OR					
11	a)	Illustrate different noise sources with examples.	L3	CO5	5 M
	b)	Illustrate the expression for Noise figure of Cascaded networks.	L3	CO5	5 M

PVP 23

Code: 23BS1304

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

PROBABILITY THEORY AND STOCHASTIC PROCESS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Explain sample space with an example.	L2	CO1
b)	State Bayes' theorem.	L2	CO1
c)	Illustrate expected value of a random variable.	L3	CO2
d)	Explain the variance of a random variable.	L2	CO2
e)	State marginal probability distribution.	L2	CO2
f)	State the Central Limit Theorem.	L2	CO2
g)	Illustrate Wide-Sense Stationarity.	L3	CO4
h)	State any two properties of autocorrelation function.	L2	CO4
i)	Explain the effective noise temperature.	L2	CO5
j)	Explain narrowband noise.	L2	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Explain probability axioms with suitable examples.	L2	CO1	5 M
	b)	Illustrate the law of total probability.	L3	CO1	5 M
		OR			
3	a)	Solve a real-world problem using Bayes' theorem.	L3	CO1	5 M
	b)	Differentiate independent and mutually exclusive events with examples.	L2	CO1	5 M
		UNIT-II			
4	a)	Compute the mean of a Binomial distribution.	L3	CO2	5 M
	b)	Apply Binomial distribution to a practical problem (e.g., coin tossing).	L3	CO2	5 M
		OR			
5	a)	Derive the MGF of Gaussian distribution.	L4	CO3	5 M
	b)	State and prove Chebyshev's inequality.	L3	CO2	5 M
		UNIT-III			
6	a)	Interpret marginal distribution from joint distribution.	L3	CO2	5 M
	b)	Show that two variables are independent if their joint PDF factorizes.	L3	CO2	5 M
		OR			
7	a)	Illustrate the Central Limit Theorem.	L3	CO2	5 M

	b)	Interpret the distribution of sum of two independent Gaussian variables.	L3	CO2	5 M
		UNIT-IV			
8	a)	Explain the classification of random processes with examples.	L2	CO4	5 M
	b)	Illustrate the properties of WSS processes.	L3	CO4	5 M
		OR			
9	a)	Statistically independent zero mean random processes X(t) and Y(t) have autocorrelation functions $R_{XX}(\tau) = e^{-\lvert\tau\rvert}$ and $R_{YY}(\tau) = \cos(2\pi\tau)$ respectively (i) Calculate autocorrelation function of the sum $W_1(t) = X(t) + Y(t)$ (ii) Calculate autocorrelation function of the difference $W_2(t) = X(t) - Y(t)$	L3	CO4	5 M
	b)	Determine ergodicity of a given random process $X(t) = A\cos(\omega_c t+\theta)$ is WSS if it is assumed that ω_c is a constant and θ is a uniformly distributed variable in the interval $(0,2\pi)$.	L3	CO4	5 M
		UNIT-V			
10	a)	Define and explain Power Spectral Density (PSD).	L2	CO1	5 M
	b)	Explain three important properties of PSD.	L2	CO1	5 M

PVP23

Code: 23BS1304

II B.Tech – I Semester – Regular/Supplementary Examinations, November 2025

PROBABILITY THEORY AND STOCHASTIC PROCESS

(Electronics & communication Engineering)

PART-A

1) a) Explain sample space with an example — 2M.

— Sample space statement — 1M.

— Writing any one example – 1M.

(b) State Baye's theorem.

— Statement of Baye's theorem – 2M.

(C). Illustrate expected value of a random variable.

— Expected value of a random variable – 2M

(d) Explain the variance of a random variable.

— Variance of a random variable – 2M.

(e) State marginal probability distribution.

— ~~The~~ statement of marginal probability distribution – 2M.

(f) State the central limit theorem.

— Statement of central limit theorem – 2M.

g) Illustrate wide-sense Stationarity.

— Explanation of WSS – 2M.

h) State any two properties of autocorrelation function.

— Any Two properties – 2M.

(i) Explain the effective noise temperature.

— Explanation of noise temperature – 2M.

(j) Explain narrowband noise.

– Explanation of narrowband noise – 2M.

Part-B

2) (a) Explain probability axioms with suitable examples.

– Axioms of probability with example – 5M

(b) Illustrate the law of total probability.

– law of Total probability – 5M.

3) a) Solve a real world problem using Baye's theorem.

