	UNIT-IV			
8	Evaluate $\int_C \frac{e^z}{(z-1)(z+3)}dz$ where C is the circle $\lvert z\rvert=\frac{3}{2}$ using Cauchy's integral formula.	L4	CO5	10 M
	OR			
9	Expand $f(z)=\frac{1}{(z-1)(z-3)}$ in Laurent's series expansion for $i)\ \lvert z\rvert<1$ $ii)\ 1<\lvert z\rvert<3$ $iii)\lvert z\rvert>3$	L3	CO3	10 M
	UNIT-V			
10	Evaluate $\int_C \frac{e^z}{(z^2+\pi^2)^2}dz$ where C is the circle $\lvert z\rvert=4$ using Cauchy's residue theorem.	L3	CO3	10 M
	OR			
11	Using calculus of residues show that $\int_0^{\pi}\frac{d\theta}{a+b\cos\theta}=\frac{\pi}{\sqrt{a^2-b^2}}$, $(a>b>0)$	L4	CO5	10 M

Code: 23BS1302

II B.Tech - I Semester – Regular Examinations - DECEMBER 2024

NUMERICAL METHODS AND COMPLEX VARIABLES
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Under what conditions, Newton-Raphson's method fails to find root of an equation?	L2	CO1
1.b)	Prove that $(1+\Delta)(1-\nabla)=1$	L2	CO1
1.c)	Using Newton's forward interpolation formula, write the formula for 2nd order derivative.	L3	CO2
1.d)	Write Simpson's ⅜ rule.	L1	CO2
1.e)	Verify whether the function $u(x,y)=e^x\cos y$ is harmonic or not?	L2	CO3
1.f)	Write Cauchy-Riemann (C-R) equations in cartesian form.	L2	CO1
1.g)	State Cauchy's integral theorem.	L2	CO1
1.h)	Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in Taylor's series about $z=0$	L3	CO3
1.i)	Find the residue of $f(z)=\frac{z^3}{z^2-1}$ at $z=1$.	L3	CO3

1.j)	Write the zeros and the poles of $f(z)=\frac{z^4+1}{z^3(1-z)}$	L2	CO1

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Apply Bisection method to find a real root of the equation $x^3-x^2-1=0$ correct to two decimal places.	L3	CO2	5 M
	b)	Using regula falsi method, find a real root of an equation $e^x \tan x = 1$	L3	CO2	5 M
OR					
3	a)	Given $\log_{10}^{654}=2.8156$, $\log_{10}^{658}=2.8182$, $\log_{10}^{659}=2.8189$, $\log_{10}^{661}=2.8202$ Estimate the value of $\log_{10}^{656}$ using Lagrange's interpolation formula.	L4	CO4	5 M
	b)	Estimate $f(3.75)$ using Newton's forward interpolation formula from the following table	L4	CO4	5 M

x	2.5	3.0	3.5	4.0
$f(x)$	24.1	22.0	20.2	18.6

UNIT-II					
4		Apply Runge-Kutta (R-K) fourth order method, to find $y(0.2)$ and $y(0.4)$. Given that $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$, $y(0)=1$.	L3	CO2	10 M
OR					
5	a)	Evaluate $\int_0^1 x^3\,dx$ with three sub intervals by using Trapezoidal rule.	L4	CO4	5 M
	b)	Estimate the approximate value of the integral $\int_0^{\pi/2}\sqrt{\cos\theta}\,d\theta$ with step size $h=\frac{\pi}{12}$ using Simpson's ⅓ rule.	L4	CO4	5 M
UNIT-III					
6	a)	Show that the function $f(z)=\sqrt{\lvert xy\rvert}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at that point.	L4	CO5	5 M
	b)	Construct an analytic function $f(z)=u+iv$ whose real part is $u=4xy-3x+2$.	L3	CO3	5 M
OR					
7		Construct an analytic function $f(z)=u+iv$ whose imaginary part is $e^x\,(x\sin y+y\cos y)$	L3	CO3	10 M

