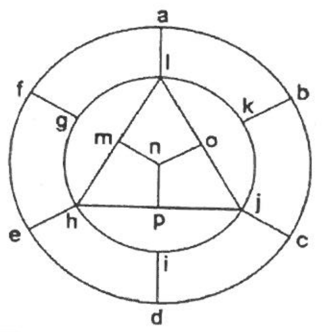
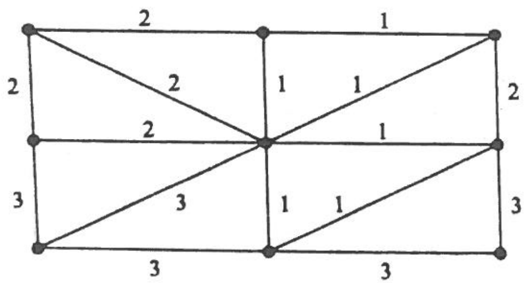
UNIT-V					
10	a)	Explain Depth First Search algorithm with an example.	L4	CO4	5 M
	b)	Show that the following graph is not Hamiltonian. 	L4	CO4	5 M
OR					
11		Define minimal spanning tree. Find the minimal spanning tree for the following weighted graph using Krushkal's algorithm: 	L4	CO4	10 M

PVP 23

Code: 23BS1305

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

DISCRETE MATHEMATICS AND GRAPH THEORY

(Common for CSE, IT, AIML, DS)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

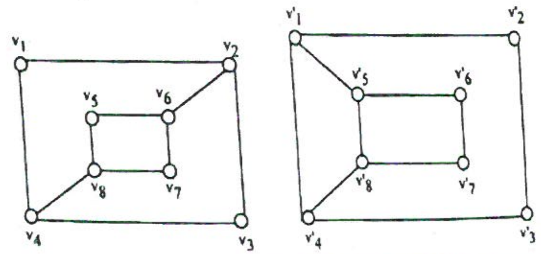
BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Find the duality of the statement $(p \to q) \to r$.	L2	CO1
1.b)	Obtain Conjunctive Normal Form (CNF) of $\sim[p \to (q \vee r)]$.	L2	CO1
1.c)	Define Predicate with an example.	L2	CO1
1.d)	State Rules of Modus Ponens and Modus Tollens.	L2	CO1
1.e)	Write the general form of a third order non-homogeneous recurrence relation.	L2	CO1
1.f)	Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = 0$.	L3	CO3
1.g)	Define partially ordered set.	L2	CO1
1.h)	Draw the Hasse diagram for positive divisors of 12.	L2	CO1
1.i)	Define planar graph with an example.	L2	CO1
1.j)	Define chromatic number of a graph.	L2	CO1

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Prove that $[P\to(Q\to R)]\to[(P\to Q)\to(P\to R)]$ is a tautology.	L3	CO2	5 M
	b)	Obtain PCNF of $\sim(P\vee Q)\leftrightarrow(P\wedge Q)$.	L3	CO2	5 M
		OR			
3	a)	Prove that $\sim(P\wedge Q)\to[\sim P\vee(\sim P\vee Q)]\Leftrightarrow(\sim P\vee Q)$.	L3	CO2	5 M
	b)	Obtain PDNF of $(\sim P\to Q)\wedge(Q\leftrightarrow P)$.	L3	CO2	5 M
		UNIT-II			
4	a)	Show that $R\wedge(P\vee Q)$ is a valid conclusion from the premises $P\vee Q, Q\to R, P\to M$ and $\sim M$.	L3	CO2	5 M
	b)	Verify the validity of the following argument: Tigers are dangerous animals. There are tigers. Therefore, there are dangerous animals.	L3	CO2	5 M
		OR			
5	a)	Write the following quantified statements in symbolic form: i. Some roses are yellow ii. All Russians are taller than all Americans iii. Some monkeys have no tail.	L2	CO2	5 M