— Baye's theorem statement – 2M

— Any Real world problem solution – 3M.

3 (b) Differentiate independent and mutually exclusive events with examples.

— Independent & mutually exclusively events — 5M.

4 a) Compute the mean of a binomial distribution.

— Mean of a binomial distribution — 5M.

(b) Apply Binomial distribution to a practical problem (e.g., coin tossing)

— Any condition for coin tossing problem Solution — 5M.

5 (a) Define the MGF of gaussian distribution

— MGF of gaussian distribution — 5M.

(b) State and prove Chebyshev's inequality.

— Statement — 2M.
— proof — 3M.

6 (a) Interpret marginal distribution from joint distribution.

— marginal distribution from joint distribution — 5M.

(b) show that two variables are independent if their joint PDF factorizes.

— proof — 5M.

7 (a) Illustrate the central limit theorem.

— Statement — 2M.
— unequal — 1M.
— equal — 2M.

(b) Interpret the distribution of sum of two independent gaussian Variables.

— Explanation 5M.

8 (a) Explain the classification of random process with examples.

— classification with examples — 5M.

(b) Illustrate the properties of WSS processes

— Any five properties — 5M.

9 (a). statistically independent zero mean random processes $X(t)$ and $Y(t)$ have auto correlation functions.

$R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = \cos(2\pi\tau)$

respectively.

(i) calculate auto correlation functions of the sum $W(t) = X(t) + Y(t)$

(ii) Calculate autocorrelation function of the difference $w_2(t) = X(t) - Y(t)$.

— ~~obt~~ writing data — 1M.
— obtaining $w_1(t)$ — 2M.
— Obtaining $w_2(t)$ — 2M

(b) Determine ergodicity of a given random process $X(t) = A\cos(\omega_c t + \theta)$ is WSS if it is assumed that ω_c is a constant and θ is a uniformly distributed variable in the interval $(0, 2\pi)$

— obtaining mean (time) — 1M
— " Autocorrelation (time) — 1M.
— " ensemble mean — 1M.
— " " Autocorrelation — 1M.
— statement for ergodicity — 1M

(10.) a) Define and explain power spectral density (PSD).

— Definition — 1M.
— Explanation — 4M.

(b) Explain three important properties of PSD.

— Any three important properties of PSD — 5M.

11 (a) Illustrate different noise sources with example

— Any 5 different noise sources with example — 5M.

(b) Illustrate the expression for noise figure of cascaded networks.

— Noise figure of cascaded Nwk — 5M.

Code : 23BS1304 PVP-23

II B.Tech - I Semester - Regular/supplementary Examinations.

NOVEMBER - 2025

PROBABILITY THEORY AND STOCHASTIC PROCESS

[ELECTRONICS & COMMUNICATION ENGINEERING]

PART - A

1.a) Explain Sample space with an example. — 2M.

Ans: The set of all possible outcomes of the experime is called sample space.

Eg: the experiment, tossing an unbiased coin has two outcomes - head (H) or tails (T). so the sample space is $\{H, T\}$.

(b) state Bayes' theorem: — 2M.

It state that if a sample space S has N mutually exclusive events B_n, $n = 1, 2, 3 \ldots, N$, such that, $B_m \cap B_n = \{\phi\}$ for $m \neq n = 1, 2, \ldots, N$, and any event A is defined on this sample space, then the conditional probability of B_n and A can be written as

$$P(B_n/A) = \frac{P(A/B_n)\,P(B_n)}{P(A/B_1)\,P(B_1) + P(A/B_2)\,P(B_2) + \cdots + P(A/B_N)\,P(B_N)}.$$

(f). Marginal probability distribution: If X and Y are two random variables with joint PDF $f_{XY}(x,y)$, then the Marginal PDF of X is $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$ &

(c) Illustrate expected Value of a random Variable — 2M

Ans: A random variable (RV) is a real function of the events of a given sample space S.

If X is a Continuous random variable with a Valid Probability density function $f_X(x)$, then the expected Value of X or the mean Value of X is defined as

$$E[X] = \overline{X} = \int_{-\infty}^{\infty} x f_X(x)\, dx.$$

for discrete RV $\Rightarrow E[X] = \sum_{i=1}^{N} x_i\, P(x_i)$.

(d) Explain the variance of a random Variable — 2M.

