9	Determine the inverse Laplace of the following functions. i) $X(s) = \frac{1}{s\,(s+1)(s+3)}$ ii) $X(s) = \frac{3s^2+8s+6}{(s+8)(s^2+6s+8)}$	L3	CO4	10 M
	UNIT-V			
10	State and prove the following properties of Z- Transform. i) Time Scaling Property. ii) Time Reversal Property. iii) Initial-value theorem. iv) Final-Value theorem.	L2	CO4	10 M
	OR			
11 a)	Compute the Z transform of $x(n) = \left(\frac{1}{3}\right)^n u(n)$ and analyze its ROC.	L4	CO4	5 M
b)	Compute the inverse z- transform of $X(z)=\frac{z}{(z+2)(z-3)}$ when the ROC is i) ROC: $\lvert z\rvert < 2$ ii) ROC: $2 < \lvert z\rvert < 3$.	L3	CO4	5 M

PVP 23

Code: 23ES1303

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

SIGNALS AND SYSTEMS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Determine whether the signal is periodic or not? $x(t) = \sin\left(\frac{\pi}{8}\right) t$	L3	CO1
1.b)	State the condition for BIBO stability of the system.	L1	CO1
1.c)	Write down the convolution integral formula to find the output of the CT systems.	L2	CO2
1.d)	State associative property of LTI system.	L1	CO2
1.e)	Write the trigonometric Fourier series representation of periodic signal with fundamental period T_0.	L2	CO3
1.f)	State the condition for convergence of Fourier series.	L1	CO3

1.g)	What is the relation between Laplace transform and Fourier transform of a signal?	L2	CO4
1.h)	Find the initial value of x(t) with $X(s) = \frac{1}{s+1}$.	L2	CO4
1.i)	Find the ROC of given signal $x(n) = \left(\frac{1}{8}\right)^n$ u(n).	L2	CO4
1.j)	Write the time-reversal property of z-transform.	L2	CO4

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Explain time shifting, time scaling and time reversal operations of a continuous time signal.	L2	CO1	5 M
	b)	Define mathematically and graphically the following continuous time signals: i) Unit Impulse Signal ii) Unit Step Signal.	L1	CO1	5 M
		OR			
3	a)	Classify the systems based on the properties and explain each with an example.	L2	CO1	5 M
	b)	Find whether the signal f(t) = 10Sin (12π)t+ 2Sin (6π)t is periodic or not? If periodic what is its fundamental period.	L2	CO1	5 M
		UNIT-II			
4	a)	What is an LTI system? Explain its properties.	L2	CO2	5 M
	b)	Compute the convolution of the following signals x(n) = (1,3,2,4) and h(n) = (2,1,3,2).	L3	CO2	5 M
		OR			
5	a)	Compare convolution sum and convolution integral with equations.	L2	CO2	5 M
	b)	Relate the impulse response and the output of an LTI system through convolution.	L2	CO2	5 M
		UNIT-III			
6	a)	Prove the linearity and time shifting properties of Fourier Transform.	L2	CO3	5 M
	b)	Write any four important properties of the continuous-time Fourier series.	L2	CO3	5 M
		OR			
7	a)	Derive the exponential Fourier series representation of a continuous-time periodic signal.	L3	CO3	5 M
	b)	Prove the time-scaling and convolution properties of the CTFT.	L2	CO3	5 M
		UNIT-IV			
8	a)	Discuss the properties of ROC of Laplace transform.	L2	CO4	5 M
	b)	Compute the Laplace transform of the signal $x(t) = te^{-2t}sin2t\,u(t)$ using properties of Laplace transform.	L3	CO4	5 M
		OR			

CUWMM

Code: 23ES1303 PVP23 ①

II B.Tech - I semester - Regular/supplementary Examination Nov 2025

Signals & systems

(ECE)

Scheme of Valuation

PART-A

1.(a) Periodicity condition 1M } - 2M
Answer 1M

(b) stability Condition — 2M

(c) Convolution formula — 2M

(d) Associative property either continuous or discrete — 2M

(e) x(t) and co-efficients — 2M

(f) Any two conditions of convergence — 2M

(g) Formulas of L.T & F.T — 1M } 2M
Relation — 1M

(h) Initial value theorem — 1M } 2M
Substitution/Answer — 1M

(i) Z-transform formula & substitution — 1M } 2M
ROC — 1M

(j) Property statement — 2M

PART B

2(a) Definitions — 3M } 5M
Explanation — 2M

2(b) Equations — 2M } 5M
Graphical Representation 3M

3(a) List — 2M } 5M
Explanation of any four types — 3M

3b) periodicity condition — 1M } 5M
substitution & simplification — 2M
Fundamental period — 2M

4a) LTI system definition & properties — 2M } 5M
Explanation — 3M

4b) Representation of $x(k)$, $h(k)$, $h(-k)$ — 2M
formulae — 1M
Simplification & Answer — 1M
} 5M

5a) Comparison any five points — 5M

5b) formulas — 2M
Explanation — 3M
} 5M

6a statements — 2M
proofs — 3M
} 5M

6b) Any four properties statements — 2M
proof — 3M
} — 5M

7a) Derivation — 5M

7b) statements — 2M
proof — 3M
} 5M

8a) Any ~~four~~ five properties — 5M

8b) formula — 4M
Substitution & Answer — 6M
} 10M

9a) partial fractions (or) any method — 2M
Simplification & Answer — 3M
} 5M

10 statements — 4M
proof — 6M
} 10M

11 a) z-transform formula — 2M
Simplification & ROC analysis — 3M

b) procedure partial fractions (or) any other method — 2M
Simplification & Answer — 3M
} 5M

II B.Tech – I semester – Regular Examinations PVP 23 ②
Code:- 23ES1303 Signals & Systems ECE Nov 2025

Scheme of valuation

Part A

1(a) $x(t) = \sin(\pi/8)t$ Determine whether it is periodic (or not

$x(t)$ is a sinusoidal signal so it is Periodic $x(t+T)=x(t)$

Compare with $\sin\omega t$; $\omega = 2\pi f = \pi/8 \Rightarrow f = \frac{1}{16}$

$\therefore$ fundamental period is 16 sec.

