.

}

(34)

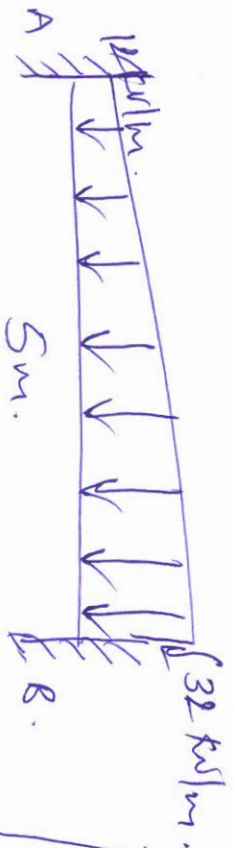
⑥ forces in members.
 $S = P + XK$

Member	P	K	L	A	$\frac{PKL}{A}$	$\frac{K^2L}{A}$	forces in members. $S = P + XK$
AB	15	0	3000	3000	0	0	15 (comp)
CF	30	15 15	3000	3000	18	$9/25$	19.8 19.8 (comp)
DE	15	$3/5$	3000	3000	9	$9/25$	4.80 4.80 (comp)
BC	20	0	4000	4000	0	0	20 (comp)
AF	0	0	4000	4000	0	0	0
CD	20	$4/5$	4000	4000	16	$16/25$	32.0 32.0 (comp)
EF	0	$4/5$	4000	4000	0	$16/25$	-13.6 (Tensile)
BF	-25	0	5000	5000	0	0	-25 (Tensile)
CE	0	-1	5000	5000	0	+1	+17 (comp)
DF	-25	-1	5000	5000	+25	+1	48 (comp)
					$\Sigma = 68 = +4$		

— (3M) —

$$X = \frac{-\sum \frac{PKL}{AE}}{\sum \frac{K^2L}{AE}} = \frac{-68}{4} = -17. \quad \text{--- (1M) ---}$$

⑥



Consider correct procedure
70% will be awarded
-7M.

i) Consider a uniformly distributed load 12 kN/m.

ii) A varying load of 0 kN/m at A and 20 kN/m at B.

$$M_a = M_b = \frac{wL^2}{12} = \frac{12 \times 5^2}{12} = 25 \text{ kNm.} \quad \text{--- (2M) ---}$$

Fixing moments due to triangular load

$$M'_a = \frac{wL^2}{30} = \frac{20 \times 5^2}{30} = 16.667 \text{ kNm.}$$

$$M'_b = \frac{wL^2}{20} = \frac{20 \times 5^2}{20} = 25 \text{ kNm.} \quad \text{--- (2M) ---}$$

$\therefore$ Total fixing moment at A = 25 + 16.667

$\therefore$ Total fixing moment at B = 25 + 25 = 50 kNm. (1M)

FEM at A = 41.667 kNm. (1M)

FEM at B = 50 kNm. (2M)

Reaction at A.

$$R_a(5) - \underbrace{41.667}_{358.333} - \frac{12 \times 5^2}{2} - 20 \times 5 \times \frac{5}{3} = -50.$$

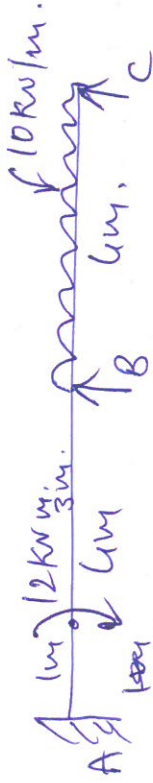
$$R_a = 61.6667 \text{ kN.}$$

$$R_a + R_b = 12 \times 5 + 20 \times \frac{5}{2} = 110.$$

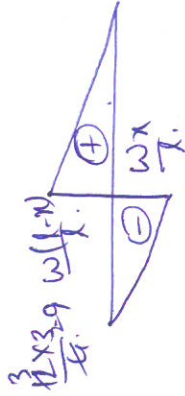
$$R_b = 110 - 61.667 = 48.333 \text{ kN.}$$

(2M)