Ans: The Variance of the density function $f_X(x)$ for a random Variable X is defined as the second Central moment μ_2 of X. It is also denoted as σ_X^2 or Var (X) and is given by $\mu_2 = \sigma_X^2 = E[(X-\overline{X})^2] = \int_{-\infty}^{\infty} (x-\overline{X})^2 f_X(x) dx$

(e) state the Central Limit Theorem. — 2M

It states that the probability density function of a sum of N independent random variables approaches the gaussian density function as N tends to infinity.

(g) Illustrate Wide-Sense Stationarity — 2M

If a random process X(t) is a second order stationary process, then it is called a wide sense stationary process (WSS) or a weak sense stationary process X. The Conditions for a wide sense stationary process are.

① $E[X(t)] = \overline{X}$ = constant. ② $E[X(t)X(t+\tau)] = R_{XX}(\tau)$ is independent of absolute time t

(h) State any two properties of auto correlation function — 2M.

Ans: (1) The mean square value of $X(t)$ is

$$E[X^2(t)] = R_{xx}(0)$$ It is equal to the power of the process $X(t)$.

(2) Auto correlation function is maximum at the origin, ie,

$$|R_{xx}(\tau)| \leq R_{xx}(0)$$

note:
Writing any two properties give — 2 Marks.

(i) Explain the effective noise temperature — 2M.

Ans:
The noise temperature is given by

$$T = \frac{P_a}{KB} \text{ kelvin.}$$

or

$$T_n = \frac{2 S_{NN}(\omega)}{K}.$$

or

for Series resistors $T_n = \frac{R_1T_1 + R_2T_2}{T_1 + T_2}$. or $T_n = \frac{R_1T_1 + R_2T_2}{R_1 + R_2}$

for parallel resistors

$$T_n = \frac{T_1R_2 + T_2R_1}{R_1 + R_2}.$$

note:
any thing valid if they written series or parallel that is also valid — 2 Marks

(j). Explain narrow band noise — 2M.

Ans: If white noise is passed through a bandpass filter whose band width is $W_N << \omega_0$, the noise process appearing at the output is called narrow band noise. The spectral components of a narrow band noise are ~~Connected~~ Concentrated about some mid-band frequency $\pm\omega_0$. — 1

$$N(t) = N_1(t)\cos(\omega_0 t) - N_Q(t)\sin(\omega_0 t) \text{ — 1M.}$$

PART-B

UNIT-I

(2) (a) Explain probability axioms with suitable examples.

Ans: The probability $P(A)$ of an event is a real number which satisfies the following axioms:

Axiom 1: $P(A) \geq 0$. ———— 1 Mark

The probility of event A is always a non-negative real number.

Axiom 2: $P(S) = 1$. ———— 1 Mark

The event S is an universal Set and it is a Sure event. therefore, its probability is always one.

$P(\phi) = 0$ ———— 1 Mark

The probability of a null event or an impossible event is always zero.

Axiom 3: ———— 1 Mark

If a Sample A_n has N events, $n = 1, 2, \ldots, N$, where N may be infinite and if all the events are mutually exclusively, i.e, $A_m \cap A_n = \{\phi\}$ for all $m \neq n$, then

$$P\left(\bigcup_{n=1}^{N} A_n\right) = \sum_{n=1}^{N} P(A_n)$$

Writing any example for each axiom ———— 1 Mark

(2) (b) Illustrate the law of total probability.

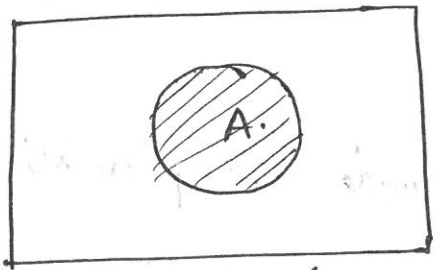
Ans: Consider a sample space S that has N mutually exclusive events B_n, $n=1,2,\dots,N$, such that $B_m \cap B_n = \{\phi\}$ for $m \neq n = 1,2\dots N$. The probability of any event A, defined on this sample space can be expressed interms of the conditional probabilities of events B_n. This probability is known as the total probability of event A. 2M

Mathematically,

$$P(A) = \sum_{n=1}^{N} P(A/B_n)\, P(B_n) \text{ —— } 1M.$$

Proof: —— 2M

The sample space S of N mutually exclusive events, B_n $n=1,2\dots,N$



The events have the properties.

$$B_m \cap B_n = \{\phi\},\quad m \neq n = 1,2,3,\dots N \quad \text{and} \quad \bigcup_{n=1}^{N} B_n = S.$$

The events $P(A \cap B_n)$ are mutually exclusive, by applying the third axiomatic definition of probability.

$$P(A) = \sum_{n=1}^{N} P(A \cap B_n).$$

From the definition of joint probability

$$P(A \cap B_n) = P(A/B_n)\, P(B_n)$$

therefore, $P(A) = \sum_{n=1}^{N} P(A/B_n)\, P(B_n)$.