1(b) A system is said to be BIBO stable if it produces a bounded o/p for every bounded i/p.

Let $x(t)$ be bounded i.e $|x(t)| \le M_x < \infty$ for all 't'

If $|y(t)| \le M_y < \infty$ i.e if $y(t)$ is also bounded then

The system is BIBO stable.

1c. Convolution integral formula to find the o/p of CT system

if $x_1(t)$ & $x_2(t)$ are two continuous signals & o/p signal is represented as $y(t)$ then

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

i.e $y(t) = x_1(t) * x_2(t)$

1d. Associative property of LTI systems

CTLTI system; $x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$

(or) (OR)

DTLTI system; $x(n) * [h_1(n) * h_2(n)] = [x(n) * h_1(n)] * h_2(n)$

(1e) Trigonometric fourier series representation of a Periodic Signal

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

where $a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt$

$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t) \cos n\omega_0 t\, dt$

$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} x(t) \sin n\omega_0 t\, dt$

(f) Convergence of fourier series

for The fourier series to exist for a periodic signal it must satisfy certain conditions.

→ The function $f(t)$ is absolutely integrable over one period i.e

$$\int_0^T |x(t)|\, dt < \infty$$

→ It has finite number of maxima & minima

→ $f(t)$ has a finite number of discontinuties.

→ $f(t)$ must be single valued function.

g) Relation b/w Laplace & fourier transform

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt \rightarrow \text{Laplace tranform where } s = \sigma + j\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt \rightarrow \text{fourier transform}$$

when The Laplace transform becomes F.T when $s = j\omega$

means $\sigma = 0$ $X(j\omega) = X(s)\big|_{s=j\omega}$.

h) find The initial value of $x(t)$ with $X(s) = 1/s+1$

initial value theorem if $x(t) \xleftrightarrow{LT} X(s)$

$$\lim_{t \to 0} x(t) = x(0) = \lim_{s \to \infty} s \cdot X(s)$$

$$\lim_{s \to \infty} s \cdot X(s) = \frac{s}{s+1} = \frac{\cancel{s}}{\cancel{s}(1+1/s)} = \frac{1}{1+\frac{1}{s}}$$

$$= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0}$$

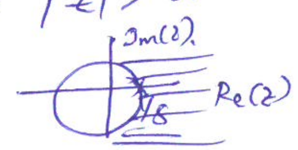
$$\boxed{= 1}$$

(i) find The ROC of The given signal $x(n) = \left(\frac{1}{8}\right)^n u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{8}\right)^n z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{8z}\right)^n = \frac{1}{1 - \frac{1}{8}z^{-1}}$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{where } |z| > a$$

So the ROC is $|z| > \frac{1}{8}$



(j) Time Reversal property of z-transform

If $x(n) \xleftrightarrow{Z.T} X(z)$

Then $x(-n) \xleftrightarrow{Z.T} X\left(\frac{1}{z}\right)$ (or) $X(z^{-1})$

Part B

(2a) Explain time shifting, time scaling and time reversal operations of a continuous time signal. (3)

Time shifting:- Let $x(t)$ be a continuous time signal. The $x(t \pm t_0)$ represents the time shifted version of the signal $x(t)$. where t_0 is constant.

$x(t-t_0)$ represents the delayed version of the signal $x(t)$. Here the signal is shifted towards right.
$x(t+t_0)$ represents the advanced version of the signal $x(t)$. Here the signal is shifted towards left.

Time scaling:-
Let $x(t)$ is continuous time signal then the time scaled version of $x(t)$ is represented as $x(at)$ where 'a' is a scaling factor. If $a>1$ Then $x(at)$ is the compressed version of $x(t)$. If a is in b/w 0 and 1 Then $x(at)$ is the expanded version of $x(t)$.

Time reversal:- Let $x(t)$ be The continuous time signal Then $x(-t)$ is mathematical time reversed version of $x(t)$. It is known as Reflection (or) folding of the given signal. It reflects The signal about y-axis.

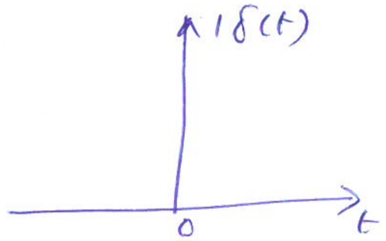
2(b) Define mathematically and graphically the following continuous time signals.
(i) unit impulse signal
(ii) unit step signal

(i) unit impulse function

It is the most widely used elementary function used in analysis of signals & systems. It is represented as $\delta(t)$. It is defined as

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \quad \text{and}$$

$$\delta(t) = 0 \quad \text{for } t \neq 0$$



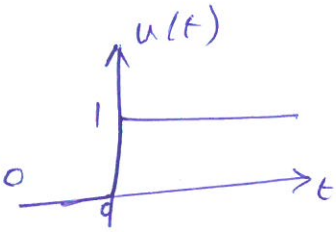
(ii) unit step signal

If a step function has unity magnitude then it is called as unit step function. It is represented as $u(t)$

$u(t)$ is defined as

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

It is discontinuous at $t = 0$



3(a) Classification of systems

(i) Continuous time & discrete time systems
(ii) static & Dynamic systems
(iii) Linear & Non linear systems
(iv) Causal & Non Causal systems
(v) Stable & unstable system
(vi) ~~In~~ Time variant & Time Invariant system.
(vii) Invertible & Non-Invertible system.

(i) Continuous time & Discrete time systems

A continuous time system is one which transforms continuous time i/p signal in to continuous time o/p signal.