7



$$EI = 1000 \text{ kN-m}^2$$



Height

$$\frac{12 \times 6}{4} = 3$$

Area

$$\frac{2}{3} \times 6 = \frac{2}{3} \times 6 \times \frac{10 \times 6^2}{8} = 53.333$$

$$\bar{x}_1 = -\frac{1}{2} \times 3 \times \frac{6}{3} + \frac{1}{2} \times 3 \times 9 \times \left(1 + \frac{6}{3}\right) = \frac{9}{2} = 4.5 \text{ m}$$

$$= -1 + 27 = 26$$

Clapeyron's theorem of three moments $M_C = 0$

$$M_A l_1 + 2M_B (l_1 + l_2) + M_C l_2 = \frac{6a_1 \bar{x}_1}{l_1} + \frac{6a_2 \bar{x}_2}{l_2}$$

$$M_A (4) + 2M_B (4+4) + 0 = \frac{6 \times 26}{4} + \frac{6 \times 53.333 \times 2}{4}$$

$$4M_A + 16M_B = 39 + 159.999$$

— (1)

Consider a imaginary beam on left side of AB.



$$0 + 2M_A (0+4) + M_B (4) = \frac{6 \times (-22)}{4}$$

$$8M_A + 4M_B = -33$$

$$2M_A + M_B = -8.25$$

$$M_B = -8.25 - 2M_A$$

— (2)

Consider correct procedure 70% marks will be awarded i.e., 7M.

7

Substituting eqn ② in eq ①.

$$4M_a + 16(-8.25 - 2M_a) = 39 + 159.999$$

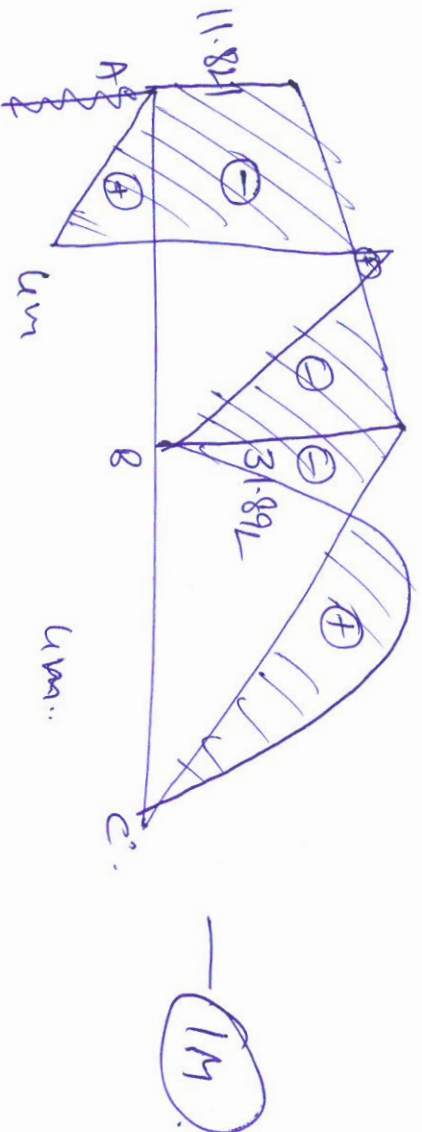
$$4M_a - 132 - 32M_a = 198.999.$$

$$-28M_a = 330.999.$$

$$M_a = -11.821 \text{ kNm.}$$

$$M_b = -8.25 - 2(-11.821)$$

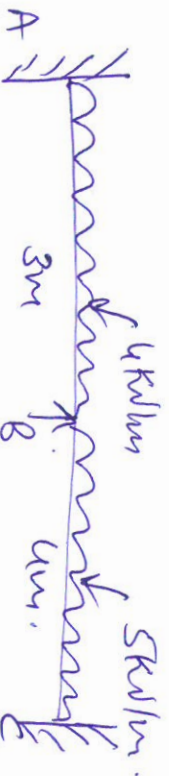
$$M_b = 31.892 \text{ kNm.}$$



BMD

Calculation of reactions R_a, R_b, R_c . } — (14)

⑧.