OR.

3) (a). Solve a real-world problem using Bayes' theorem.

Ans:
Baye's theorem: ———————— 2M

It states that if a sample space S has N mutually exclusive events, B_n, $n=1,2,---N$, such that, $B_m \cap B_n = \{\phi\}$ for $m \neq n = 1,2----,N$, and any event A is defined on this sample space, then the conditional probability of B_n and A can be written as

$$P(B_n|A) = \frac{P(A|B_n)\,P(B_n)}{P(A|B_1)\,P(B_1) + P(A|B_2)\,P(B_2) + --- + P(A|B_N)\,P(B_N)}$$

Any example explanation/ problem ——— 3M.

3. (b) Differentiate independent and mutually exclusive events with examples.

Ans: Definition Independent Events: Two events A and B are independent if the occurrence of one event does NOT affect the occurrence of the other.

$$P(A \cap B) = P(A)\,P(B).$$

eg: Event A: A coin shows Heads.
Event B: A dice shows 4.
The coin result does not affect the die result.
$P(A) = 1/2$, $P(B) = 1/6$, $P(A \cap B) = 1/2 \times 1/6 = 1/12$

2. Mutually Exclusive Events:

Definition:

Two events A and B are mutually exclusive if they cannot occur at the same time

mathematical condition:

$$P(A \cap B) = 0.$$

eg: Event A: Rolling a 2 on a die

Event B: Rolling a 5 on a die.

You cannot get 2 and 5 at the same time on a single roll $P(A \cap B) = 0$. So A and B are mutually exclusively.

UNIT- II

(4) (a) Compute the mean of a Binomial distributio

Ans: A binomial (n, P) distribution gives the probability of getting x successes in n dependent Bernoulli trails where

probability of success = p.

probability of failure $q = 1-p$.

A binomial random Variables X takes Values

$0, 1, 2, - - -, n$.

with probability mass function (PMF)

$$P(X=x) = \binom{n}{x} p^x q^{n-z}.$$

finally $\boxed{E(X) = np}$

The mean of a binomial (n, P) distribution is np because each of the 'n' independent bernoulli trials contributes P expected success.

4 (b) Apply Binomial distribution to a practical problem (e.g: coin tossing). NOTE: Any thing Corresponding Coin tossing Game. 5M.

Ans: The distribution can be applied to many games of chance, detection problems in radar and sonar.

A fair coin is tossed 12 times.
find the probability of getting
1. Exactly 7 heads. (2) At least 9 heads.

$p = P(\text{head}) = \frac{1}{2}$ $q = 1-p = \frac{1}{2}$

No. of trials $n = 12$.

Let X = no. of heads in 12 tosses.

$$X \sim B(n, p) = B(12, 0.5)$$

$$P(X=x) = \binom{12}{x} p^x q^{12-x}$$ (probability mass function).

(1) probability of getting exactly 7 heads.

$$P(X=7) = \binom{12}{7} \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^5$$

$$P(X=7) = 792 \times \frac{1}{4096} \approx 0.193$$

(2) probability of getting at least 9 heads.

$$P(X \geq 9) = P(X=9) + P(X=10) + P(X=11) + P(X=12)$$

term (1) $P(X=9) = \frac{220}{4096}$

(2) $P(X=10) = 66/4096$

(3) $P(X=11) = 12/4096$.

(4) $P(X=12) = 1/4096$

$$P(X \geq 9) = \frac{220 + 66 + 12 + 1}{4096} \approx 0.073$$

5(a) Derive the MGF of Gaussian distribution :- 5M

Ans:

let $X \sim N(\mu, \sigma^2)$

The PDF of X is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

The moment generating function (MGF) is defined as

$$M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx.$$

Substitute the PDF

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx.$$

Combine the exponent terms:

$$M_X(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(tx - \frac{(x-\mu)^2}{2\sigma^2}\right) dx.$$

Complete the square in the component.

expand the quadratic term:

$$tx - \frac{(x-\mu)^2}{2\sigma^2} = -\frac{1}{2\sigma^2}\left[(x-\mu)^2 - 2\sigma^2 tx\right]$$

thus.