$\therefore y(t) = T\{x(t)\}$ $x(t)$ is i/p signal, $y(t)$ is o/p signal

A Discrete time system is one which trans-forms the discrete time i/p signal into discrete time o/p signal ④

$y(n) = T\{x(n)\}$ $x(n) \rightarrow$ i/p signal
$y(n) \rightarrow$ o/p signal

<u>Static & Dynamic systems</u>

Static systems are also called as memoryless systems. It is defined as if its o/p at any instant of time depends only on the i/p at that instant of time.

$$o/p = F\{\text{present i/p}\}$$
$$y(t) = F\{x(t)\} \text{ - continuous}$$
$$y(n) = F\{x(n)\} \text{ - discrete}$$
$$Ex:- \ y(t) = x(t) + u(t+1)$$

Dynamic system are defined as its o/p depends on past (or) future values of i/p.

$$Ex:- \ y(t) = x(2t)$$

<u>Linear & Non linear systems</u>

A system is said to be linear if it obeys the principle of superposition. So it states that the response of the system to the weighted sum of i/p signals is equal to the weighted sum of responses of the system to each of the individual o/ps.

If CT system is linear $T\{a x_1(t) + b x_2(t)\} = aT\{x_1(t)\} + bT\{x_2(t)\}$
Similarly for discrete system also.

$$Ex:- \ y(t) = t \cdot x(t)$$

A system that does not obey the principle of superposition is said to be non-linear system. Similarly for discrete system also.

$$Ex:- \ y(t) = x(t) + u(t+1)$$

<u>Causal & Non causal systems</u>

A system is said to be causal if its o/p depends only present & past values of i/p at any instant of time

Ex:- $y(t) = t \cdot x(t)$; $y(n) = 2x(n)+3$

A system is said to be non causal if its o/p at any instant of time depends on the future values of i/p

Ex:- $y(t) = x(t+2)$

Stable & unstable systems

If a system is said to be BIBO stable if it produces bounded o/p for every bounded i/p

If $|x(t)| \le M_x < \infty$ then $|y(t)| \le M_y < \infty$ Then The system is BIBO stable. Other wise unstable.

Ex:- $y(t) = x(t) \cdot u(t) \rightarrow$ stable

$y(t) = t \cdot x(t) \rightarrow$ unstable $\rightarrow$ for a bounded i/p it produces unbounded o/p.

Time invariant & Time variant

A system is ~~to~~ said to be time invariant if a shift (or) delay in i/p signal causes a corresponding shift (or) delay in o/p signal.

$y(t-t_0) = T\{x(t-t_0)\} \rightarrow$ T. Invariant

Ex:- $y(t) = x(t-2)$

If a shift in the i/p signal does not produce a corresponding shift in the o/p signal the system is time variant system

Ex:- $y(t) = x(2t)$

Invertible and Non Invertible :- A system is said to be invertible if the i/p is recoverable from the system o/p. Other wise it is non Invertible.

Ex:- $y(t) = 2x(t) \rightarrow$ Invertible; $y(t) = x^2(t) \rightarrow$ Non invertible

3b) $f(t) = 10\sin(12\pi)t + 2\sin(6\pi)t$ find it is periodic (or) not

Let $f(t) = f_1(t) + f_2(t)$; $f_1(t) = 10\sin(12\pi)t$; $f_2(t) = 2\sin(6\pi)t$

Compare $f_1(t)$ with $\sin \omega_1 t$

$\omega_1 = 12\pi$

$\cancel{2\pi} f_1 = \cancel{12}^{6}\pi$

$T_1 = \frac{1}{6}$ sec

Compare $f_2(t)$ with $\sin \omega_2 t$

$\omega_2 = 6\pi$

$\cancel{2\pi} f_2 = \cancel{6\pi}^{3}$

$T_2 = 1/3$ sec

The ratio of two periods $\frac{T_1}{T_2} = \frac{1}{\cancel{6}_2} \times \frac{\cancel{3}}{1} = \frac{1}{2}$ so it is a rational number; so it is "Periodic" with a period of $T = 2T_1 = \frac{1}{3}$ sec

4a) What is an LTI system Explain its properties (5)

LTI system :- A system which is linear as well as time invariant is called a LTI system.

<u>Properties of LTI systems</u>

(i) <u>Commutative Property</u>

$$x(t) * h(t) = h(t) * x(t)$$

i.e the o/p of an LTI system with i/p $x(t)$ & unit impulse response $h(t)$ is identical to the o/p of an LTI system with input $h(t)$ and impulse response $x(t)$.

(ii) <u>Distributive property</u>

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

i.e two systems with impulse responses $h_1(t)$ & $h_2(t)$ connected in parallel can be replaced by a single system with impulse response $h_1(t) + h_2(t)$.

(iii) <u>Associative property</u> :-

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

i.e series connection of two systems is equivalent to a single system.

(iv) <u>Static (or) memory less & dynamic property</u> :-

A continuous time LTI system is static if $h(t) = 0$ for $t \neq 0$
" " " dynamic if $h(t) \neq 0$; for $t \neq 0$

(or)

A DT LTI system is static if $h(n)$ or $h(k) = 0$ for k (or) $n \neq 0$
" " is dynamic " " $\neq 0$ for k (or) $n \neq 0$

(v) <u>Causality</u>

Its o/p depends only on the past & present values of i/p but not on future i/ps.