Subject Code: 23BS1302 PVP23

II B.Tech. I Semester Regular Examinations - DECEMBER-2024

NUMERICAL METHODS and COMPLEX VARIABLES

Key & Scheme of Evaluation

1.a) By Newton-Raphson's method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for $n = 0, 1, 2, \cdots\cdots$ — 1

This method fails when $f'(x) = 0$ to find root of an equation $f(x) = 0$. — 1

1.b) We know that $\Delta = E - 1$ and $\nabla = 1 - E^{-1}$ — 1

Then $(1+\Delta)(1-\nabla) = EE^{-1} = 1$ — 1

1.c) $\left(\frac{d^2y}{dx^2}\right)_{x = x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\cdots\right]$ — 2

1.d) $\int_a^b y\,dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 \cdots\cdots) + 2(y_3 + y_6 + y_9 \cdots\cdots)\right]$ — 2

1.e) $u_x = e^x \cos y\,,\ u_{xx} = e^x \cos y$

$u_y = -e^x \sin y\,,\ u_{yy} = -e^x \cos y$ — 1

Then, $u_{xx} + u_{yy} = e^x \cos y - e^x \cos y = 0$

$\Rightarrow u$ is harmonic. — 1

1.f) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}\quad (u_x = v_y)$ and — 1

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}\quad (v_x = -u_y)$ — 1

1.g) If $f(z)$ is analytic within and on a closed curve C, then $\int_C f(z)\,dz = 0$. — 2

1.h) $\frac{1}{(z-1)(z-2)} = \frac{1}{z-2} - \frac{1}{z-1}$ — 1

$= -\frac{1}{2}\left(1 - \frac{z}{2}\right)^{-1} + (1-z)^{-1}$

$= -\frac{1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{4} + \cdots\cdots\right) + (1 + z + z^2 + \cdots\cdots)$ — 1

1.i) $\text{Res} f(1) = \lim_{z\to 1}(z-1)f(z)$ — 1

$= \lim_{z\to 1}\frac{z^3}{z+1} = \frac{1}{2}$ — 1

1.j) Zeros of $f(z)$ are $\frac{1\pm i}{\sqrt{2}}, \frac{-1\pm i}{\sqrt{2}}$ — 1

Poles of $f(z)$ are $0, 1$ — 1

2.a) Let $f(x) = x^3 - x^2 - 1 = 0$

$f(1.4) = -0.216 < 0$

$f(1.5) = 0.125 > 0$

Root lies between 1.4 and 1.5 — 2

a	b	$f(a)$	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$
1.4	1.5	-0.216	0.125	1.45	-0.0539
1.45	1.5	-0.0539	0.125	1.475	0.0334
1.45	1.475	-0.0539	0.0334	1.4625	-0.0107
1.4625	1.475	-0.0107	0.0334	1.4688	0.0113

— 2

$\therefore$ 1.4688 is a root for the given equation corrected to two decimal places. — 1

Note:-Marks can be awarded for alternate procedure(s) or values (s) and/or root(s) or solution(s)

2.b) Let $f(x) = e^x \tan x - 1 = 0$

$f(0.5) = -0.0993 < 0$

a	b	$f(a)$	$f(b)$	$c = \frac{af(b)-bf(a)}{f(b)-f(a)}$	$f(c)$
0.5	0.6	-0.0993	0.2466	0.5287	-0.0088
0.5287	0.6	-0.0088	0.2466	0.5312	-0.0006
0.5312	0.6	-0.0006	0.2466	0.5314	0

2

$f(0.6) = 0.2466 > 0$

Root lies between 0.5 and 0.6 2

∴ 0.5314 is a root for the given equation. 1

Note:-Marks can be awarded for alternate procedure(s) or values (s) and/or root(s) or solution(s)