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left[-\frac{(x-(\mu+\sigma^2 t))^2}{2\sigma^2}\right]$$

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

for a gaussian random Variable $X \sim N(\mu, \sigma^2)$

$$M_X(t) = E[e^{tX}] = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

5(b). state and prove chebyshev's inequality — 5M

Ans: Statement: — 2M.

For a given random variable X with mean value $\overline{X}$ and Variance σ_X^2, it states that $P\{|X-\overline{X}| \geq \epsilon\} \leq \sigma_X^2/\epsilon^2$.

where ϵ is a very small positive number. — 2M

Proof — 3M

we know that the probability density function of a random variable X is given by

$$P\{X \leq x\} = F_X(x) = \int_{-\infty}^{\infty} f_X(x)\,dx.$$

Now expand $P\{|X-\overline{X}| \geq \epsilon\} = P\{(X-\overline{X}) \leq -\epsilon\} + P\{(X-\overline{X})$

$$\therefore P\{|X-\overline{X}| \geq \epsilon\} = \int_{|X-\overline{X}| \geq 0}^{\infty} f_X(x)\,dx.$$

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x-\overline{X})^2 f_X(x)\,dx.$$

$$\sigma_X^2 \geq \int_{|X-\overline{X}| \geq \epsilon}^{\infty} (x-\overline{X})^2 f_X(x)\,dx.$$

if $X-\overline{X} = \epsilon$,

$$\sigma_X^2 \geq \epsilon^2 P\{|X-\overline{X}| \geq \epsilon\}.$$

$$\therefore P\{|X-\overline{X}| \geq \epsilon\} \leq \frac{\sigma_X^2}{\epsilon^2} \text{ proved.}$$

6(a) Interpret marginal distribution from joint distribution

(6(a)) ~~Derive the MGF of Gaussian distribution~~. — 5M

Ans: The marginal distribution of a random variable is the probability distribution of that variable alone.

Def: If X and Y are two random variables with joint distribution $f_{X,Y}(x, y)$, then the marginal distribution of a variable is found by eliminating the other variable.

Discrete random variables with joint PMF

$$P_{XY}(x,y) : \quad P_X(x) = \sum_{y} P_{XY}(x,y)$$

$$P_Y(y) = \sum_{x} P_{XY}(x,y).$$

This means the probability for each value of X is obtained by summing all joint probabilities overall values of Y

Continuous random variables with joint PDF

$$f_{X,Y}(x,y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx.$$

Here the unwanted variable is removed by integration.

UNIT-III

(6). (a) Interpret marginal distribution from joint distribution

Ans:

Joint distribution – Behaviour of both variables together

Marginal distribution – behaviour of one variable independently.

Therefore, marginal distribution is obtained by summing or integrating the joint distribution over the other variable, giving the probability distribution of a single variable independently.

6 (b) Show that two variables are independent if their joint PDF factorizes. —— 5M

Ans: Let X and Y be two continuous random variable with joint PDF

$$f_{X,Y}(x,y) = g(x)\,h(y) \text{ for all } x, y,$$

where $g, h \geq 0$ and $\iint f_{XY}(x,y)\,dx\,dy = 1$

(1) Compute the marginal of X

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_{-\infty}^{\infty} g(x)\,h(y)\,dy$$

(2) Compute the marginal of Y

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_{-\infty}^{\infty} g(x)\,hy\,dx.$$

(3) normalization of the ~~joint~~ joint PDF give.

$$1 = \iint f_{XY}(x,y)\,dx\,dy) = \left(\int g(x)\,dx\right)\left(\int h(y)\,dy\right)$$

so $GH = 1$.

Multiply the Marginals.

$$f_X(x)\,f_Y(y) = (g(x)\,H)\,(h(y)\,G)) = (GH)\,g(x)\,h(y)$$

$$f_X(x)\,f_Y(y) = f_{X,Y}(x,y).$$

If the joint PDF factorizes as

$$f_{X,Y}(x,y) = g(x)\,h(y)$$

then it can be written as

$$f_{X,Y}(x,y)\ f_X(x)\,f_Y(y).$$

7 (a) Illustrate the central limit theorem. — 5M.

Ans: It states that the probability density function of a sum of N independent random variables approaches the gaussian density function as N tends to infinity. — 2M.