A CT LTI system is causal if $h(t) = 0$ for $t < 0$
" anti causal if $h(t) \neq 0$ for $t < 0$

(iii) A DT LTI system is causal $h(n)$ (or) $h(k) = 0$ for n (or) $k < 0$

" " anti causal $h(n)$ (or) $h(k) \neq 0$ for n (or) $k < 0$

Stability

For a Continuous-time LTI system to be BIBO stable its impulse response $h(t)$ must be absolutely integrable i.e

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty \rightarrow \text{CT LTI system}$$

$$\text{(or)} \quad \sum_{n=-\infty}^{\infty} |h(n)| < \infty \rightarrow \text{DT LTI system}$$

Invertibility

A CT LTI system with impulse response $h(t)$ is said to be invertible, if an inverse system with impulse response $h_1(t)$ which when connected in series with the original system produces an o/p equal to the i/p of the first system.

$$h(t) * h_1(t) = \delta(t) \quad \text{for CT LTI system}$$

$$h(n) * h_1(n) = \delta(n) \quad \text{" DT LTI system}$$

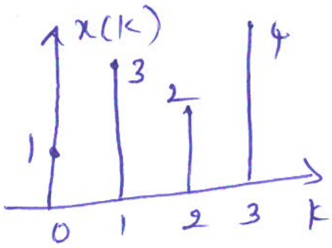
4(b) Compute the convolution of the following signals

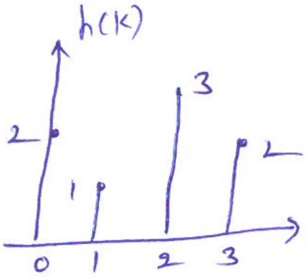
$x(n) = (1, 3, 2, 4)$ and $h(n) = (2, 1, 3, 2)$

Let $y(n) = x(n) * h(n)$

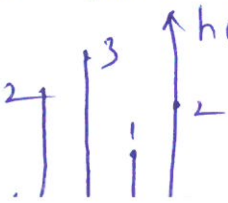
$$= \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

(i) Represent $x(n)$ & $h(n)$ in terms of indexed scale $x(k)$ & $h(k)$





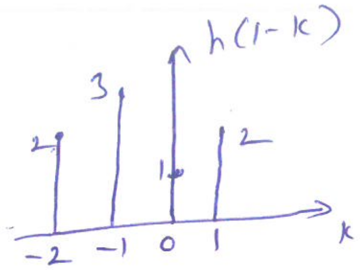
(ii) fold the signal $h(k)$ to obtain $h(-k)$ (or) $h(n-k)$ at $n=0$



$y(n)$ in calculated from $n=0$ to 6

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$$

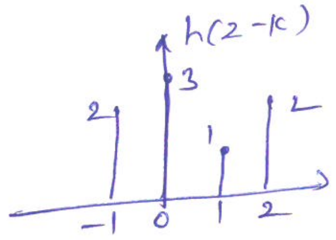
$$y(0) = 1 \times 2 = 2$$



$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

$$= (1 \times 1) + (2 \times 3)$$

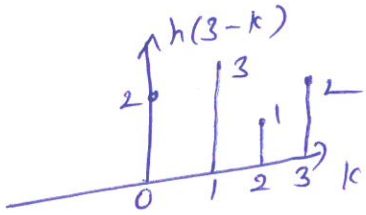
$$= 1 + 6 = 7$$



$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$$

$$= (1 \times 3) + (3 \times 1) + (2 \times 2)$$

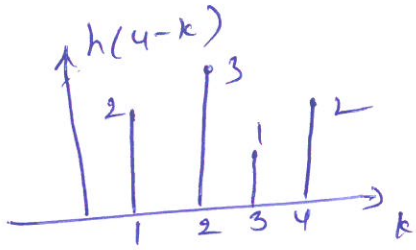
$$= 3 + 3 + 4 = 10$$



$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$$

$$= (1 \times 2) + (3 \times 3) + (2 \times 1) + (4 \times 2)$$

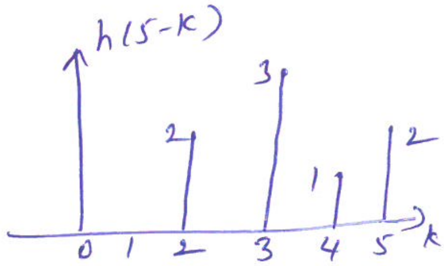
$$= 2 + 9 + 2 + 8 = 21$$



$$y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$$

$$= (3 \times 2) + (2 \times 3) + (4 \times 1)$$

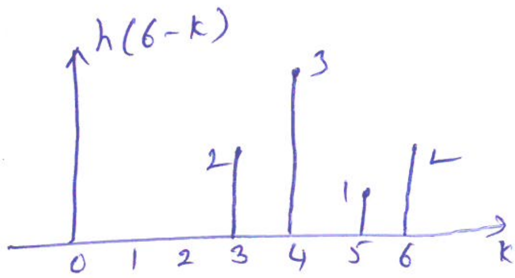
$$= 6 + 6 + 4 = 14$$



$$y(5) = \sum_{k=-\infty}^{\infty} x(k)\, h(5-k)$$

$$= (2 \times 2) + (4 \times 3)$$

$$= 4 + 12 = 16$$

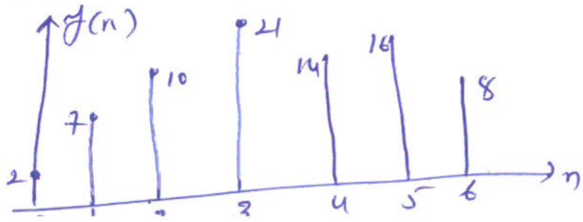


$$y(6) = \sum_{k=-\infty}^{\infty} x(k)\, h(6-k)$$

$$= (4 \times 2)$$

$$= \cancel{\infty}\ 8$$

$$\therefore\ y(n) = (\underset{\uparrow}{2}, 7, 10, 21, 14, 16, 8)$$



5(a) Compare Convolution Sum and Convolution integral with equation.