Step-1)- calculation of fixed end moments.

$$\bar{M}_{ab} = -\frac{wL^2}{12} = -\frac{4 \times 12^2}{12} = -3 \text{ kNm.}$$

$$\bar{M}_{ba} = +\frac{wL^2}{12} = +3 \text{ kNm}$$

$$\bar{M}_{bc} = -\frac{wL^2}{12} = -\frac{5 \times 4^2}{12} = -6.67 \text{ kNm,}$$

$$\bar{M}_{cb} = +\frac{wL^2}{12} = +\frac{5 \times 4^2}{12} = +6.67 \text{ kNm.}$$

314

Step-2:- Slope deflection equations

$$\theta_A = 0, \theta_C = 0, \theta_B = ? , \delta = 0.$$

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l} (2\theta_A + \theta_B - \frac{3\delta}{l})$$

$$M_{ab} = -3 + \frac{2}{3} EI (\theta_B) \quad \text{--- (1)}$$

$$M_{ba} = \bar{M}_{ba} + \frac{2EI}{l} (\theta_A + 2\theta_B - \frac{3\delta}{l})$$

$$M_{ba} = +3 + \frac{4EI}{3} \theta_B \quad \text{--- (2)}$$

$$M_{bc} = \bar{M}_{bc} + \frac{2EI}{l} (2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$M_{bc} = -6.67 + EI \theta_B \quad \text{--- (3)}$$

$$M_{cb} = \bar{M}_{cb} + \frac{2EI}{l} (\theta_B + 2\theta_C - \frac{3\delta}{l})$$

$$M_{cb} = +6.67 + 0.5 EI \theta_B \quad \text{--- (4)}$$

Step-3:- Condition of equilibrium

$$M_{ba} + M_{bc} = 0.$$

$$-3 + \frac{4}{3} EI \theta_B - 6.67 + EI \theta_B = 0.$$

$$\frac{7}{3} EI \theta_B = 3.67.$$

$$EI \theta_B = 1.57286.$$

Step-4:- Calculation of final moments.

$$M_{ab} = -3 + \frac{2}{3} (1.57286) = -1.95 \text{ kNm.}$$

$$M_{ba} = +3 + \frac{4}{3} (1.57286) = +5.10 \text{ kNm.}$$

$$M_{bc} = -6.67 + 1.57286 = -5.10 \text{ kNm.}$$

$$M_{cb} = +6.67 + \frac{1}{2} (1.57286) = +7.46 \text{ kNm.}$$

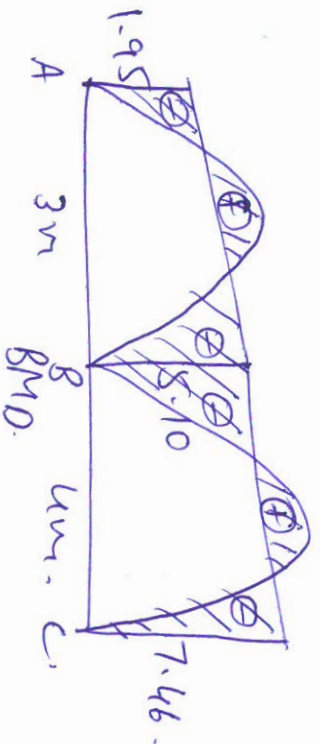
⑧

Consider correct procedure 70% marks will be awarded i.e. 7M.

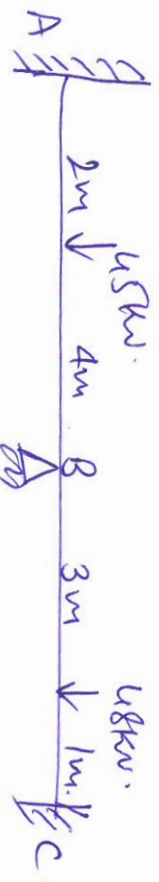
3M

2M

2M



⑨.



Consider correct procedure 70% marks will be awarded i.e., 71.

Step-1:- Calculation of fixed end moments.

$$M_{ab} = -w_{ab} \frac{l^2}{12} = -45 \times \frac{2 \times 2^2}{12} = -40 \text{ kNm.}$$

$$M_{ba} = +w_{ab} \frac{l^2}{12} = +45 \times \frac{2^2 \times 2}{6} = +20 \text{ kNm.}$$

$$M_{bc} = -w_{bc} \frac{l^2}{12} = -48 \times \frac{3^2 \times 1}{4} = -9 \text{ kNm.}$$

$$M_{cb} = +w_{bc} \frac{l^2}{12} = +48 \times \frac{3^2 \times 1}{4} = +27 \text{ kNm.}$$