3.a) Let

$x_0 = 654 \quad x_1 = 658 \quad x_2 = 659 \quad x_3 = 661$

$y_0 = 2.8156 \quad y_1 = 2.8182 \quad y_2 = 2.8189 \quad y_3 = 2.8202$ 1

Lagrange's Interpolation formula,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \cdots\cdots + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$ 2

$$= \frac{(-2)(-3)(-5)}{(-4)(-5)(-7)}(2.8156) + \frac{(2)(-3)(-5)}{(4)(-1)(-3)}(2.8182) + \frac{(2)(-2)(-5)}{(5)(1)(-2)}(2.8189) + \frac{(2)(-2)(-3)}{(7)(3)(2)}(2.8202)$$

2

$= 2.8168$

3.b)

x	y	Δ	Δ^2	Δ^3
x_0 **2.5**	y_0 **24.1**			
		-2.1		
x_1 3	y_1 22		**0.3**	
		-1.8		**-0.1**
x_2 **3.5**	y_2 20.2		0.2	
		-1.6		
x_3 **4**	y_3 18.6			

2

By Newton's forward interpolation formula

$$y = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots\cdots$$

where $p = \frac{x-x_0}{h} = \frac{3.75-2.5}{0.5} = 2.5$ 2

$$= 24.1 + 2.5(-2.1) + \frac{2.5(1.5)}{2}(0.3) + \frac{2.5(1.5)(0.5)}{6}(-0.1)$$

$= 19.3813$ 1

Note:- Marks can be awarded for alternate procedure(s) or method(s) and/or solution(s)

4) Given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2} \ni y(0) = 1$

Here $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$, $x_0 = 0$, $y_0 = 1$ $\quad h = 0.2$ 2

$x_1 = x_0 + h = 0.2 \qquad x_2 = x_1 + h = 0.4$

To find y_1:

$k_1 = h.f(x_0, y_0) = (0.2).f(0,1) = 0.2$

$k_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.2).f(0.1, 1.1) = 0.1967$

$k_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.2).f(0.1, 1.0984) = 0.1967$ 3

$k_4 = h.f(x_0 + h, y_0 + k_3) = (0.2).f(0.2, 1.1967) = 0.1891$

	$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.2 + 0.3934 + 0.3934 + 0.1891) = 0.1959$	1
	$\therefore y_1 = y(0.2) = y_0 + k = 1 + 0.1959 = 1.1959$	
	To find y_2:	
	$k_1 = h.f(x_1, y_1) = (0.2).f(0.2, 1.1959) = (0.2).(0.9456) = 0.1891$ $k_2 = h.f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right) = (0.2).f(0.3, 1.2904) = 0.1795$ $k_3 = h.f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right) = (0.2).f(0.3, 1.2857) = 0.1793$ $k_4 = h.f(x_1 + h, y_1 + k_3) = (0.2).(0.844) = 0.1688$	3
	$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.1891 + 0.359 + 0.3586 + 0.1688) = 0.1793$	1
	$\therefore y_2 = y_1 + k = 1.1959 + 0.1793 = 1.3752$	
5.a)	Here step size $h = 1/3$	

x	0	1/3	2/3	1
$y = x^3$	0	1/27	8/27	1
	y_0	y_1	y_2	y_3

2

By Trapezoidal rule, $\int_0^1 y\, dx = \frac{h}{2}[(y_0 + y_3) + 2(y_1 + y_2)]$ 2

$$= \frac{(1/3)}{2}\left[(0+1) + 2\left(\frac{1}{27} + \frac{8}{27}\right)\right]$$

$$= \frac{1}{6}\left[1 + \frac{18}{27}\right] = 0.2778$$ 1

5.b)

θ	0	$\frac{\pi}{12}$	$\frac{2\pi}{12}$	$\frac{3\pi}{12}$	$\frac{4\pi}{12}$	$\frac{5\pi}{12}$	$\frac{6\pi}{12} = \frac{\pi}{2}$
$y = \sqrt{\cos\theta}$	1	0.9828	0.9306	0.8409	0.7071	0.5087	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

