PVP 23

Code: 23CS3301, 23AM3301, 23DS3301

II B.Tech - I Semester – Regular Examinations - DECEMBER 2024

ADVANCED DATA STRUCTURES AND ALGORITHM ANALYSIS

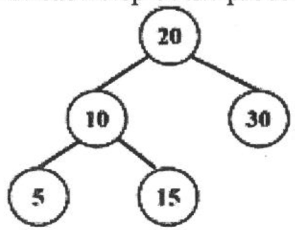
(Common for CSE, AIML, DS)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Given the growth rate of standard function as shown below, arrange them in ascending order of time complexity. i)n^3 ii) nlog(n/5) iii) 2^{n+1} iv) log(n+1) e) n/2	L2	CO1
1.b)	Given the AVL tree in figure, insert the node 27 to the tree and rebalance if the tree is unbalanced, mark the balance factors in each step of the process. 	L3	CO3
1.c)	Define heap data structure. What are the operations on heap.	L2	CO3
1.d)	Define the path and cycle of a graph. Give an example.	L1	CO3
1.e)	What is the time complexity of Quick Sort and Merge Sort algorithm.	L2	CO1

2

1.f)	Packets in a buffer on a router have arrived in the following order: i)Pkt12; t=0.2745s ;P5 ii)Pkt06 ; t=1.1376s ;P3 iii)Pkt21; t=1.8953s ;P2 iv)Pkt24 ; t=2.0012s ;P1 v)Pkt17; t= 1.003s; P4 P1(Highest),P2,P3,P4,P5(Lowest) indicate the priority of the packets and t is the time arrival of the packets. What is the order in which these packets are arranged in a priority queue?	L3	CO4
1.g)	Match the following: i) Knapsack a) $O(2^k)$ ii) Binary Search b) O(n log(n)) iii) Merge sort c) $O(n^3)$ iv) Matrix Multiplication d) O(log(n))	L2	CO4
1.h)	Give any two differences between Dijkstra's algorithm and Bellman-Ford algorithm.	L2	CO1
1.i)	Give examples for NP complete and NP hard problems.	L2	CO1
1.j)	What is the time complexity of n-queen problem?	L2	CO1

PART – B

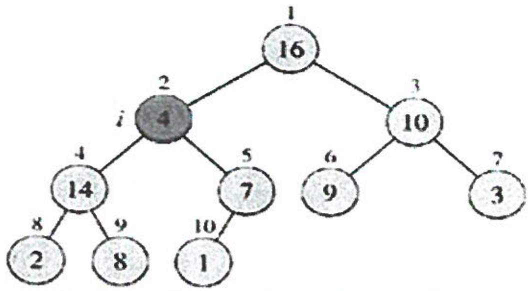
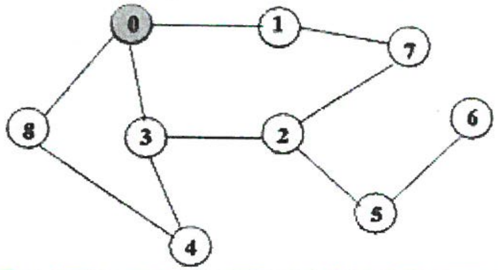
			BL	CO	Max. Marks
		UNIT-I			
2	a)	Construct an AVL tree by inserting the following elements in the given order. 37, 63, 108, 21, 85, 99.Now delete 108 form this tree. Rebalance if necessary.	L3	CO3	5 M

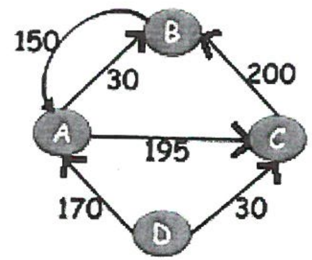
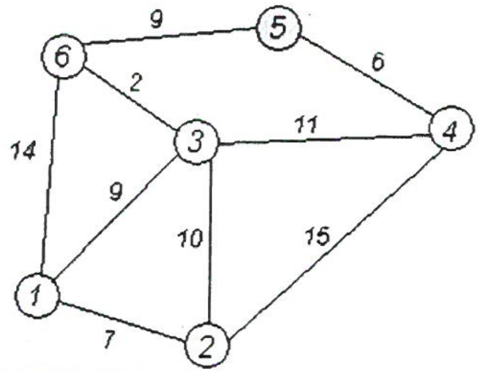
	b)	Define i) Time Complexity and ii) Space Complexity. For the given Algorithm, find the worst-case time complexity. Express your answer in big O notation. Show each step of the calculations.
		Algorithm sum(a,n)
		{
		s=0.0;
			L2	CO1	5 M

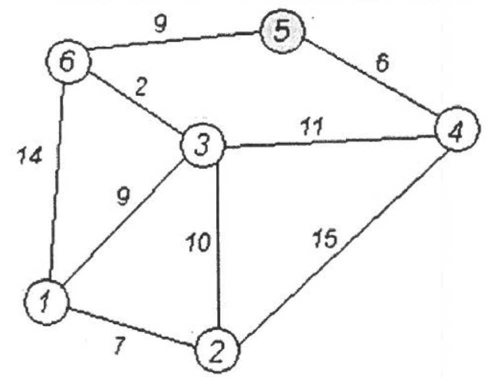
10

8/

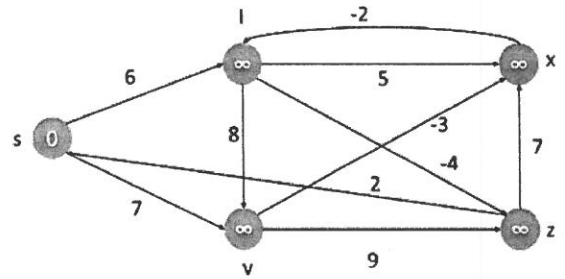
		for i=1 to n do
		s=s+a[i];
		return s;
		}			