	b)	Prove that $(\forall x)(P(x)\vee(\forall x)(Q(x)\Leftrightarrow(\forall x)(P(x)\vee Q(x))$ is logically valid.	L3	CO2	5 M
		UNIT-III			
6		Slove $a_{n+2}-5a_{n+1}+6a_n=3n+4$ with $a_0=2,\ a_1=5$	L3	CO3	10 M
		OR			
7		Solve $a_r-7a_{r-1}+16a_{r-2}-12a_{r-3}=0,\ r\geq 3$ with $a_0=1,\ a_1=4$ and $a_2=8$ by the method of characteristic roots.	L3	CO3	10 M
		UNIT-IV			
8		Define an equivalence relation. Let $\mathbb{Z}$ denote the set of integers and the relation R on $\mathbb{Z}$ be defined by aRb if and only if $a-b$ is an integer. Prove that R is an equivalence relation.	L3	CO4	10 M
		OR			
9	a)	Let R be a relation defined on $A=\{1,2,3,4,5\}$ by $R=\{(x,y)/x\geq y\}$. Verify whether R is partial ordered relation or not?	L4	CO4	5 M
	b)	Verify whether the following graphs are isomorphic or not? 	L4	CO4	5 M

II B.Tech - I Semester - Regular Examinations

DISCRETE MATHEMATICS AND GRAPH THEORY.
(Common for CSE, IT, AIML, DS).

PART- A

1a) Find the duality of the statement

Let $A = (P \to Q) \to r$.

$A = (\sim P \vee Q) \to r$.

$= \sim(\sim P \vee Q) \vee r = (P \wedge \sim Q) \vee r$. ——— 1M

Duality of the statement $\Rightarrow A^* = (P \vee \sim Q) \wedge r$. ——— 1M

b) obtain CNF of $\sim[P \to (Q \vee r)]$.

$\sim[\sim P \vee (Q \vee r)]$

$\Rightarrow P \wedge \sim(Q \vee r)$

$\Rightarrow P \wedge (\sim Q \wedge \sim r)$. ——— 2M

c). Define predicate with an example.

The part of a sentence that describes the action or state of the subject. ——— 1M

Ex:- Indhu is a student. ——— 1M

subject is "Indhu" & predicate is " is a student".

d) State rules of Modus Ponens & Modus Tollens.

Modus Ponens :: $P, P \to Q \Leftrightarrow Q$. ——— 1M

Modus Tollens :- $\sim Q, P \to Q \Leftrightarrow \sim P$. ——— 1M

e). write the general form of a 3^{rd} order non-homogeneous recurrence relation.

$a_n + c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} = f(n)$, $n \geq 3$.

$f(n) \neq 0$, c_1, c_2, c_3 are constants. ——— 2M

f) solve $a_n - 4a_{n-1} + 4a_{n-2} = 0$

characteristic equation is $t^2 - 4t + 4 = 0$.

$(t-2)^2 = 0 \Rightarrow t = 2, 2$ — 1M

$a_n^{(h)} = (c_1 + c_2 n)(2)^n$.

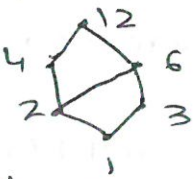
——— 1M

g) Define Partially ordered set.

A relation R on a set 'P' is called a partial order relation iff R is reflexive, antisymmetric & Transitive. It is denoted by the symbol $\leq$. — 2M

h) Draw the Hasse diagram for positive divisors of 12.

$D_{12} = \{1, 2, 3, 4, 6, 12\}$.

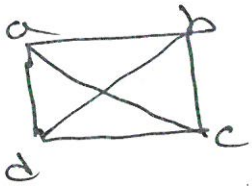


— 2M

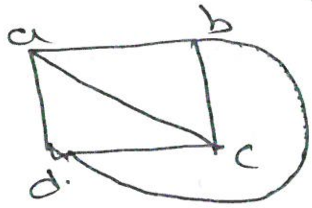
i). Define planar graph with an example.

A graph 'G' is said to be planar if it can be drawn in a plane so that its edges do not cross over. — 1M

Ex:-



planar graph



— 1M

j. Define chromatic number of a graph.

It is the minimum number of colours required to colour the vertices of G, such that no two adjacent vertices receive the same colour. It is denoted by $\chi(G)$. — 2M

PART-B

2 a) P.T $[P\to(Q\to R)] \to [(P\to Q)\to(P\to R)]$ is a tautology.

P	Q	R	$\overset{a}{P\to Q}$	$\overset{b}{P\to R}$	$\overset{c}{Q\to R}$	$\overset{d}{P\to c}$	$\overset{e}{a\to b}$	$d\to e$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

— 5M

$\Rightarrow$ Given formula is a Tautology.