The central limit theorem tells us that for large N, the distribution of the sum $Y = X_1 + X_2 + \cdots + X_N$ is approximatel normal regardless of the form of the distribution of the individual random variables X_N. In practice, where ever an observed random variable is known to be a sum of a larger number of random variables.

Unequal distribution: — 1M.

Let N random variables X_n, $n = 1, 2, 3, \ldots, N$ have probability density functions, with mean $\bar{X}_n$ and variance σ_X^2. The central limit theorem states that the sum of the random variables $Y_N = X_1 + X_2 + X_3 + \cdots + X_N$, with mean $\bar{Y}_N = \bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \cdots + \bar{X}_N$ and variance $\sigma_Y^2 = \sigma$ $+ \sigma_{X_2}^2 + \cdots + \sigma_{X_N}^2$, have a probability distribution functi asymptotically approaching a Gaussian distribution as N tends to infinity.

The sufficient conditions are

$$\sigma_{X_N}^2 > B_1 > 0 \quad n = 1, 2, 3 \ldots N.$$

$$\& \; E\left[\left| X_n - \bar{X}_n \right|^3\right] < B_2 \quad n = 1, 2, 3 \ldots N$$

Equal distributions: — 2M.

Consider N continuous random variables X_n, $n=1,2,\dots N$ have the same distribution and density functions.

let $Y = X_1 + X_2 + \dots + X_N$.

$$W = \frac{Y - \bar{Y}}{\sigma_y}.$$

$$W = \frac{\sum_{n=1}^{N} X_n - \sum_{n=1}^{N} \bar{X}_n}{\left[\sum_{n=1}^{N} \sigma_x^2\right]^{1/2}}.$$

Since all random variables have the same distribution,

$$W = \frac{1}{\sqrt{N}\,\sigma_x} \sum_{n=1}^{N} (x_n - \bar{x}).$$

7(b) Interpret the distribution of sum of two independent Gaussian variables. — 5M.

Ans: The central limit theorem states that the sum of the random variables $Y_N = X_1 + X_2 + X_3 + \dots + X_N$ and with mean $\bar{Y}_N = \bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \dots + \bar{X}_N$ and variance $\sigma_y^2 = \sigma_{x_1}^2 + \sigma_{x_2}^2 + \dots + \sigma_{x_N}^2$ have a probability distribution function asymptotically approaching gaussian distribution as N tends to infinity.

mean $\Rightarrow E[Z] = E[X] + E[Y] = \mu_X + \mu_Y$

Variance $\Rightarrow$

$$Var[Z] = Var(X) + Var(Y) = \sigma_X^2 + \sigma_Y^2$$

Resulting distribution.

$$Z \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2).$$

(8) 10

(8) (a) Explain the classification of random processes with examples. — 5M

Ans: Random processes are mainly classified into four types based on time t and amplitude of the random variable X

(1) Continuous Random processes:
A random process is said to be continuous if both the random variable X and time are continuous over the entire time.
eg: the fluctuations of a noise voltage in any network is a continuous Random process.

(2) Discrete Random processes:
In a discrete random process, the random variables X has only discrete values while time t is continuous.
eg: A digital encoded signal has only two discrete values.

(3) Continuous Random Sequences:
A random process for which the random variable X is continuous but time t has discrete values is called a continuous random sequence.

eg. A continuous random signal is defined only at discrete (sample) time time intervals.

(4) Discrete Random Sequence: In a discrete random sequence, both random variables X and time t are discrete
A discrete random sequence can be obtained by samplin

and quantizing a random signal. Eg: Digital Signal processing App

& (b) Illustrate the properties of WSS processes. - 5M

Ans: If a random process $X(t)$ is a second order stationary process, then it is called a wide sense stationary process (WSS) or a weak sense stationary process X. However the converse is not true. 2M

the conditions for a WSS are. - 3M

(1) $E[x(t)] = \bar{x}$ = constant.

(2) $E[x(t)\, x(t+\tau)] = R_{xx}(\tau)$ is independent of absolute time t.

(3). Autocorrelation depends only on time difference

(4) Auto covariance also depends only on time difference. $C_x(t_1, t_2) = C_x(\tau)$.

(5) Auto correlation is Even symmetric

$R_x(\tau) = R_x(-\tau)$.