		OR			
3		Compare and contrast B-trees with Binary trees and give their time complexities. Show the results of inserting the keys 6, 19, 17, 11, 3, 12, 8, 20, 22 in the given order into an empty B-Tree with minimum degree t=2.	L3	CO3	10 M
		UNIT-II			
4	a)	Using the property of Min Heap, Heapify the above give graph to get the min heap. Give the complete details step-by-step. 	L3	CO3	5 M
	b)	For the given graph, find the breadth first search traversal. Give the details of each step. 	L3	CO3	5 M
		OR			
5	a)	Define the following terms with examples i)Graph ii)Vertex of a graph	L2	CO1	5 M

		iii)Isolated graph iv)Forest v)Graph with negative weight. Mention what does negative weight indicate.			
	b)	For the given graph, write the adjacency matrix and adjacency list. 	L3	CO3	5 M
		UNIT-III			
6	a)	Given the sequence of numbers as follows: 23, 89, 45, 32, 9, 56, 84, 93, 66, 19, 83, 64. sort these numbers using Merge Sort.	L3	CO4	5 M
	b)	For the given graph, determine minimum spanning tree using Kruskal's algorithm. Write the complete details of the steps. 	L3	CO3	5 M
		OR			
7	a)	Find the shortest path between the vertex 6 and vertex 4 using all-pair shortest path algorithm for the following graph.	L3	CO3	5 M



	b)	Explain with example, the job sequencing scheduling based on first-cum-first-serve principle.	L2	CO4	5 M
UNIT-IV					
8	a)	Using Bellman-Ford algorithm, determine the shortest path between I and Z.	L3	CO4	5 M



	b)	Find an optimal solution using dynamic programming for the following 0/1 knapsack instance n=4, m=5, (p1,p2,p3,p4)=(4,5,3,7) and (w1,w2,w3,w4)=(2,3,1,4)	L3	CO4	5 M
OR					
9	a)	Give two sequences X=(A, B, C, B, D, A, B) and Y=(B, D, C, A, B, A). use dynamic programming approach to find the Longest Common Sequence(LCS) of X and Y.	L3	CO4	5 M
	b)	Explain the principle of optimality with example and state why this principle is necessary?	L2	CO4	5 M

UNIT-V					
10	a)	Explain the following with examples: P class; NP class and NP-complete class with examples.	L2	CO1	5 M
	b)	Explain n-queens problem using backtracking approach.	L2	CO4	5 M
OR					
11	a)	Explain graph coloring problem with an example.	L2	CO4	5 M
	b)	Write backtracking algorithm for sum of subsets problem.	L2	CO4	5 M

II B.Tech - I Sem - Reg Examination - DEC - 2024

Advanced Data Structures and Algorithm Analysis

Short Scheme

Part - A

1 (a) Ascending order [2M] (b) AVL Tree [2M] (c) heap definition - [1M] operation - [1M]

(d) path, cycle [1M] Example [1M] (e) Time complexity Quicksort [1M], Mergesort [1M]

(f) Maxheap [2M] (g) Matching [2M] (h) Differences [2M]

(i) Examples [2M] (j) Time complexity [2M]

Part B

UNIT - I

2 (a) AVL Tree Construction - 3M
Deletion - 2M

(b) Time complexity [1M], Space complexity [1M]
Algorithm time complexity [3M]

3 Differences - [4M]
B - Tree - [6M]

UNIT - II

4 (a) Min heap - [5M]
(b) BFS - [5M]

5 (a) (i) Graph [1M] (ii) vertex of Graph [1M] (iii) Isolated vertex [1M] (iv) Forest [1M]
(v) negative weight - [1M]

(b) Adjacency Matrix - 2 1/2
Adjacency List - 2 1/2

UNIT- III

6

(a) Merge sort - [5M]

(b) Kruskal's [5M]

(7)
(a) All pair's shortest path - [5M] (b) Job sequence dedline - 1m
Algorithm - 3m
Example - 1m

(8) (a) Bellman ford - [5M]

(b) Optimal solution - 2m, Knapsack solution vector - [3m]

(9) (a) LCS table - 5M

(b) principle optimality - [2m]
Applications - [2M]
Example - [1M]

10

(a) Definition P class, NP class - 2m
Applications - [3m]

(b) N-Queen's problem - [1M]
state space Tree - [3m]
Algorithm - [1m]

11

(a) Definition - 1m
State space Tree - 3m
Example - [1m]

(b) Algorithm - 3m
time complexity [1m]
Example - [1m]

ADS & AA SOLUTIONS.

PART - A

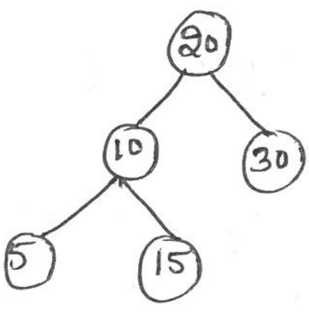
1.a) Given the growth rate of standard function as shown below, arrange them in ascending order of time complexity.

i) n^3 ii) $n\log(n/5)$ iii) 2^{n+1} iv) $\log(n+1)$
v) $n/2$.

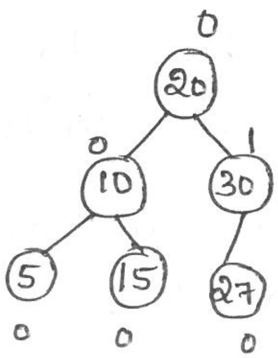
Sol:- i) $\log(n+1)$ ii) $n/2$ iii) $n\log(n/5)$
iv) n^3 v) 2^{n+1}

1.b) Given an AVL tree in figure, insert node 27 to tree and rebalance if tree is unbalanced, mark the balance factors in each step of the process.

Sol:-



Insert node (27) ⇒



1.c) Define heap data structure. What are the operations on Heap.

Sol:- A Heap data structure is a type of binary tree where the value of each parent node is either less than (or) equal to min heap (or) greater than equal to max heap to it's child nodes.

1, Insertion: Adds an element to the heap while maintaining the Heap property.

2, Deletion: Removes the root node by replacing root with last element.

(3, Heapify: Converts an unordered array into a valid Heap.

(4, Peek: Retrives, but does not remove, the root element of the heap.

---

d) Define the path and cycle of a graph. Give an Example.

Sol:- A path is a sequence of nodes that are connected by edges, where no vertex appears more than once.

A cycle is a path has a same first & last vertex, and no vertex appears more than once.

1.e) What is Time Complexity of Merge Sort and Quick sort ?