3M

Step-2:- Slope deflection equations

$$\theta_A = 0$$

$$M_{ab} = M_{ab} + \frac{2EI}{l} (\theta_A + \theta_B)$$

$$\theta_C = 0.$$

$$= -40 + \frac{2EI}{6} (0 + \theta_B)$$

$$= -40 + \frac{1}{3} EI \theta_B \quad \text{--- (1)}$$

$$M_{ba} = M_{ba} + \frac{2EI}{l} (\theta_A + \theta_B)$$

$$= 20 + \frac{2EI}{6} (\theta_B + 0) = 20 + \frac{1}{3} EI \theta_B \quad \text{--- (2)}$$

$$M_{bc} = M_{bc} + \frac{2EI}{l} (\theta_B + \theta_C)$$

$$= -9 + \frac{2EI}{4} (\theta_B + 0) = -9 + EI \theta_B \quad \text{--- (3)}$$

(9)

$$M_{cb} = \bar{M}_{cb} +$$

$$\begin{aligned} M_{cb} &= M_{cb} + \frac{2EI}{L} (\theta_B + 2\theta_C) \\ &= +27 + \frac{2EI}{4} (\theta_B + 2\theta_C) \\ &= +27 + \frac{1}{2} EI\theta_B \quad \text{--- (4)} \end{aligned}$$

Step-3:-

Condition of equilibrium.

$$\boxed{M_{ba} + M_{bc} = 0.}$$

$$\left. \begin{aligned} 20 + \frac{2}{3} EI\theta_B - 9 + EI\theta_B &= 0. \\ \frac{5}{3} EI\theta_B &= -11. \end{aligned} \right\} \quad (2M)$$

Step-4:-

Final moments

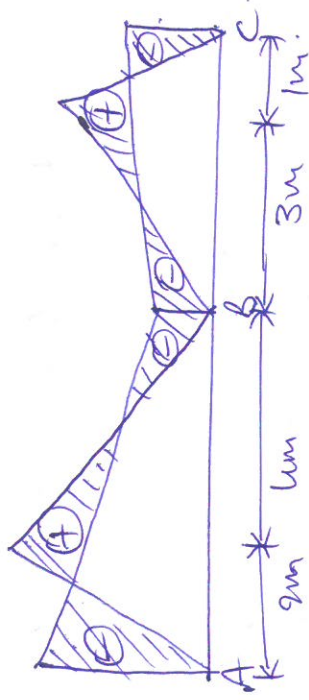
$$EI\theta_B = -6.60.$$

$$M_{bb} = -40 + \frac{1}{3} (-6.6) = -42.20 \text{ kNm}$$

$$M_{ba} = +20 + \frac{2}{3} (-6.6) = +15.6 \text{ kNm}$$

$$M_{bc} = -9 + (-6.6) = -15.60 \text{ kNm}$$

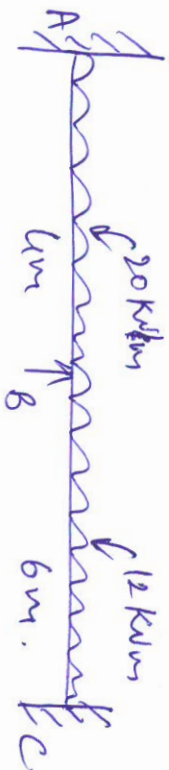
$$M_{cb} = +27 + \frac{1}{2} (-6.600) = +23.70 \text{ kNm}$$



BMD.

(2M)

10.



Consider correct procedure
70% notes will be awarded
ie, 7M.

Step-1:- calculation of fixed end moments

$$\bar{M}_{ab} = -\frac{wL^2}{12} = -\frac{20 \times 12^2}{12} = -26.67 \text{ kNm.}$$

$$\bar{M}_{ba} = +\frac{wL^2}{12} = +\frac{20 \times 12^2}{12} = +26.67 \text{ kNm.}$$

$$\bar{M}_{bc} = -\frac{wL^2}{12} = -\frac{12 \times 6^2}{12} = -36 \text{ kNm.}$$

$$\bar{M}_{cb} = +\frac{wL^2}{12} = +\frac{12 \times 6^2}{12} = +36 \text{ kNm.}$$

3M

Step-2:- calculation of distribution factors

Joint Number	Relative stiffness	Total Relative stiffness	Distribution factor
B	$\frac{I}{L} = \frac{I}{12}$ $\frac{I}{L} = \frac{I}{6}$	$\frac{5I}{12}$	$\frac{3}{5}$
			$\frac{2}{5}$