(a)(a) Statistically independent zero mean random processes $X(t)$ and $Y(t)$ have auto correlation functions.

$$R_{XX}(\tau) = e^{-|\tau|} \quad \& \quad R_{YY}(\tau) = \cos(2\pi\tau) \text{ respectively}$$

① $W_1(t) = X(t) + Y(t)$ 2M ② $W_2(t) = X(t) - Y(t)$. 2M

DATA – 1M

Sol: Given $X(t)$ and $Y(t)$ are two statistically independent zero mean random processes. the auto correlation functions are $R_{XX}(\tau) = e^{-|\tau|}$.

$$R_{YY}(\tau) = \cos(2\pi\tau)$$

① $W_1(t) = X(t) + Y(t)$

$$R_{W_1W_1}(\tau) = E[W_1(t) W_1(t+\tau)]$$

$$= E\{(X(t) + Y(t))(X(t+\tau) + Y(t+\tau))\}$$

$$\Rightarrow E[(X(t) X(t+\tau) + X(t) Y(t+\tau) + X(t+\tau) Y(\tau) + Y(t) Y(t+\tau)]$$

$$\Rightarrow E[X(t) X(t+\tau)] + E[X(t) Y(t+\tau)] + E(Y(t) X(t+\tau)] + E[Y(t) Y(t+\tau)]$$

$$\therefore R_{W_1W_1}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}$$

Since $X(t)$ and $Y(t)$ are statistically independent

$$R_{XY}(\tau) = R_{YX}(\tau) = 0$$

$$\therefore R_{W_1W_1}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$$

$$= e^{|\tau|} + \cos 2\pi\tau.$$

(b) $W_2(t)$ is

$$R_{W_2W_2}(\tau) = E\left[W_2(t)W_2(t+\tau)\right].$$

$$= E\left[(x(t)-Y(t))(x(t+\tau)-Y(t+\tau))\right]$$

$$\Rightarrow E\left[x(t)x(t+\tau)\right]-E\left[x(t)Y(t+\tau)\right]-E\left[Y(t)\,x(t+\tau)\right]$$

$$+E\left[Y(t)Y(t+\tau)\right].$$

$$R_{W_2W_2}(\tau) = R_{XX}(\tau) - R_{XY}(\tau) - R_{YX}(\tau) + R_{YY}(\tau)$$

$$= R_{XX}(\tau) + R_{YY}(\tau)$$

$$\underline{R_{W_2W_2}(\tau) = e^{-|\tau|} + \cos(2\pi\tau)}.$$

9(b). Determine ergodicity of a given random process $X(t) = A\cos(\omega_c t+\theta)$ is WSS if it is assumed that ω_c is a constant and θ is a uniformly distributed ~~vals~~ variable in the interval $(0, 2\pi)$.

Sol: Random process $X(t) = A\cos(\omega_c t+\theta)$

① Mean — 1M

$$E\left[\cos(\omega_c t+\theta)\right] = \frac{1}{2\pi}\int_0^{2\pi}\cos(\omega t+\theta)\,d\theta = 0.$$

$$m_X(t) = 0 \text{ (constant)}$$

② Auto correlation function. — 1M

$$R_X(\tau) = E\left[x(t)x(t+\tau)\right].$$

$$R_X(\tau) = A^2 E\left[\cos(\omega t+\theta)\cos(\omega(t+\tau)+\theta)\right]$$

$$= A^2/2\,\cos(\omega\tau)$$

It depend only on τ, so the process is wss

(3) Time - average mean. (for ergodicity in mean) — 1M.

$$\bar{x} = \lim_{T \to \infty} \frac{1}{T} \int_0^T A \cos(\omega_c t + \theta) dt = 0.$$

$$= 0.$$

(4) Time Average auto correlation function) — 1M

$$R(\tau) = \frac{A^2}{2} \cos(\omega \tau)$$

This equals the ensemble auto correlation.

Since the time averages of mean and autocorrelation are equal to the ensemble averages, so the process is ergodic. — 1M.

10 (a) Define and explain power spectral density (PSD)

Ans: Definition: — 2M

The power spectrum density of a wss random process $X(t)$ is defined as the fourier transform of the auto correlation function $R_{XX}(\tau)$ of $X(t)$. It can be expressed as $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$.

The auto correlation function from the power spectral density by taking the inverse fourier transform $R_{XX}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) e^{j\omega\tau} d\omega$

therefore, the power density spectrum $S_{XX}(\omega)$ and the autocorrelation function $R_{XX}(\tau)$ are fourier transform pairs.