2b) obtain PCNF of $\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$P \wedge Q$	$\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$
T	T	T	F	T	F
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	F	F

— 3M

Min terms are $(P \wedge \neg Q)$, $(\neg P \wedge Q)$. — 1M

P.d.n.f $= (P \wedge \neg Q) \vee (\neg P \wedge Q) = A$.

$\neg A = (P \wedge Q) \vee (\neg P \wedge \neg Q)$.

$\neg(\neg A) =$ P.C.N.F ~~of~~ ~~A~~ $= (\neg P \vee \neg Q) \wedge (P \vee Q)$. — 1M

OR

3a) P.T $\neg(P \wedge Q) \rightarrow [\neg P \vee (\neg P \vee Q)] \Leftrightarrow (\neg P \vee Q)$

P	Q	$P \wedge Q$	$\overset{a}{\neg(P \wedge Q)}$	$\neg P$	$\neg P \vee Q$	$\overset{b}{\neg P \vee (\neg P \vee Q)}$	$a \rightarrow b$
T	T	T	F	F	T	T	T
T	F	F	T	F	F	F	F
F	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T

— 5M

$\because$ 6^{th} & 8^{th} columns of truth tables are same.

$\therefore$ $\neg(P \wedge Q) \rightarrow [\neg P \vee (\neg P \vee Q)] \Leftrightarrow (\neg P \vee Q)$

b) obtain PDNF of $(\neg P \rightarrow Q) \wedge (Q \Leftrightarrow P)$

P	Q	$\neg P$	$\neg P \rightarrow Q$	$Q \Leftrightarrow P$	$(\neg P \rightarrow Q) \wedge (Q \Leftrightarrow P)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	F
F	F	T	F	T	F

— 4M

$\therefore$ PDNF of $(\neg P \rightarrow Q) \wedge (Q \Leftrightarrow P)$ is $(P \wedge Q)$ — 1M

UNIT - II.

4 a) S.T $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \to R$, $P \to M$ & $\sim M$

{1} 1) $P \to M$ Rule P.

{2} 2) $\sim M$ Rule P.

{1,2} 3) $\sim P$ Rule T (1), (2) Modus Tollens.

{4}. 4) $P \vee Q$ Rule P.

{4} 5) $\sim P \to Q$ Rule T Implication. — 5M

{1,2,4} 6). Q. Rule T (3), (5) Modus Ponens.

{7} 7) $Q \to R$ Rule P.

{1,2,4,7} 8) R Rule T (6), (7). Modus Ponens.

{1,2,4,7}. 9) $R \wedge (P \vee Q)$ Rule T (4), (8)

b). Let $T(x)$: x is a Tiger.

$D(x)$: x is a dangerous animal.

The given premises are $(\forall x)\{T(x) \to D(x)\}$, $(\exists x)\, T(x)$.

T.S.T $(\exists x)\, D(x)$.

{1} 1. $(\exists x)\, T(x)$ Rule P.

{1} 2. $T(y)$ Rule. ES (1).

{3} 3. $(\forall x)(T(x) \to D(x))$ Rule P. — 5M

{3} 4. $T(y) \to D(y)$ Rule US (3).

{1,3}, 5. $D(y)$. Rule P (2), (4) Modus Ponens.

{1,3} 6. $(\exists x)\, D(x)$ Rule EG.

$\therefore$ Conclusion $(\exists x)\, D(x)$. is valid from the given premises.

OR.

5 a). Some roses are yellow.

Let $R(x)$ be "x is a rose"

Let $Y(x)$ be "x is yellow".

Then $(\exists x)\,[R(x) \wedge Y(x)]$. — 2M

5 ii) All Russians are taller than all Americans.

Let $R(x)$ be "x is Russian"

$A(y)$ be "y is American"

$T(x,y)$ be "x is taller than y"

Then $(\forall x)(\forall y)\ (R(x) \wedge A(y)) \rightarrow T(x,y)$. —— 1M

5 iii) Some monkeys have no tail.