Sol:-

Merge Sort :-

Best case :- $O(n\log n)$
Average Case :- $O(n\log n)$
Worst case :- $O(n\log n)$

Quick sort :-

Best case :- $O(n\log n)$
Average Case :- $O(n\log n)$
Worst case :- $O(n^2)$

1.f) packets in a buffer on a router have arrived in the following order.

i) Pkt12 ; t = 0.2745s ; P5
ii) Pkt06 ; t = ~~0~~.1376s ; P3
iii) Pkt21 ; t = 1.8953S ; P2
iv) Pkt24 ; t = 2.0012S ; P1
v) Pkt ~~27~~ ; t = 1.003S ; P4

P1 (Highest) P2 P3 P4 P5 (Lowest) indicate the priority of the packet & t is time interval of packets. What is the order ?

Sol:- It is the Max Heap order for which the priority of the packets is arranged in a priority Queue, where P1 has a highest value and P5 has a Least value which can be as Max-Heap structure.

1.g) Match the following:-

Sol:-

i) KnapSack	a) $O(2^K)$
ii) Binary Search	b) $O(n \log n)$
iii) Merge sort	c) $O(n^3)$
iv) Matrix Multiplication	d) $O(\log n)$

1.h) Give any two differences b/w Dijkstra's Algorithm & Bellman ford Algorithm.

Sol:-

1, Edge weights:-

Dijkstra's algorithm only works with non-negative edge weights, while Bellman ford algorithm can handle negative weights.

2, Time complexity:-

Dijkstra's algorithm has lower time complexity than Bellman ford Algorithm.

i) Give examples of NP complete & NP Hard problems.

Sol:-

NP complete:-

Hamiliton Cycle, 3-SAT Problem, Travelling Salesman Problem

NP-Hard:-

Halting Problem, Travelling Salesman Problem.

1.j) What is time complexity of n-queens Problem.

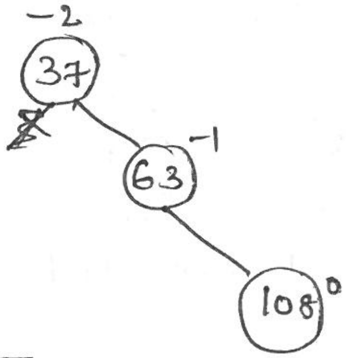
Sol:- Time complexity of n-Queens :- $O(N!)$

PART-B

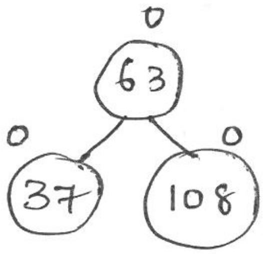
2a) Construct an AVL tree by inserting the following elements in given order. 37, 63, 108, 21, 85, 99
Now delete 108 from tree & rebalance it.

Sol:- Step-1 :-

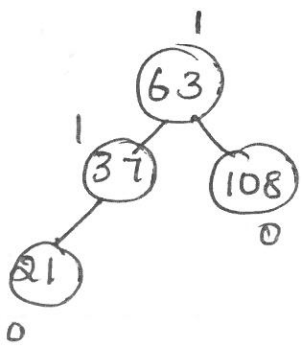
Insert (37, 63, 108) :-



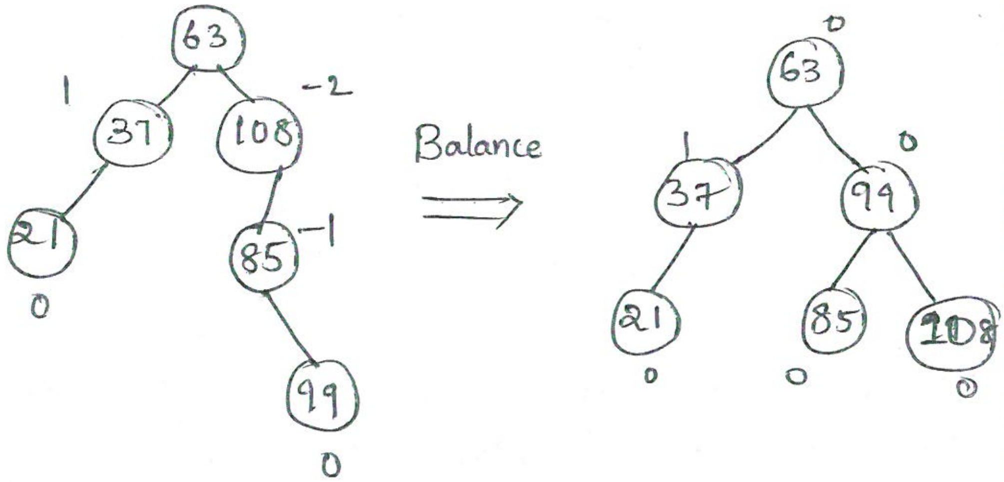
Balance the AVL Tree :-



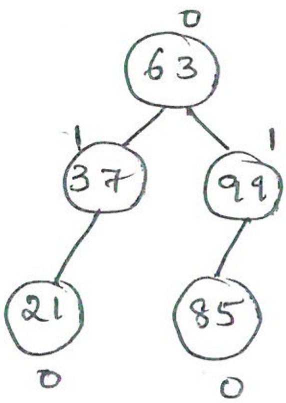
Step-2 :- Insert 21



Step-3 :- Insert (85, 99) ⇒

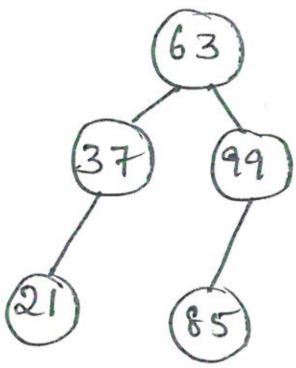


Delete (108) :-



∵ No rebalancing Needed Hence It is Balanced

final Resultant AVL Tree is :-



2 b) Define i) Time Complexity and ii) Space Complexity. For given Algorithm, find worst case time complexity. Express your answer in Big O Notation.

Sol:- Time complexity is defined as amount of time for an program how an algorithm takes time is called time complexity.

time complexity (t_p) = Compile time + run time

Space complexity is defined as amount of memory an algorithm uses to solve a problem.