Convolution Sum	Convolution integral
(i) Here i/p signal $x(n)$ & impulse response $h(n)$ & o/p signal $y(n)$ all are in discrete nature	Here i/p signal $x(t)$ & impulse response $h(t)$ & o/p signal $y(t)$ all are in discrete nature.
(ii) $y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$ $y(n) = x(n) * h(n)$	$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$ $= x(t) * h(t)$
(iii) Used in CT LTI system	Used in DT LTI systems
(iv) It involves the folding, shifting mutiplication & summation operation	It also involves the folding, shifting multiplication & summation operation
(v) It involves the Summation	It involves the integration
(vi) It is used in audio, continuous physical systems	It is used in DSP, Filters & image processing
(vii) It satisfies Commutative, Associate, distributive, static or dynamic stability etc properties	It also statisfies all the properties ex:- commutative, Associative, distributive, stability, causality etc

5b) Relate the impulse response & o/p of LTI system using Convolution

If The system is discrete.

$$x(n) \longrightarrow \boxed{\text{DT LTI system}} \longrightarrow y(n)$$

Response of DT LTI system is an impulse function and is considered as unit impulse response of a function

$$\delta(n) \longrightarrow \boxed{\text{DT LTI system}} \longrightarrow h(n)$$

$\delta(n) \rightarrow$ unit impulse function

$h(n) \rightarrow$ unit impulse response of the system

$$T\{\delta(n)\} = h(n)$$

$$y(n) = T\{x(n)\}$$

Any arbitrary DT signal can be represented as a weighted sum of shifted impulses i.e

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$y(n) = T\left\{\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right\}$$

Since the system is linear

$$y(n) = \sum_{k=-\infty}^{\infty} T\{x(k)\,\delta(n-k)\}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,T\{\delta(n-k)\}$$

If system is time - invariant

$$T\{\delta(n)\} = h(n) \text{ so}$$

$$T\{\delta(n-k)\} = h(n-k)$$

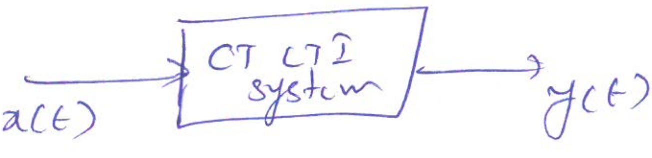
$$\therefore\ y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$ It can also be represented as

$$\boxed{y(n) = x(n) * h(n)}$$

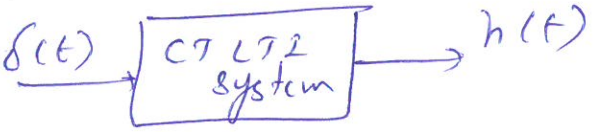
$\therefore$ Response of of DTLTI system to any arbitrary i/p signal is the Convolution of applied i/p and the impulse response of the system

(OR)

If the system is Continuous



If the i/p impulse function then $x(t) = \delta(t)$ & o/p $y(t) = h(t)$



$$h(t) = T\{\delta(t)\}$$

$$\therefore y(t) = T\{x(t)\}$$

Any arbitrary CT signal can be represented as weighted sum of shifted impulses i.e

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$\therefore y(t) = T\left\{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right\}$$

Using the linearity property of the system

$$y(t) = \int_{-\infty}^{\infty} T\{x(\tau)\,\delta(t-\tau)\,d\tau\}$$

$$= \int_{-\infty}^{\infty} x(\tau)\, T\{\delta(t-\tau)\}\,d\tau$$

If the system time invariant

$$T\{\delta(t)\} = h(t)$$

$$T\{\delta(t-\tau)\} = h(t-\tau)$$

$$\therefore y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

It can also be represented as $y(t) = x(t) * h(t)$

$\therefore$ The response of CT LTI system to any arbitrary i/p signal is the convolution of applied i/p and the impulse response of the system.

6(a) Linearity and time shifting properties of Fourier transform (8)

Linearity property:-

It states that the F.T of a weighted sum of two signals is equal to the weighted sum of their individual Fourier transform

if $x_1(t) \xleftrightarrow{F.T} X_1(\omega)$ & $x_2(t) \xleftrightarrow{F.T} X_2(\omega)$

Then $a x_1(t) + b x_2(t) \xleftrightarrow{F.T} a X_1(\omega) + b X_2(\omega)$

proof

$$F[a x_1(t) + b x_2(t)] = \int_{-\infty}^{\infty} [a x_1(t) + b x_2(t)] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} a x_1(t) e^{-j\omega t} dt + \int_{-\infty}^{\infty} b x_2(t) \cdot e^{-j\omega t} dt$$

$$= a X_1(\omega) + b X_2(\omega).$$

$$\boxed{\therefore \; a x_1(t) + b x_2(t) \xleftrightarrow{F.T} a X_1(\omega) + b X_2(\omega)}$$

Time shifting property:-

If $x(t) \xleftrightarrow{F.T} X(\omega)$

Then $x(t - t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} X(\omega)$

proof:-

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$F.T[x(t - t_0)] = \int_{-\infty}^{\infty} x(t - t_0) e^{-j\omega t} dt$$

Let $p = t - t_0 \Rightarrow t = p + t_0$ & $dt = dp$

$$F.T[x(t - t_0)] = \int_{-\infty}^{\infty} x(p) e^{-j\omega(p + t_0)} dp$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} x(p) e^{-j\omega p} dp$$

$$= e^{-j\omega t_0} X(\omega)$$

$$\boxed{\therefore\ x(t-t_0) \xleftrightarrow{F.T} e^{-j\omega t_0} X(\omega)}$$

Similarly $x(t+t_0) \xleftrightarrow{F.T} e^{j\omega t_0} X(\omega)$

6b) Write any four important properties of CT fourier Series.

i) Linearity property
(ii) Time shifting property
(iii) Time reversal "
(iv) Time Scaling "
(v) Time Differentiation property
(vi) Time Integration "
(vii) Convolution property "
(viii) Conjugate property. "

Linearity property

It states That if $x_1(t) \xleftrightarrow{F.S} a_k$ and $x_2(t) \xleftrightarrow{F.S} b_k$

Then $A x_1(t) + B x_2(t) \xleftrightarrow{F.S} A a_k + B b_k$

Time Shifting Property

It states That if $x(t) \xleftrightarrow{F.S} a_k$ Then
$x(t-t_0) \xleftrightarrow{F.S} a_k \cdot e^{-jk\omega_0 t_0}$

Time reversal property:-

It states That if $x(t) \xleftrightarrow{F.S} a_k$ Then
$x(-t) \xleftrightarrow{F.S} a_{-k}$.