$(\exists x)\ (M(x) \wedge \sim T(x))$. —— 2M

5b) P.T $(\forall x)\ (P(x) \vee (\forall x)\ Q(x)) \Leftrightarrow (\forall x)\ (P(x) \vee Q(x))$ is logically valid.

$\{1\}$ 1) $(\forall x)\ P(x)$ Rule P.

$\{1\}$ 2) $P(y)$ Rule US (1).

$\{3\}$ 3) $(\forall x)\ Q(x)$ Rule P.

$\{3\}$ 4) $Q(y)$ Rule US. (3)

$\{1,3\}$ 5) $P(y) \vee Q(y)$ Rule T. (2), (4).

$\{1,3\}$ 6). $(\forall x)\ (P(x) \vee Q(x))$ Rule UG.

—— 5M

UNIT III

6. Solve $a_{n+2} - 5a_{n+1} + 6a_n = 3n+4$, $a_0 = 2$, $a_1 = 5$.

Let $n+2 = r \Rightarrow n = r-2$.

$a_r - 5a_{r-1} + 6a_{r-2} = 3(r-2)+4 = 3r-2$

Characteristic equation is $t^2 - 5t + 6 = 0$.

$(t-2)(t-3) = 0$

$t = 2, 3$. —— 4M

$a_r^{(h)} = c_1(2)^r + c_2(3)^r$. —— 1M

$P.S = Q_0 + Q_1 r$. —— 1M

$(Q_0 + Q_1 r) - 5[Q_0 + Q_1(r-1)] + 6[Q_0 + Q_1(r-2)] = 3r-2$.

$2Q_0 - 7Q_1 + 2Q_1 r = 3r - 2$

Comparing coefficients on both sides.

$2Q_1 = 3 \Rightarrow \boxed{Q_1 = \frac{3}{2}}$.

$2Q_0 - 7Q_1 = -2$.

$\boxed{Q_0 = \frac{17}{4}}$

P.S $= Q_0 + Q_1 r = \frac{17}{4} + \left(\frac{3}{2}\right) r$.

G.S $a_r = a_r^{(h)} + a_r^{(P)}$.

$a_r = C_1(2)^r + C_2(3)^r + \frac{17}{4} + \frac{3}{2} r$.

If $r = 0$, $a_0 = 2 = C_1 + C_2 + \frac{17}{4}$.

$\Rightarrow \boxed{C_1 + C_2 = -\frac{9}{4}} \rightarrow ①$.

if $r = 1$, $a_1 = 5 = 2C_1 + 3C_2 + \frac{17}{4} + \frac{3}{2}$.

$\Rightarrow \boxed{2C_1 + 3C_2 = -\frac{3}{4}} \rightarrow ②$.

— (3M)

By solving ① & ② we get $\boxed{C_1 = -6, \; C_2 = \frac{15}{4}}$

$\boxed{a_r = (-6)(2)^r + \frac{15}{4}(3)^r + \frac{17}{4} + \frac{3}{2} r}$ — (1M)

OR

7) Solve $a_r - 7a_{r-1} + 16a_{r-2} - 12a_{r-3} = 0$, $r \geq 3$, $a_0 = 1$, $a_1 = 4$, $a_2 = 8$.

characteristic equation is $t^3 - 7t^2 + 16t - 12 = 0$.

$(t-2)(t^2 - 5t + 6) = 0$

$(t-2)(t-2)(t-3) = 0$

$t = 2, 2, 3$. — (5M)

$\boxed{a_n^{(h)} = C_1(3)^n + (C_2 + C_3 n)(2)^n}$. — (1M)

if $n = 0$, $a_0 = C_1 + C_2 = 1 \rightarrow ①$.

if $n = 1$, $a_1 = 4 = 3C_1 + 2C_2 + 2C_3 \rightarrow ②$.

if $n = 2$, $a_2 = 8 = 9C_1 + 4C_2 + 8C_3 \rightarrow ③$.

By solving ①, ② & ③ we get..

$\boxed{c_1 = -4,\ c_2 = 5,\ c_3 = 3}$ — 3M

$\boxed{a_n = (-4)(3)^n + (5+3n)(2)^n}$. — 1M

UNIT - IV

8) Equivalence Relations :- A relation R on a set A is said to be equivalence relation on A if.

i) Reflexive on A if $(a,a) \in R \ \forall a \in A$

ii) symmetric on A if $(a,b) \in R \Rightarrow (b,a) \in R \ \forall (a,b) \in A$

iii) Transitive on A if $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \ \forall (a,b,c)$

— Si

i) Reflexive :- $(a,a) \in R \ \forall a \in A$.