2M

Step-3:- Moment distribution method

A B C

Member.	AB	BA	BC	CB.
DF	1	$\frac{3}{5}$	$\frac{2}{5}$	1
FEM	-26.67	+26.67	-36	+36
Balancing		+5.60	+3.73	
Carryover	+2.80			+1.87
Total	-23.87	+32.27	-32.27	+37.87

3M

(10)

Step-4:- Final moments:-

$$M_{ab} = -23.87 \text{ kNm},$$

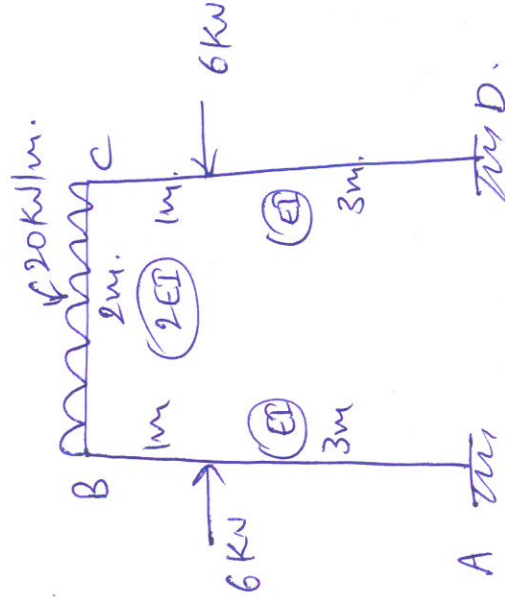
$$M_{ba} = 32.27 \text{ kNm},$$

$$M_{bc} = -32.27 \text{ kNm},$$

$$M_{cb} = +37.87 \text{ kNm}.$$

} (2M)

(11)



Consider correct procedure
70% mistakes will be avoided
ie, 7M.

Step-1:- Calculation of fixed end moments.

$$\bar{M}_{ab} = -\frac{wab^2}{l^2} = -\frac{6 \times 3 \times 1^2}{4^2} = -1.12 \text{ kNm},$$

$$\bar{M}_{ba} = +\frac{wab^2}{l^2} = +\frac{6 \times 3^2 \times 1}{4^2} = +3.37 \text{ kNm},$$

$$\bar{M}_{bc} = -\frac{wl^2}{12} = -\frac{20 \times 1^2}{12} = -6.67 \text{ kNm},$$

$$\bar{M}_{cb} = +\frac{wl^2}{12} = +\frac{20 \times 1^2}{12} = +6.67 \text{ kNm},$$

$$\bar{M}_{cd} = -\frac{wab^2}{l^2} = -\frac{6 \times 1 \times 3^2}{4^2} = -3.37 \text{ kNm},$$

$$\bar{M}_{dc} = +\frac{wab^2}{l^2} = +\frac{6 \times 1^2 \times 3}{4^2} = +1.12 \text{ kNm}.$$

} (3M)

Step 2:- Calculation of distribution factors.

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution factor
B	BA	$I/L = 2I/4$	$5I/4$	$1/5$
	Bc	$I/L = 2I/2$		
	Bc	$I/L = 2I/2$		
C	CB	$I/L = 2I/2$	$5I/4$	$1/5$
	CD	$I/L = I/4$		

(24)

Step 3:- Moment distribution method

	A	B	C	D
Member	AB	BA	BC	CB
DF	1	$\frac{1}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
FEM	-112	+337	-667	+667
Balancing		+0.66	+2.64	-2.64
Carry over	-0.33	-1.32	+1.32	-0.33
Balance		+0.26	+1.06	-1.06
Carry over	+0.13	-0.53	+0.53	-0.13
Balance		+0.11	+0.44	-0.44
Carry over	+0.06	-0.21	+0.21	-0.06
Balance		+0.04	+0.17	-0.17
Carry over	0.02	-0.085	+0.085	-0.02
Total	-0.60	+4.44	-4.44	+4.44

(34)

Step 4:- Final moments.

$$M_{ab} = -0.6 \text{ kNm}$$

$$M_{ba} = +4.44 \text{ kNm}$$

$$M_{bc} = -4.44 \text{ kNm}$$

$$M_{cb} = +4.44 \text{ kNm}$$

$$M_{cd} = -4.44 \text{ kNm}$$

$$M_{dc} = +0.6 \text{ kNm}$$