Using Count variable Method:-

```
Algorithm sum (a, n) {  ----> 0
    s = 0.0;            ----> 1
    for i = 1 to n do   ----> n
        s = s + a[i];   ----> n
    return s;           ----> 1
}                       ----> 0
```

Time Complexity :- $1+1+n+n+0+0 = 3+2n$

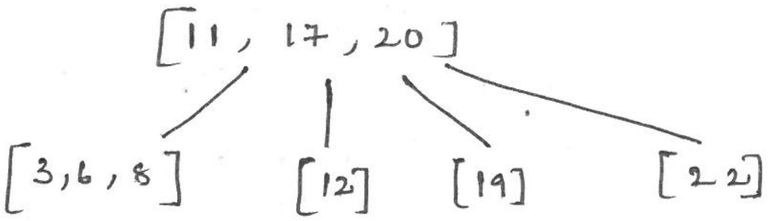
∴ Overall time Complexity :- $O(n)$.

3. Compare and contrast B-trees with Binary Trees and give their time complexity. Show the results of inserting keys 6, 19, 17, 11, 3, 12, 8, 20, 22 in given order into an empty B-tree with minimum degree $t = 2$.

Sol:-

B - Tree	Binary Tree.
(1) A balanced tree with nodes that can have multiple keys.	(1) A hiearchial structure where each node has atmost 2 children
2) Optimized for disk access; store more keys.	2) Not optimized for the data access.
3) Used in Database Systems, file System applications, requiring range queries.	3) Used in memory, simple data structure.
Each node atmost	can have $= 2t-1$ keys $= 3$ keys.
4) It is a Self Balanced Binary Search Tree.	(4) It has no Balancing Mechanisms

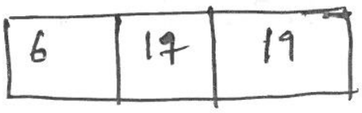
final Solution :-



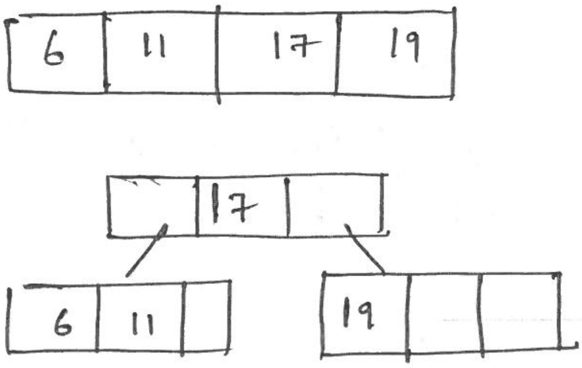
~~Unit~~

Proof :-

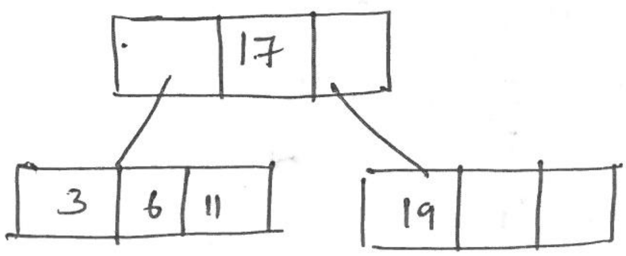
① Insert 6, 19, 17.



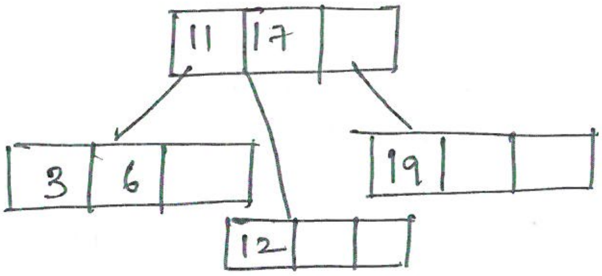
② Insert 11



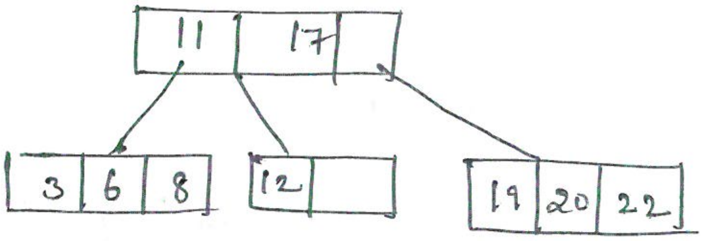
③ Insert 3,



④ Insert 12



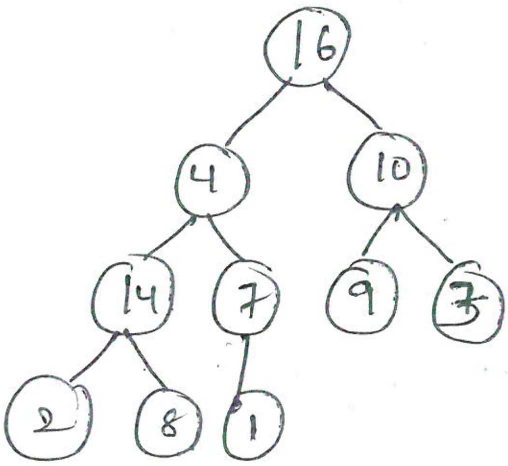
Insert (8, 20, 22) :-



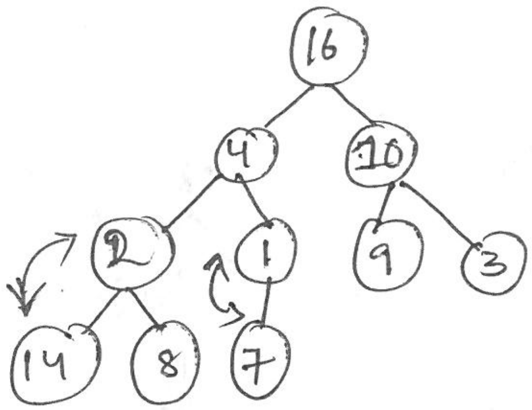
Unit - II

4a) Using property of Min heap, Heapify above graph to get min heap.