2

By Simpson's ⅓ rd rule, $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$ 2

$$= \frac{\left(\frac{\pi}{12}\right)}{3}[(1+0) + 4(0.9828 + 0.8409 + 0.5087) + 2(0.9306 + 0.7071)]$$

$$= \frac{\pi}{36}[13.605] = 1.1873$$ 1

6.a) Let $z = x + iy$ Given $f(z) = xy + iy$

Here $u(x, y) = \sqrt{|xy|}$, $v(x, y) = 0$

$$\left(\frac{\partial u}{\partial x}\right)_{(0,0)} = lim_{x\to 0}\ \frac{u(x,0) - u(0,0)}{x} = lim_{x\to 0}\ \frac{0-0}{x} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)_{(0,0)} = lim_{y\to 0}\ \frac{u(0,y) - u(0,0)}{y} = lim_{y\to 0}\ \frac{0-0}{y} = 0$$

$$\left(\frac{\partial v}{\partial x}\right)_{(0,0)} = lim_{x\to 0}\ \frac{v(x,0) - v(0,0)}{x} = lim_{x\to 0}\ \frac{0-0}{x} = 0$$

$$\left(\frac{\partial v}{\partial y}\right)_{(0,0)} = lim_{y\to 0}\ \frac{v(0,y) - v(0,0)}{y} = lim_{y\to 0}\ \frac{0-0}{y} = 0$$ 2

$\left(\frac{\partial u}{\partial x}\right)_{(0,0)} = \left(\frac{\partial v}{\partial y}\right)_{(0,0)}$ and $\left(\frac{\partial v}{\partial x}\right)_{(0,0)} = -\left(\frac{\partial u}{\partial y}\right)_{(0,0)}$

$\therefore f(z)$ satisfies C-R equations at the origin.

Now, $f'(0) = lim_{z\to 0}\ \frac{f(z) - f(0)}{z - 0} = lim_{z\to 0}\ \frac{f(z)}{z}$ 1

Suppose $z \to 0$ along the path $y = mx$

Then $\lim_{\substack{y\to mx \\ x\to 0}} \frac{f(z)}{z} = \lim_{\substack{y\to mx \\ x\to 0}} \frac{\sqrt{|xy|}}{x + iy}$

$$= lim_{x\to 0}\ \frac{\sqrt{|x \cdot mx|}}{x + imx}$$

$$= lim_{x\to 0} \frac{\sqrt{|mx^2|}}{x+imx}$$

$$= lim_{x\to 0} \frac{x\sqrt{|m|}}{x(1+im)}$$

$$= lim_{x\to 0} \frac{\sqrt{|m|}}{1+im} = \frac{\sqrt{|m|}}{1+im} \text{ which is not unique}$$

$\Rightarrow f(z)$ is not analytic at the origin. 2

6.b) Let $f(z) = u + iv$ be an analytic function where $u(x,y) = 4xy - 3x + 2$

Then $\frac{\partial u}{\partial x} = 4y - 3$ and $\frac{\partial u}{\partial y} = 4x$ 2

Since, $f(z)$ is an analytic function, $f(z)$ satisfies C-R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\therefore f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$ 2

$$= (4y - 3) - i(4x)$$

By Milne-Thomson's method, replace x by z and y by 0 to obtain

$$f'(z) = (-3) - i(4z)$$

$$\Rightarrow f(z) = -3z - 2iz^2 + A$$ 1

7) Let $f(z) = u + iv$ be an analytic function where $v(x,y) = e^x(x\sin y + y\cos y)$

Then $\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y + \sin y)$ and 2

$\frac{\partial v}{\partial y} = e^x(x\cos y + \cos y - y\sin y)$ 2

Since, $f(z)$ is an analytic function, $f(z)$ satisfies C-R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\therefore f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$$ 2

$$= e^x(x\cos y + \cos y - y\sin y) + ie^x(x\sin y + y\cos y + \sin y)$$ 2

By Milne-Thomson's method, replace x by z and y by 0 to obtain

$$f'(z) = e^z(z+1) - i(0)$$

$$\Rightarrow f(z) = ze^z + A$$ 2

Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s)..