Time Scaling Property:-

It states That if $x(t) \xleftrightarrow{F.S} a_k$ Then
$x(at) \xleftrightarrow{F.S} a_k$ with $\omega_0 \rightarrow a\omega$

Time differentiation property

It states That if $x(t) \xleftrightarrow{F.S} a_k$ Then

Then $\frac{d}{dt}x(t) \overset{F.S}{\longleftrightarrow} jk\omega_0 a_k$

(vi) Time integration property

It states That if

$x(t) \overset{F.S}{\longleftrightarrow} a_k$ Then

$\int_{-\infty}^{t} x(\tau)d\tau \overset{F.S}{\longleftrightarrow} \frac{a_k}{jk\omega_0}$ if $a_0 = 0$

(vii) Convolution property

It states That $x_1(t) \overset{F.S}{\longleftrightarrow} a_k$ & $x_2(t) \overset{F.S}{\longleftrightarrow} b_k$

Then $x_1(t) * x_2(t) \overset{F.S}{\longleftrightarrow} T a_k \cdot b_k$.

(viii) Conjugation property:-

It states That $x(t) \overset{F.S}{\longleftrightarrow} a_k$ Then

$x^*(t) \overset{F.S}{\longleftrightarrow} a^*_{-k}$

7(a) Derivation of exponenential fourier series representation of a Continuous - time periodic signal

The exponential Fourier series is The most widely used form of Fourier Series. In This The function $x(t)$ is expressed as a weighted sum of The complex exponential functions.

The set of complex exponential functions forms a closed orthogonal set over an interval (t_0, t_0+T) where $T = (2\pi/\omega_0)$ for any value of t_0, and therefore it can be used as a fourier series. Using Euler's identity

$$A_n \cos(n\omega_0 t + \theta_n) = A_n \left[\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}}{2} \right]$$

Substituting this in the definition of the cosine Fourier transform representation, we obtain

$$x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2}\left[e^{j(n\omega_0 t+\theta_n)} + e^{-j(n\omega_0 t+\theta_n)}\right]$$

$$= A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2}\left[e^{jn\omega_0 t}\cdot e^{j\theta_n} + e^{-jn\omega_0 t}\cdot e^{-j\theta_n}\right]$$

$$= A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{jn\omega_0 t}\cdot e^{j\theta_n}\right) + \left(\sum_{n=1}^{\infty}\frac{A_n}{2} e^{-jn\omega_0 t}\cdot e^{-j\theta_n}\right)$$

$$= A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{j\theta_n}\right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{j(-\theta_n)}\right) e^{-jn\omega_0 t}$$

Let $n = -k$ in the second summation of the above equ we have

$$x(t) = A_0 + \sum_{n=1}^{\infty}\left(\frac{A_n}{2} e^{j\theta_n}\right) e^{j\omega_0 n t} + \sum_{k=-1}^{-\infty}\left(\frac{A_k}{2} e^{j\theta_k}\right) e^{jk\omega_0 t}$$

Comparing the above two equ for $x(t)$ we get

$A_n = A_k$; $(-\theta_n) = \theta_k$ if $n > 0$, $k < 0$

Let us define

$$C_0 = A_0; \quad C_n = \frac{A_n}{2} e^{j\theta_n}, \quad n > 0$$

$$\therefore x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j\theta_n} e^{j\omega_0 n t} + \sum_{n=-1}^{-\infty} \frac{A_n}{2} e^{j\theta_n} e^{jn\omega_0 t}$$

i.e $x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$.

This is known as exponential form of fourier series.

So the exponential series from cosine series is

$$C_0 = A_0$$

$$C_n = \frac{A_n}{2} e^{j\theta_n}$$

Determination of The co-efficients

We have $x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$; $\omega_0 = \frac{2\pi}{T}$

Multiplying both sides by $e^{-jk\omega_0 t}$ and integrating over one period

$$\int_{t_0}^{t_0+T} x(t)\, e^{-jk\omega_0 t}\, dt = \int_{t_0}^{t_0+T} \left(\sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \right) e^{-jk\omega_0 t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} C_n \int_{t_0}^{t_0+T} e^{jn\omega_0 t}\, e^{-jk\omega_0 t}\, dt$$

We know That

$$\int_{t_0}^{t_0+T} e^{jn\omega_0 t} \cdot e^{-jk\omega_0 t}\, dt = \begin{cases} 0 & \text{for } k \neq n \\ T & \text{for } k = n \end{cases}$$

$$\therefore \int_{t_0}^{t_0+T} x(t) \cdot e^{-jk\omega_0 t}\, dt = T \cdot C_k$$

or $C_k = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\, e^{-jk\omega_0 t}\, dt$ (or) $\boxed{C_n = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\, e^{-jn\omega_0 t}\, dt}$

& $c_0 = A_0 = \frac{1}{T} \int_{t_0}^{} x(t)\, dt.$

(7b) Time Scaling and Convolution properties of CTFT

It states That if $x(t) \xleftrightarrow{F.T} X(\omega)$ Then

$$x(at) \xleftrightarrow{F.T} \frac{1}{|a|} X(\omega/a)$$

Proof $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$F[x(at)] = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

Let $at = p \Rightarrow t = p/a$ & $dt = \frac{dp}{a}$

$$F[x(at)] = \int_{-\infty}^{\infty} x(p)\, e^{-j\omega (p/a)}\, \frac{dp}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} x(p)\, e^{-j(\omega/a)p}\, dp$$

Case 1 $a > 0$ $F[x(at)] = \frac{1}{a} X(\omega/a)$

Case 2 When $a<0$

$$F[at] = \frac{1}{-a}\int_{-\infty}^{\infty} x(p)\, e^{j(\omega/a)p}\, dp = \frac{1}{-a}\int_{-\infty}^{\infty} x(p)\, e^{-j\left(\frac{-\omega}{a}\right)p}\, dp$$

$$= \frac{-1}{a} X\left(\frac{-\omega}{a}\right)$$

$$= \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$$\boxed{\therefore\ x(at) \xleftrightarrow{F.T} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)}$$