Sol:-



1


16
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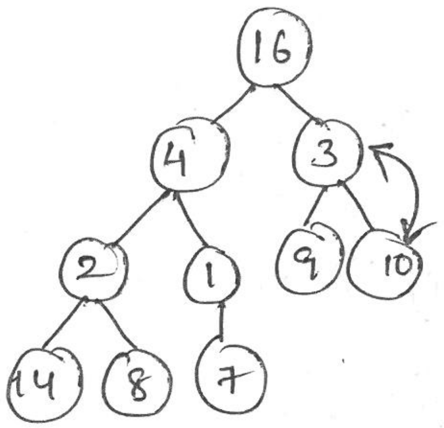
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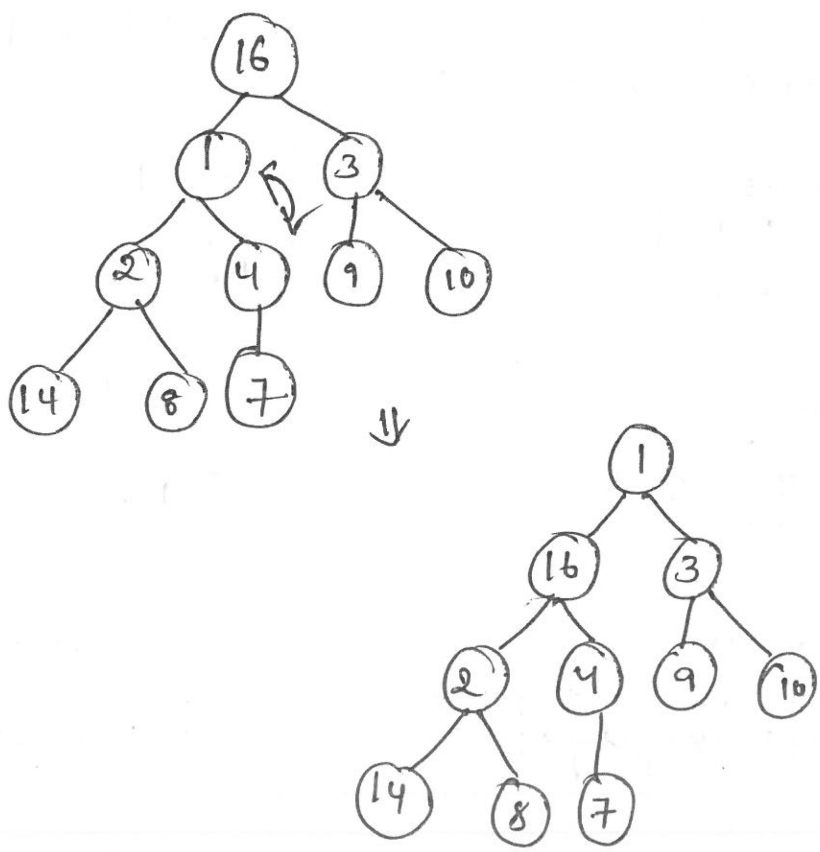
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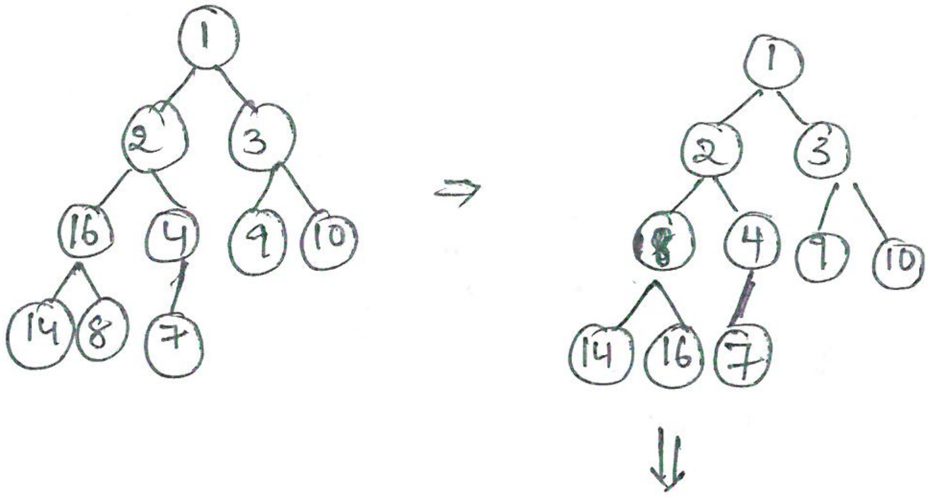
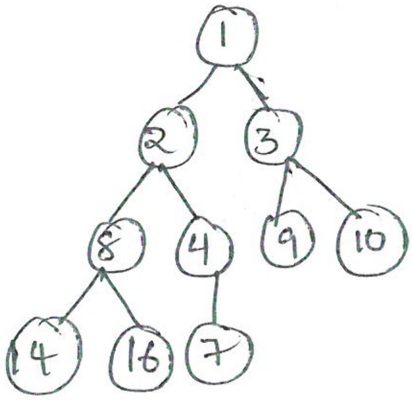


Hence min Heap is sastify all Nodes in Tree :-

∴



4 b) Sol:-

Source vertex :- 1 . .

Note: student can take any vertex as the source vertex

① Queue

1			

Visited

[1.]

② Queue

~~1~~	7	2.

Visited

[1, 7]

③ Queue

~~1~~	~~7~~	~~2~~	~~0~~	8	3

Visited

[1, 7, 0, 2]

(4) Queue

~~1~~	~~7~~	~~2~~	~~0~~	8	3	4

Visited
[1, 7, 0, 2 ; 8]

(5) Queue

~~1~~	~~7~~	2	~~0~~	~~8~~	~~3~~	4

Visited
[1, 7, 0, 2, 8, 3]

(6) Queue

~~1~~	~~7~~	~~2~~	~~0~~	~~8~~	~~3~~	~~4~~	5.

Visited
[1, 7, 0, 2, 8, 3, 4]

(7) Queue

~~1~~	~~7~~	~~2~~	~~0~~	~~8~~	~~3~~	~~4~~	~~5~~	~~6~~

Visited
[1, 7, 0, 2, 8, 3, 4, 5, 6]

---

5 a) Define the following terms with Examples.

i) Graph

Sol:- A graph is a collection of vertices connected by edges. It can be represented as $G = (V, E)$ and where V is set of vertices & E is edges.

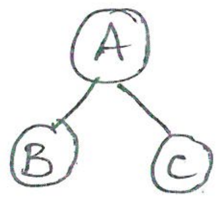
Ex:- $V = (A, B, C)$
$E = \{(A, B), (B, C)\}$.

