**Duration: 3 hours** 

## II B.Tech - I Semester – Regular Examinations - DECEMBER 2024

## DISCRETE MATHEMATICS AND GRAPH THEORY (Common for CSE, IT, AIML, DS)

Note: 1. This question paper contains two Parts A and B.

- 2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
- 3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
- 4. All parts of Question paper must be answered in one place.

| -             | -    | <br>-               |
|---------------|------|---------------------|
| BL – Blooms L | evel | CO – Course Outcome |

## $\mathbf{PART} - \mathbf{A}$

|      |  | BL | CO  |
|------|--|----|-----|
| 1.a) | Define Proposition.  | L1 | CO1 |
| 1.b) | What is the Difference between CNF and PCNF.               | L1 | CO1 |
| 1.c) | Let $Q(x)$ be the statement " $x < 2$ ." What is the       | L1 | CO2 |
|      | truth value of the quantification $\forall x Q(x)$ , where |    |     |
|      | the domain consists of all real numbers?                   |    |     |
| 1.d) | Explain existential quantifier.                            | L1 | CO2 |
| 1.e) | Define non-homogeneous recurrence relation of              | L2 | CO3 |
|      | order three.   |    |     |
| 1.f) | Solve $a_n + 4a_{n-1} = 2$ .                               | L2 | CO3 |
| 1.g) | Write Warshall's Algorithm.                                | L2 | CO4 |
| 1.h) | Define a Directed Graph.                                   | L1 | CO4 |
| 1.i) | Define a minimal spanning tree.                            | L1 | CO4 |
| 1.j) | Define Hamiltonian Graph.                                  | L1 | CO4 |

Max. Marks: 70

PART – B

|   |     |   | BL | CO  | Max.<br>Marks  |
|---|-----|---|----|-----|----------------|
|   |     | UNIT-I  |    |     | <b>IVIUIII</b> |
| 2 | a)  | Show that the proposition $(p \lor \neg q) \land$                     | L2 | CO1 | 5 M            |
|   | ,   | $(\neg p \lor \neg q) \lor q$ is a tautology.                         |    |     | •              |
|   | b)  |   | L3 | CO2 | 5 M            |
|   | - / | that $[(p \lor q) \to r] \Leftrightarrow [(p \to q) \land (p \to r)]$ |    |     |                |
|   |     | OR  |    |     |                |
| 3 | a)  | Construct the truth table of the compound                             | L2 | CO1 | 5 M            |
|   | ,   | proposition   |    |     |                |
|   |     | $(p \lor \neg q) \rightarrow (p \land q)$                             |    |     |                |
|   | b)  | Obtain CDNF of the following  | L3 | CO2 | 5 M            |
|   |     | $P \rightarrow ((P \rightarrow Q) \land \neg (\neg Q \lor \neg P))$   |    |     |                |
|   | I   | UNIT-II   |    |     | I              |
| 4 | a)  | Consider these statements "All lions are                              | L2 | CO1 | 5 M            |
|   |     | fierce", "Some lions do not drink coffee",                            |    |     |                |
|   |     | "Some fierce creatures do not drink                                   |    |     |                |
|   |     | coffee" Let $P(x)$ , $Q(x)$ , and $R(x)$ be the                       |    |     |                |
|   |     | statements "x is a lion", "x is fierce" and                           |    |     |                |
|   |     | "x drinks coffee" respectively. Assuming                              |    |     |                |
|   |     | that the domain consists of all creatures                             |    |     |                |
|   |     | express the statement in the argument                                 |    |     |                |
|   |     | using quantifiers and $P(x)$ , $Q(x)$ and $R(x)$ .                    |    |     |                |
|   | b)  | Assume that "For all positive integers n,                             | L3 | CO2 | 5 M            |
|   |     | if n is greater than 4, then $n^2$ is less than                       |    |     |                |
|   |     | $2^{n}$ , is true. Use universal modus ponens                         |    |     |                |
|   |     | to show that $100^2 < 2100$ .   |    |     |                |
|   | OR  |   |    |     |                |
|   |     |   |    |     |                |