8) Clearly $z = 1, -3$ are singular points of $\frac{e^z}{(z-1)(z+3)}$ of which $z = 1$ only lies inside $C: |z| = \frac{3}{2}$. 3

Here $f(z) = \frac{e^z}{z+3}$, $a = 1$ 1

By Cauchy's integral formula, $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a} dz$ 2

$$\Rightarrow f(1) = \frac{1}{2\pi i}\int_C \frac{\left(\frac{e^z}{z+3}\right)}{z-1} dz$$ 2

$$\Rightarrow \int_C \frac{e^z}{(z-1)(z+3)} dz = 2\pi i \cdot f(1) = 2\pi i\left(\frac{e}{4}\right) = \frac{\pi e i}{2}$$ 2

Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).

Q	Solution	Marks
9)	$f(z) = \dfrac{1}{(z-1)(z-3)} = \dfrac{1}{2}\left(\dfrac{1}{z-3} - \dfrac{1}{z-1}\right)$	2
	$\underline{\lvert z\rvert < 1:}$ $f(z) = \frac{1}{2}\left[-\frac{1}{3}\left(1-\frac{z}{3}\right)^{-1} + (1-z)^{-1}\right]$	1
	$= \dfrac{1}{2}\left[-\dfrac{1}{3}\left(1+\left(\dfrac{z}{3}\right)+\left(\dfrac{z}{3}\right)^2+\cdots\cdots\right)+(1+z+z^2+\cdots\cdots)\right]$	1
	$\underline{1 < \lvert z\rvert < 3:}$ $f(z) = \frac{1}{2}\left[-\frac{1}{3}\left(1-\frac{z}{3}\right)^{-1} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}\right]$	1
	$= \dfrac{1}{2}\left[-\dfrac{1}{3}\left(1+\left(\dfrac{z}{3}\right)+\left(\dfrac{z}{3}\right)^2+\cdots\cdots\right)-\dfrac{1}{z}\left(1+\left(\dfrac{1}{z}\right)+\left(\dfrac{1}{z}\right)^2+\cdots\cdots\right)\right]$	2
	$\underline{\lvert z\rvert > 3:}$ $f(z) = \frac{1}{2}\left[\frac{1}{z}\left(1-\frac{3}{z}\right)^{-1} - \frac{1}{z}\left(1-\frac{1}{z}\right)^{-1}\right]$	1
	$= \dfrac{1}{2}\left[\dfrac{1}{z}\left(1+\left(\dfrac{3}{z}\right)+\left(\dfrac{3}{z}\right)^2+\cdots\cdots\right)-\dfrac{1}{z}\left(1+\left(\dfrac{1}{z}\right)+\left(\dfrac{1}{z}\right)^2+\cdots\cdots\right)\right]$	2
10)	Let $f(z) = \frac{e^z}{(z^2+\pi^2)^2}$	
	Clearly, $z = \pm i\pi$ are singular points order 2 and both lies inside the circle $C: \lvert z\rvert = 4$	2
	$\lim\limits_{z\to i\pi}(z-i\pi)^2 f(z) = \lim\limits_{z\to i\pi}\dfrac{e^z}{(z+i\pi)^2} = \dfrac{e^{i\pi}}{(2i\pi)^2} \neq 0 \quad \left(=\dfrac{1}{4\pi^2}\right)$	
	$\therefore z = i\pi$ is a pole order 2.	1
	Then, $\text{Res} f(i\pi) = \frac{1}{(2-1)!} \cdot lim_{z\to i\pi}\left\{\frac{d}{dz}[(z-i\pi)^2 f(z)]\right\}$	
	$= lim_{z\to i\pi}\left\{\frac{d}{dz}\left[\frac{e^z}{(z+i\pi)^2}\right]\right\}$	
	$= lim_{z\to i\pi}\left\{\frac{((z+i\pi)^2)e^z - 2e^z(z+i\pi)}{(z+i\pi)^4}\right\} = \frac{2-2i\pi}{(2i\pi)^3}$	2
	$\lim\limits_{z\to -i\pi}(z+i\pi)^2 f(z) = \lim\limits_{z\to -i\pi}\dfrac{e^z}{(z-i\pi)^2} = \dfrac{e^{-i\pi}}{(-2i\pi)^2} \neq 0 \quad \left(=\dfrac{1}{4\pi^2}\right)$	
	$\therefore z = -i\pi$ is a pole order 2.	1
	Then, $\text{Res} f(-i\pi) = \frac{1}{(2-1)!} \cdot lim_{z\to -i\pi}\left\{\frac{d}{dz}[(z+i\pi)^2 f(z)]\right\}$	
	$= lim_{z\to -i\pi}\left\{\frac{d}{dz}\left[\frac{e^z}{(z-i\pi)^2}\right]\right\}$	
	$= lim_{z\to -i\pi}\left\{\frac{((z-i\pi)^2)e^z - 2e^z(z-i\pi)}{(z-i\pi)^4}\right\} = \frac{2+2i\pi}{(-2i\pi)^3} = -\left[\frac{2+2i\pi}{(2i\pi)^3}\right]$	2
	$\therefore$ By Cauchy-Residue theorem,	
	$\int_C f(z)dz = 2\pi i\,[\text{Sum of residues at singular points within C}]$	1
	$= 2\pi i\,[\text{Res} f(i\pi) + \text{Res} f(-i\pi)]$	
	$= 2\pi i\left[\dfrac{2-2i\pi}{(2i\pi)^3} - \dfrac{2+2i\pi}{(2i\pi)^3}\right]$	
	$= 2\pi i\left[\dfrac{2-2i\pi}{(2i\pi)^3} - \dfrac{2+2i\pi}{(2i\pi)^3}\right]$	
	$= 2\pi i\left[\dfrac{-4i\pi}{(2i\pi)^3}\right] = \dfrac{i}{\pi}$	1
11)	Put $z = e^{i\theta}$. Then $\cos\theta = \frac{z^2+1}{2z}$, $d\theta = \frac{dz}{iz}$	2
	$\therefore \int_0^{2\pi}\dfrac{d\theta}{a+b\cos\theta} = \int_C \dfrac{(dz/_{iz})}{\left[a+b\left(\frac{z^2+1}{2z}\right)\right]}$	
	$= \frac{2}{i}\int_C \frac{dz}{bz^2+2az+b}$	
	$= \frac{2}{bi}\int_C \frac{dz}{z^2+\frac{2a}{b}z+1} = \frac{2}{bi}\int_C f(z)dz$ where $f(z) = \frac{1}{z^2+\frac{2a}{b}z+1}$	2