O/P :- A — B — C.

ii) Vertex of a Graph :-

A vertex (or) node is a fundamental unit of a graph. It represents an entity in graph.

Ex :-

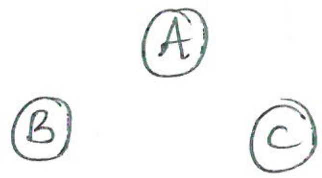


Here A, B, C are vertices.

iii) Isolated Graph :-

An isolated graph is a graph where no edges exist b/w any vertices.

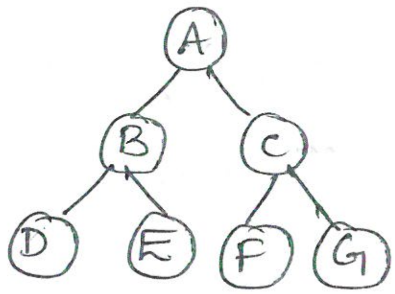
Ex :-



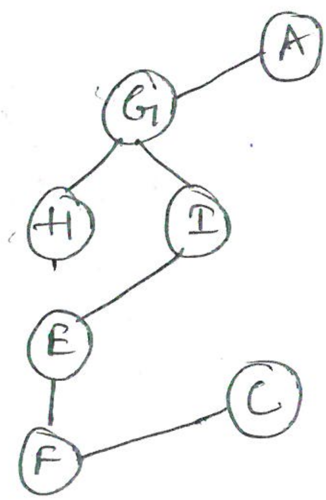
iv) Forest :-

A forest is collection of ~~collecti~~ one (or) more disjoint trees. A tree is acyclic connected Graph.

Ex :-



(or)

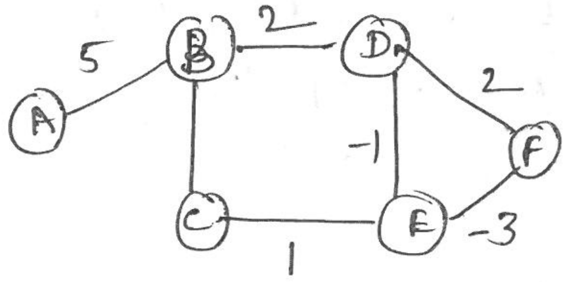


(or)

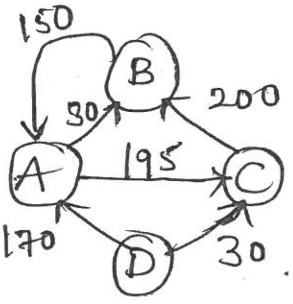
v) Graph with Negative Weights :-

A negative weight graph is a graph where some of the edges have a negative weight, which represents a cost (or) loss.

Ex :-



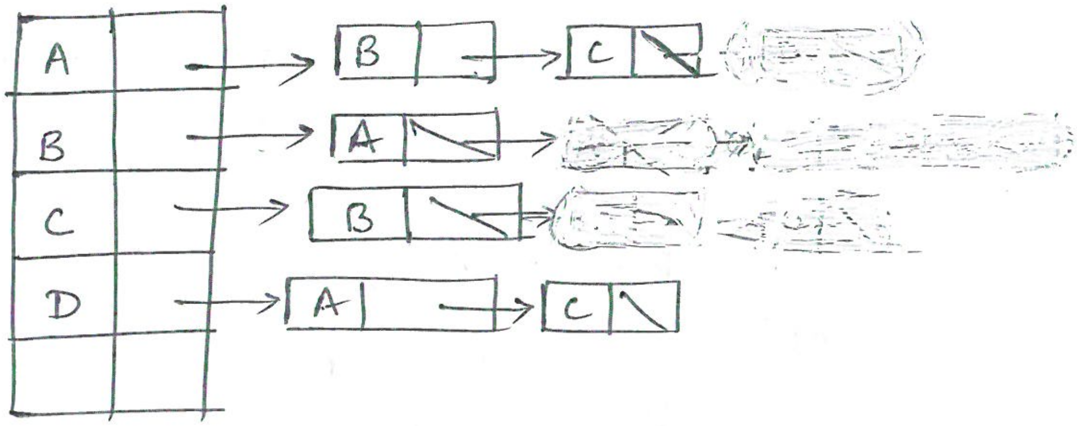
5b) for given graph, write the Adjancency matrix & adjacency List.



Sol:- Adjacency matrix :-

$$
\begin{array}{c|c}
 & \begin{array}{cccc} A & B & C & D \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \end{array} &
\begin{bmatrix} 0 & 30 & 195 & 0 \\ 150 & 0 & 200 & 0 \\ 195 & 0 & 0 & 0 \\ 170 & 0 & 30 & 0 \end{bmatrix}
\end{array}
$$