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|----------|----|--|----|-----|--------------|
| 5        | a) | Show that the premises "A student in this                      | L3 | CO2 | 5 M          |
|          |    | class has not read the book," and                              |    |     |              |
|          |    | "Everyone in this class passed the first                       |    |     |              |
|          |    | exam" imply the conclusion "Someone                            |    |     |              |
|          |    | who passed the first exam has not read                         |    |     |              |
|          |    | the book."   |    |     |              |
|          | b) | Use contraposition show that if $x$ and $y$                    | L2 | CO1 | 5 M          |
|          |    | are integers and both $xy$ and $x + y$ are                     |    |     |              |
|          |    | even, then both <i>x</i> and <i>y</i> are even.                |    |     |              |
| UNIT-III |    |  |    |     |              |
| 6        | a) | Solve the recurrence relation                                  | L2 | CO1 | 5 M          |
|          |    | $a_n = 7a_{n-1} - 10a_{n-2}$ with $a_0 = 2$ and                |    |     |              |
|          |    | $a_1 = 3.$   |    |     |              |
|          | b) | Solve the recurrence relation of                               | L3 | CO3 | 5 M          |
|          |    | Fibonacci sequence of numbers                                  |    |     |              |
|          |    | $F_{n+2} = F_{n+1} + F_n$ for $n \ge 0$ given that $F_0 = 0$ , |    |     |              |
|          |    | $F_1=1.$   |    |     |              |
| -        |    | OR   | L  |     |              |
| 7        | a) | Solve the following recurrence relation                        | L3 | CO3 | 5 M          |
|          |    | using characteristic roots.                                    |    |     |              |
|          |    | $a_n + 4a_{n-1} + 6a_{n-2} = 0$ and $a_0 = 2$ , $a_1 = -7$ .   |    |     |              |
|          | b) | Solve  | L3 | CO3 | 5 M          |
|          |    | $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0,$                  |    |     |              |
|          |    | for $n \ge 3$ .  |    |     |              |
| UNIT-IV  |    |  |    |     |              |
| 8        | a) | Draw the Hasse diagram representing the                        | L2 | CO1 | 5 M          |
|          |    | positive divisors of 36.                                       |    |     |              |
|          | b) | Show that the following graphs are                             | L4 | CO4 | 5 M          |
|          |    | isomorphic.  |    |     |              |
|          |    | 1  |    |     |              |

|    |  | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$                 |    |     |      |
|----|--|---|----|-----|------|
|    |  | $G_1$ $G_2$   |    |     |      |
|    |  | OR  | T  | 1   |      |
| 9  | a)   | Show that congruence modulo m is an equivalence relation on integers. | L2 | CO1 | 5 M  |
|    | b)   | Draw the Hasse diagram representing the                               | L4 | CO4 | 5 M  |
|    |  | partial ordering {(a, b)/a divides b}                                 |    |     |      |
|    |  | on {1, 2, 3, 4, 6, 8, 12}.  |    |     |      |
|    |  | UNIT-V  | 1  |     |      |
| 10 | a)   | Write about Euler's circuit and                                       | L2 | CO1 | 5 M  |
|    |  | Hamiltonian cycle with suitable examples.                             |    |     |      |
|    | b)   | Explain DFS algorithm to find spanning                                | L4 | CO4 | 5 M  |
|    |  | tree of a graph with suitable example.                                |    |     |      |
|    |  | OR  |    |     |      |
| 11 | Sho  | ow step by step Kruskal's algorithm on the                            | L4 | CO4 | 10 M |
|    | foll   | owing connected weighted graph and also                               |    |     |      |
|    | calculate sum of the weights of the minimal spanning tree? |   |    |     |      |
|    |  |   |    |     |      |
|    |  |   |    |     |      |