The singular points of $f(z)$ are $z = \alpha,\ \beta$ where $\alpha = \frac{-a+\sqrt{a^2-b^2}}{b},\ \beta = \frac{-a-\sqrt{a^2-b^2}}{b}$ Since $a > b > 0,\ \lvert\alpha\rvert < 1$ and $\lvert\beta\rvert > 1$	2
Also $z = \alpha$ is a simple pole. $\text{Res}f(\alpha) = \lim_{z\to\alpha}(z-\alpha)f(z)$ $= \lim_{z\to\alpha}(z-\alpha)\cdot\frac{1}{(z-\alpha)(z-\beta)}$ $= \frac{1}{\alpha-\beta} = \frac{b}{2\sqrt{a^2-b^2}}$	1
∴ By Cauchy-Residue theorem, $\int_C f(z)dz = 2\pi i$ [Sum of residues at singular points within C] $= 2\pi i\,[\text{Res}f(\alpha)] = 2\pi i\left[\frac{1}{\alpha-\beta}\right] = \frac{2\pi i}{\alpha-\beta}$	2
$\therefore \int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2}{bi}\int_C f(z)dz$ $= \frac{2}{bi}\left(\frac{2\pi i}{\alpha-\beta}\right)$ $= \frac{4\pi}{b(\alpha-\beta)} = \frac{2\pi}{\sqrt{a^2-b^2}}$ $\Rightarrow \int_0^{\pi} \frac{d\theta}{a + b\cos\theta} = \frac{\pi}{\sqrt{a^2-b^2}}$	1

Note:- Marks can be awarded for alternate procedure(s) or method(s) and/or solution(s) and/or root(s).