(ii) Convolution property:-

It states that if $x_1(t) \xleftrightarrow{F.T} X_1(\omega)$ & $x_2(t) \xleftrightarrow{F.T} X_2(\omega)$

Then $x_1(t) * x_2(t) \xleftrightarrow{F.T} X_1(\omega)\cdot X_2(\omega)$

We know that the convolution of two signals $x_1(t)$ & $x_2(t)$ is

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$\therefore F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_1(\tau)(x_2(t-\tau))\, d\tau\right] e^{-j\omega t}\, dt$$

Interchanging the order of integration

$$= \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(t-\tau)\, e^{-j\omega t}\, dt\right] d\tau$$

Let $t-\tau = p$; $t = p+\tau$; $dt = dp$

$$\therefore F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(p)\, e^{-j\omega(p+\tau)}\, dp\right] d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(p)\, e^{-j\omega p}\, dp\right] e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, X_2(\omega)\, e^{-j\omega\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= X_1(\omega)\cdot X_2(\omega)$$

$$\boxed{\therefore\ F.T[x_1(t) * x_2(t)] \xleftrightarrow{F.T} X_1(\omega)\cdot X_2(\omega)}$$

8.(a) properties of ROC of Laplace transform

i) If $x(t)$ is a two-sided signal, The ROC of $X(s)$ is a strip parallel to $j\omega$ axis in s-plane.

(ii) ROC does not contain any poles

(iii) for a finite duration signal the ROC of Laplace transform is entire s-plane.

(iv) for a right sided signal the ROC of $X(s)$ extends to the right of the right most ~~plane~~ pole and no pole is located inside the ROC.

(v) For a Left sided signal the ROC of $X(s)$ extends to the Left of the Left most pole and no pole is located inside The ROC.

(vi) If $x(t)$ is a two sided signal, The ROC of $X(s)$ is a strip in the s-plane bounded by poles and no pole is located inside The ROC.

8b) Compute The Laplace transform of The Signal $x(t) = t \cdot e^{-2t} \sin 2t \; u(t)$ using properties of Laplace transform

We know that $L.T$ of $\sin\omega_0 t \cdot u(t) = \frac{\omega_0}{s^2+\omega_0^2}$

Similarly $\sin 2t \; u(t) = \frac{2}{s^2+4}$

using frequency shifting property

$$L.T\left[e^{-2t} \sin 2t \; u(t)\right] = L.T\left[\sin 2t \cdot u(t)\right]\Big|_{s=s+2}$$

$$= \frac{2}{s^2+4}\Big|_{s=s+2}$$

$$= \frac{2}{(s+2)^2+4} = \frac{2}{s^2+4s+8}$$

Using differentiation in s- domain property

$$L.T\left[t \cdot e^{-2t} \cdot \sin 2t \; u(t)\right] = -\frac{d}{ds}\left(L.T\left[e^{-2t}\sin 2t \, u(t)\right]\right)$$

$$= -\frac{d}{ds}\left(\frac{2}{s^2+4s+8}\right)$$

$$\boxed{= \frac{4(s+2)}{(s^2+4s+8)^2}}$$

9. Determine the inverse Laplace of the following functions

(i) $X(s) = \frac{1}{s(s+1)(s+3)}$

(ii) $X(s) = \frac{3s^2+8s+6}{(s+8)(s^2+6s+8)}$

(i) $X(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$

$$= \frac{A(s+1)(s+3) + B \cdot s \cdot (s+3) + C \cdot s \cdot (s+1)}{s(s+1)(s+3)}$$

$1 = A(s+1)(s+3) + B(s)(s+3) + C \cdot s \cdot (s+1)$

if $s=0$ $3A = 1 \Rightarrow A = 1/3$

if $s=-1 \Rightarrow 1 = B(-1)(2) \Rightarrow B = -1/2$

if $s=-3 \Rightarrow 1 = C(-3)(-2) \Rightarrow C = 1/6$

$$\therefore X(s) = \frac{\frac{1}{3}}{s} - \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{6}}{s+3}$$

Taking inverse Laplace transform

$$\therefore\ x(t) = \frac{1}{3}u(t) - \frac{1}{2}e^{-t}u(t) + \frac{1}{6}e^{-3t}u(t)$$

(ii) $$X(s) = \frac{3s^2+8s+6}{(s+8)(s^2+6s+8)}$$

$$= \frac{3s^2+8s+6}{(s+8)(s+2)(s+4)}$$

$$= \frac{A}{s+8} + \frac{B}{s+2} + \frac{C}{s+4}$$

$$= \frac{A(s+2)(s+4) + B(s+4)(s+8) + C(s+2)(s+8)}{(s+8)(s+2)(s+4)}$$

$$3s^2+8s+6 = A(s+2)(s+4) + B(s+4)(s+8) + C(s+2)(s+8)$$

if $s=-8$

$$3(-8)^2 + 8(-8) + 6 = A(-6)(-4)$$

$$134 = 24A$$

$$\Rightarrow A = \frac{134}{24} \Rightarrow \frac{67}{12}$$

if $s=-2$

$$3(-2)^2 + 8(-2) + 6 = B(6)(2)$$

$$2 = 12B$$

$$\Rightarrow B = 1/6$$

if $s=-4$

$$3(-4)^2 + 8(-4) + 6 = C(4)(-2)$$

$$22 = -8C$$

$$\Rightarrow C = \frac{-22}{8} = \frac{-11}{4}$$

$$\therefore X(s) = \frac{\frac{67}{12}}{s+8} + \frac{\frac{1}{6}}{s+2} - \frac{\frac{11}{4}}{s+4}$$