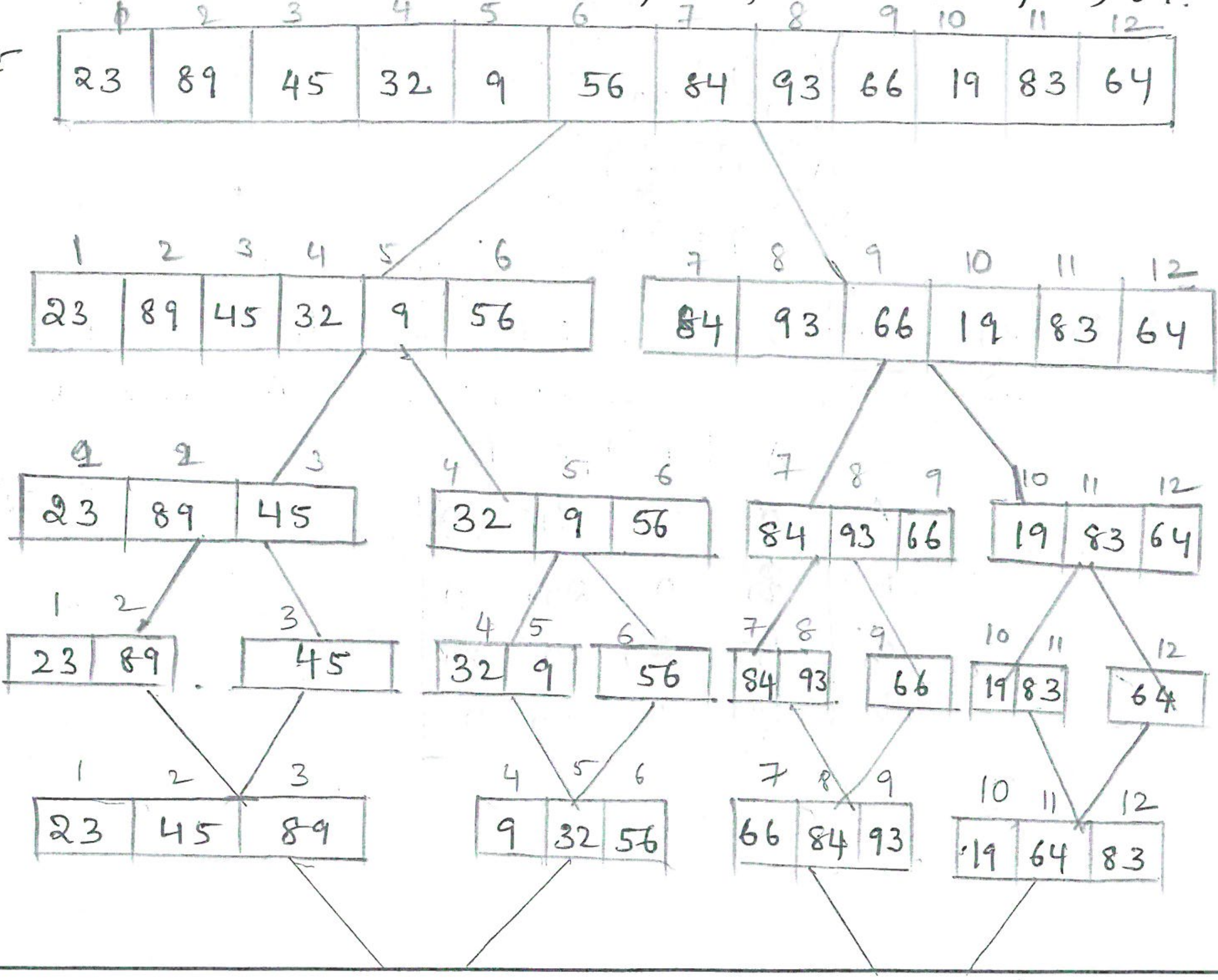
Adjacency List :-



Unit - III

a) Given the Sequence of numbers as follows:
23, 89, 45, 32, 9, 56, 84, 93, 66, 19, 83, 64.

Sol:-



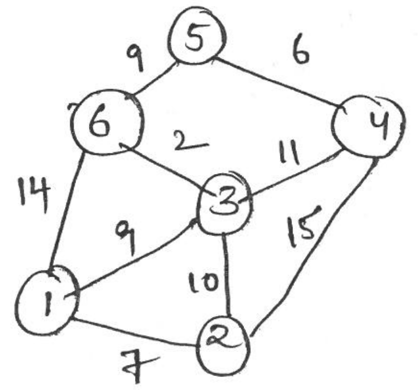
1	2	3	4	5	6
9	23	32	45	56	89

7	8	9	10	11	12
19	64	66	83	84	93

1	2	3	4	5	6	7	8	9	10	11	12
9	19	23	32	45	56	64	66	83	84	89	93

6 b) for given graph, determine Minimum Spanning Tree using Kruskal Algorithm.

Sol:-



~~Step~~ Write the Edges & it's weights in Ascending Order:-

✓ $\boxed{6 - 3 \Rightarrow 2}$

✓ $\boxed{5 - 4 \Rightarrow 6}$

✓ $\boxed{1 - 2 \Rightarrow 7}$

✓ $\boxed{6 - 5 \Rightarrow 9}$

✓ $\boxed{1 - 3 \Rightarrow 9}$

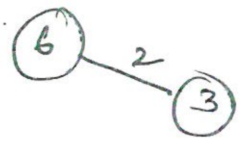
✗ $\boxed{2 - 3 \Rightarrow 10}$

✗ $3 - 4 \Rightarrow 11$

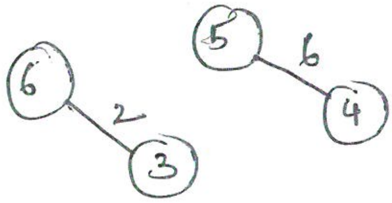
✗ $1 - 6 \Rightarrow 14$

✗ $2 - 4 \Rightarrow 15$.

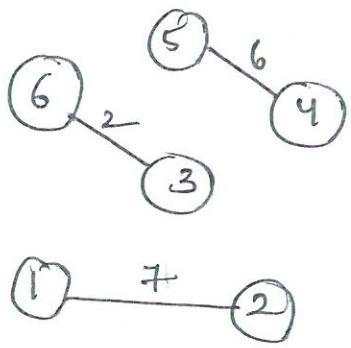
Step-1 :-



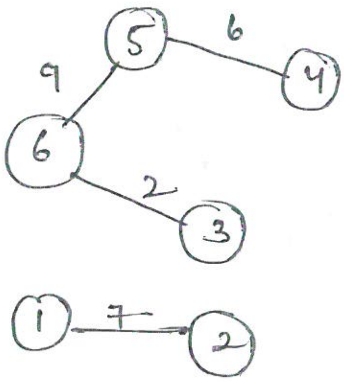
Step-2 :-



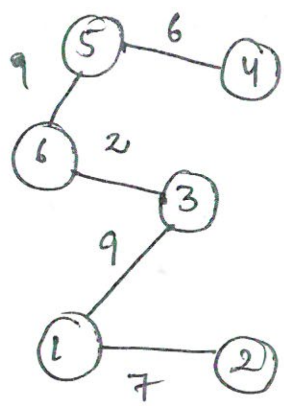
Step-3 :-



Step-4 :-



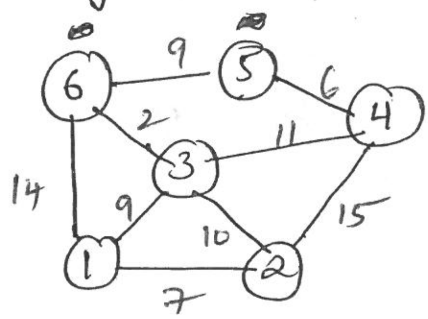
Step-5 :-



MST = 33

Hence no more edges left to form spanning Tree for given graph using Kruskal Algorithm.