Definition 2 — 3M.

the power spectral density can also be defined as

$$S_{XX}(\omega) = \lim_{T \to \infty} \frac{E\left[|X_T(\omega)|^2\right]}{2T}.$$

where $X_T(\omega)$ is a Fourier transform of $X(t)$ in the interval $[-T, T]$.

(10) (b) Explain three important properties of PSD.

Ans: NOTE: Any three properties — 5M.

The properties of the power density spectrum $S_{XX}(\omega)$ for a WSS random process $X(t)$ are given as.

① $S_{XX}(\omega) \geq 0$.

proof: From the definition, the expected value of a non-negative function $E\left[|X_T(\omega)|^2\right]$ is always non-negative.

Hence $S_{XX}(\omega) \geq 0$.

② The power spectral density at zero frequency is equal to the area under the curve of the autocorrelation $R_{XX}(\tau)$.

that is, $S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\,d\tau$.

proof :-

from the definition, we know that

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau.$$

At $\omega = 0$.

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\,d\tau.$$

(3) the power density spectrum of a real process $X(t)$ is an even function ie $S_{XX}(-\omega) = S_{XX}(\omega)$ $X(t)$ is real.

proof :

Consider a WSS real process $X(t)$

then $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$.

also $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$

substitute $\tau = -\tau$ then.

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)\, e^{-j\omega\tau}\, d\tau.$$

Since $X(t)$ is real, from the properties of autocorrelation of $R_{XX}(-\tau) = R_{XX}(\tau)$

$$\therefore S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)\, e^{-j\omega\tau}\, d\tau.$$

$$S_{XX}(-\omega) = S_{XX}(\omega)$$

$\therefore S_{XX}(\omega)$ is an even function.

11 (a) Illustrate different noise sources with examples.

Ans: Any Five — 5M

(1) Thermal noise:

- caused by random motion of electrons in resistors, wires and semiconductor devices.
- power spectral density is white.

Eg: Noise generated in a resistor.

(2) Shot noise

- caused by random arrival of charge carriers across a barrier.

Eg: Diodes, transistors, photodiodes.

(3) Flicker noise (Pink noise)

- Dominant at low frequencies (<100

Eg: low frequency noise in MOSFETs.

(4) Transit-time noise:

- occurs at high frequencies when the transit time of carriers through a device is comparable to the signal period.

Eg: High frequency transistors, μwave devi

(5) Burst noise:

- Sudden step-like pulses appearing randomly.
- caused by impurities or defects in semiconductor junctions

Eg: old

11 (b) Illustrate the expression for Noise figure of cascaded nwks. — 5M.

Ans: Noise figure:

Consider two amplifiers that are cascaded.

G_{a1} : power gain of amplifier 1

G_{a2} : " " " 2

T_{e1} : Equivalent i/p noise temperature of amplifier 1

T_{e2} : " 2

F_1 : Noise figure of amplifier 1

F_2 : " " " 2

The overall of gain is

$$G_a = G_{a1} \times G_{a2}$$

The total o/p noise power available is

$$N_o = N_{o1} + N_{o2} + N_{o3}$$

$$\therefore N_{o1} = G_{a1} G_{a2} N_i = N_i G_a$$

$$N_{o2} = N_{sys1} = G_{a1} N_i (F_1 - 1) G_{a2}$$

$$= G_a N_i (F_1 - 1).$$

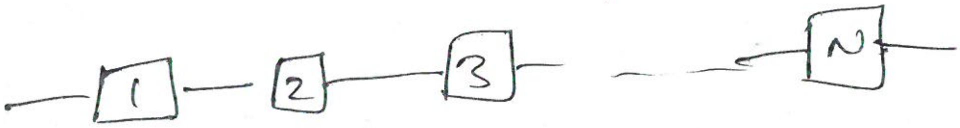
$$G_a N_i F = G_a N_i + G_a N_i (F_1 - 1) + G_{a2} N_i (F_2 - 1)$$

$$F = 1 + F_1 - 1 + \frac{(F_2 - 1) G_{a2}}{G_{a1} + G_{a2}}$$

$$\text{or } F = F_1 + \frac{F_2 - 1}{G_{a1}}$$

Similarly, for three amplifiers cascaded

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}}$$



overall noise figure is

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}} + \cdots + \frac{F_n - 1}{G_{a1} G_{a2} \cdots G_{a}}$$