Apply inverse Laplace transform to the X(S)

$$x(t) = \frac{67}{12}e^{-8t}u(t) + \frac{1}{6}e^{-2t}u(t) - \frac{11}{4}e^{-4t}u(t)$$

10) State and prove the following properties of z transform:

i) Time Scaling property

(ii) Time Reversal property

(iii) Initial value theorem

(iv) Final-value theorem

(i) Time Scaling property

If $x(n) \xleftrightarrow{Z.T} X(z)$

Then $a^n . x(n) \xleftrightarrow{Z.T} X\left(\frac{z}{a}\right)$ where 'a' is a complex number

proof
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$z\left[a^n . x(n)\right] = \sum_{n=-\infty}^{\infty} a^n x(n) . z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left(\frac{z}{a}\right)^{-n}$$

$$= X(z/a)$$

$$\therefore a^n x(n) \xleftrightarrow{Z.T} X(z/a)$$

(ii) Time Reversal property

If $x(n) \xleftrightarrow{Z.T} X(z)$

Then $x(-n) \xleftrightarrow{Z.T} X(1/z)$

proof
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n) z^{-n}$$

Let $p=-n$ in the above summation

$$z[x(-n)] = \sum_{p=-\infty}^{\infty} x(p) (z^{-1})^{-p}$$

$$= X(z^{-1}) = X\left(\frac{1}{z}\right)$$ $\therefore x(-n) \xleftrightarrow{Z.T} X(1/z)$

(iii) Initial Value Theorem

The initial value theorem of z-transform states That for a causal signal $x(n)$

If $x(n) \xleftrightarrow{Z.T} X(z)$

Then $\underset{n \to 0}{Lt}\, x(n) = x(0) = \underset{z \to \infty}{Lt}\, X(z)$

for a causal signal

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = x(0) + x(1) z^{-1} + x(2) z^{-2} + \cdots$$

$$= x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \cdots$$

Taking The limit $z \to \infty$ on both sides

$$\underset{z \to \infty}{Lt}\, X(z) = \underset{z \to \infty}{Lt}\left[x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} + \cdots\right]$$

$$= x(0)$$

$$\boxed{\therefore x(0) = \underset{z \to \infty}{Lt}\, X(z) = \underset{n \to 0}{Lt}\, x(n)}$$

(iv) Final value theorem :-

It states That for a causal signal

If $x(n) \xleftrightarrow{Z.T} X(z)$ and if $X(z)$ has no poles outside The unit circle, and it has no double (or) higher order poles on the unit circle centered ~~at The origin~~ of The z-plane Then

$$x(\infty) = \underset{z \to 1}{Lt}\, (z-1) X(z)$$

for a causal signal

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$Z.T\left[x(n+1)\right] = \sum_{n=0}^{\infty} x(n+1) z^{-n} = zX(z) - z\,x(0)$$

$$Z.T \text{ of } [x(n+1)] - Z.T \text{ of } [x(n)] = z\cdot X(z) - z\,x(0) - X(z)$$

$$= \sum_{n=0}^{\infty} x(n+1)\, z^{-n} - \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$X(z)[z-1] - z\,X(0) = \sum_{n=0}^{\infty} [x(n+1) - x(n)]\, z^{-n}$$

i.e $$(z-1)X(z) - z\,x(0) = [x(1) - x(0)]z^{-0} + [x(2) - x(1)]z^{-1} + \dots$$

Taking limit $z \to 1$ on both sides

$$\lim_{z\to 1} (z-1)X(z) - x(0) = [\cancel{x(1)} - x(0) + \cancel{x(2)} - \cancel{x(1)} + x(3) - \cancel{x(2)} + \dots x(\infty) - x(\infty - 1)]$$

$$= x(\infty) - x(0)$$

$$\boxed{\therefore\ x(\infty) = \lim_{z\to 1} (z-1)X(z)}$$

11 (a) Compute the z-transform of $x(n) = \left(\frac{1}{3}\right)^n u(n)$ and analyze its ROC

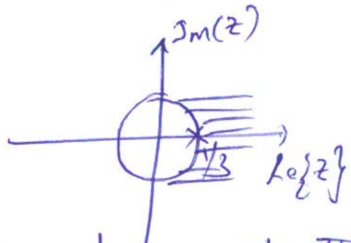
$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u(n)\cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} \qquad \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n \qquad \text{~~If~~ It converges if } |z| > |a|$$

$$= \frac{1}{1 - \frac{1}{3}z^{-1}}$$



It implies that the ROC is exterior to the circle.

11b) Compute The inverse z - transform of $X(z) = \frac{z}{(z+2)(z-3)}$ when the ROC is (14)

i) ROC : $|z| < 2$

(ii) ROC : $2 < |z| < 3$.

$$X(z) = \frac{z}{(z+2)(z-3)}$$

$$\frac{X(z)}{z} = \frac{1}{(z+2)(z-3)}$$

$$= \frac{A}{z+2} + \frac{B}{z-3}$$

$$\frac{X(z)}{z} = \frac{A(z-3) + B(z+2)}{(z+2)(z-3)}$$

if $z = -2$; $1 = A(-2-3) \Rightarrow A = \frac{-1}{5}$

if $z = 3$; $1 = B(5) \Rightarrow B = 1/5$

$$\frac{X(z)}{z} = \frac{\frac{-1}{5}}{z+2} + \frac{\frac{1}{5}}{z-3}$$

$$X(z) = \frac{-1}{5} \cdot \frac{z}{z+2} + \frac{1}{5} \frac{z}{z-3}$$

(i) if ROC : $|z| < 2$

Taking inverse 'z' transform

$\therefore x(n) = \frac{1}{5} (-2)^n u(-n-1) - \frac{1}{5} \cdot (3)^n u(-n-1)$

(ii) if ROC : $2 < |z| < 3$

Taking inverse z transform

$x(n) = \frac{-1}{5} (-2)^n u(n) - \frac{1}{5} (3)^n u(-n-1)$