7 a) find the shortest path b/w vertex 6 and vertex 4 using all pairs shortest Path algorithm.



Sol:-

$$A = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 14 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 14 & \infty & 2 & \infty & 9 & 0 \end{bmatrix} \end{array}$$

(K=1) Intermediate vertex :-

$A^1[6,3] = \min\{A[6,3], A[6,1] + A[1,3]\}$

$= \min\{2, 14\} = 2$

$A^1[3,6] = \min\{2, \infty\} = 2$

$A^1[3,2] = \min\{10, 9+14\} = 10$

$A^1[2,3] = 10$

$A^1[6,5] = 9$; $A^1[5,6] = 9$

$A^1[4,5] = 6$, $A^1[2,4] = 15$, $A^1[3,4] = 11$

$$A^1 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 14 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 14 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

Similarily,

$$A^2 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 14 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 14 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 11 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 11 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 11 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 11 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 11 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 11 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} 0 & 7 & 9 & \infty & \infty & 11 \\ 7 & 0 & 10 & 15 & \infty & \infty \\ 9 & 10 & 0 & 11 & \infty & 2 \\ \infty & 15 & 11 & 0 & 6 & \infty \\ \infty & \infty & \infty & 6 & 0 & 9 \\ 11 & \infty & 2 & \infty & 9 & 0 \end{bmatrix}$$

Shortest Path B/w vertex 6 & vertex 4 are:-

6-3 ⇒ 2	6-5 ⇒ 9
3-4 ⇒ 11.	5-4 ⇒ 6

---

7 b) Explain with example, the job Sequencing scheduling based on first-cum-first serve principle.

Sol:- Given a set of jobs, associated with each job (i) is an integer. deadline (di > 0) and a profit (Pi > 0) . for any job, i is completed within the Deadline. The profit (Pi) is earned, Ex:-

Let $n=4$, $(P1, P2, P3, P4) = (100, 10, 15, 27)$

$(D1, D2, D3, D4) = (2, 1, 2, 1)$

Sol:-

Step-1 :- place the profit in decreasing order

$(P1\ P4\ P3\ P2) = (100, 27, 15, 10)$

$(d1\ d4\ d3\ d2) = (2, 1, 2, 1)$

Step-2 :- Draw the gant chart

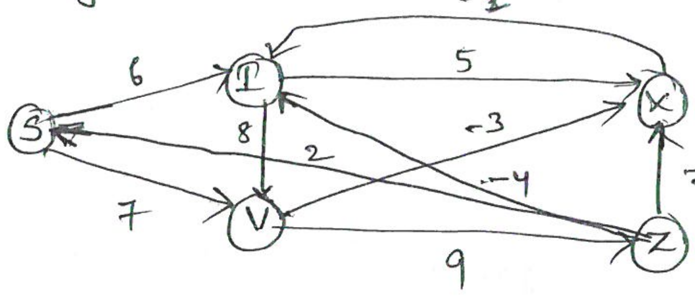
0 – 1	1 – 2
J4	J1

Step-3 :- Optimal Solution = $\{4, 1\} \Rightarrow 100 + 27 = 127$

Unit -IV

8 a) Using Bellman ford algorithm, determine the shortest path between I and z.

Sol:-



Iteration-1 :-

S	I	X	V	Z
0	∞	∞	∞	∞
0	6	5	∞	∞
0	3	5	11	20

Iteration-2 :-

S	I	X	V	Z
0	3	5	11	20
0	3	5	7	20

Iteration-3 :-

S	I	X	V	Z
0	3	5	7	16
0	3	$\cancel{4}$	7	16

Iteration -4 :-

S	I	X	V	Z
0	3	4	7	16
0	3	4	7	16

∵ final Solution :-

S	I	X	V	Z.
0	2	4	7	-2

$\boxed{I \text{ to } Z = -2}$

8b) find an optimal solution for 0/1 Knapsack usin Dynamic programming approach. $n=4$, $m=5$
$(P1, P2, P3, P4) = (4, 5, 3, 7)$ & $(W1, W2, W3, W4)$
$\Downarrow$
$(2, 3, 1, 4)$

Sol:-

$S_1^1 = \{(0,0)\} + (4,2)\}$

$S^1 = \{(0,0)\ (4,2)\}$

$S_1^2 = \{(0,0)(4,2)\} + (5,3)$

$= \{(5,3)\ (9,5)\}$

$S^2 = \{(0,0)\ (4,2)\ (5,3)\ (9,5)\}$

$S_1^3 = \{(0,0)\ (4,2)(5,3)\ (9,5)\} + (3,1)$

$\Rightarrow \{(3,1)\ (7,3)\ (8,4)\ (12,6)\}$

$S^3 = \{(0,0)(4,2)(5,3)(9,5)(3,1)(7,3)(8,4)(12,6)\}$

$S^4_1 = \{(0,0)(4,2)(5,3)(9,5)(3,1)(7,3)(8,4)(12,6)\} + (7,4)$

$S^4_1 = \{(7,4)(11,6)(12,7)(16,9)(10,5)(14,7)(15,8)(19,10)\}$

$S^4 = \{(0,0)(4,2)(5,3)(9,5)(3,1)(7,3)(8,4)(12,6)(7,4)(11,6)(12,7)(16,9)(10,5)(14,7)(15,8)(19,10)\}$

Eliminate the pairs which exceeds the maximum capacity.

$S^4 = \{(0,0)(4,2)(5,3)(9,5)(3,1)(7,3)(8,4)(7,4)(10,5)\}$

$(10,5) \in S^4$ and $(10,5) \notin S^3$.

$x_4 = 1$

$(10-7, 5-4)$

$\Downarrow$

$(3,1)$

$(3,1) \in S^3$ and $(3,1) \notin S^2$.

$x_3 = 1$

$(3-3, 1-1)$

$\Downarrow$

$(0,0)$

$$\boxed{\begin{array}{l}\Sigma p_i x_i = 10 \\ \Sigma w_i x_i = 5\end{array}}$$

Hence $x_2 = 0 \mid x_1 = 0$

Solution :- $\boxed{(x_1, x_2, x_3, x_4) = (0, 0, 1, 1)}$