$(a,a) \in R \Rightarrow (a-a)$ is an integer.

$\Rightarrow 0$ is an integer.

$\therefore$ R is Reflexive — 2M

ii) symmetric :- $(a,b) \in R \Rightarrow (b,a) \in R \ \forall (a,b) \in A$.

$(a,b) \in R \Rightarrow (a-b)$ is an integer.

$\Rightarrow -(b-a)$ is also an integer.

$\Rightarrow (b,a) \in R$.

$\therefore$ R is symmetric — 2M

iii) Transitive : $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \ \forall (a,b,c) \in$

$(a,b) \in R \Rightarrow (a-b)$ is an integer

$(b,c) \in R \Rightarrow (b-c)$ is an integer.

$(a-b)+(b-c) = (a-c)$ is also an integer.

$\Rightarrow (a,c) \in R$.

$\therefore$ R is transitive. — 1M

$\because$ R satisfies all 3 reflexive, symmetric & transi

$\therefore$ R is an equivalence relation on z.

OR.

9 (a) $A = \{1,2,3,4,5\}$ $R = \{(x,y) / x \geq y\}$, ~~R~~

A relation R on a set A is a partial ordered relation if it satisfies the following 3 conditions $\forall (x,y,z) \in A$

i) Reflexive :- $(x,x) \in R$ $\forall x \in A$.

For the given relation $R = \{(x,y) / x \geq y\}$.

$x \geq x$ is always true for any no x.

$\therefore$ Relation is reflexive — 2M

ii) Anti-symmetry :- if $(x,y) \in R$ & $(y,x) \in R$ then $x = y$.

if $x \geq y$ & $y \geq x$ $\Rightarrow$ that x must be equal to y.

$\therefore$ Relation is antisymmetric — 1M

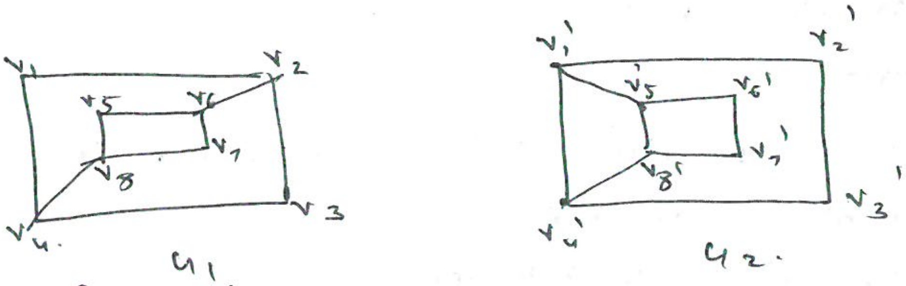
iii) Transitive :- if $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$.

if $x \geq y$ & $y \geq z$ $\Rightarrow$ $x \geq z$.

$\therefore$ Relation is transitive. — 2M

$\because$ the relation R satisfies all 3 conditions, it is a partial ordered relation.

9b)



Given No of vertices.

$|V_1| = 8$ $|V_2| = 8$

$|V_1| = |V_2|$.

No of edges.

$|E_1| = 10$, $|E_2| = 10$

$|E_1| = |E_2|$. — 2M

$\Rightarrow$ No of vertices & edges are same in both the graphs.

Degree sequences.

G_1

vertex	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
degree	2	3	2	3	2	3	2	3.

$deg(G_1) = \{3, 3, 3, 3, 2, 2, 2, 2\}$.

G_2

vertex	v_1'	v_2'	v_3'	v_4'	v_5'	v_6'	v_7'	v_8'
degree	3	2	2	3	3	2	2	3.

2M

~~vertex~~ $deg(G_2) = \{3, 3, 3, 3, 2, 2, 2, 2\}$.

$d(v_1) = v_2'$

$d(v_2) = v_1'$

$d(v_6) = v_5'$

$d(v_5) = v_6'$

$d(v_8) = v_8'$

$d(v_7) = v_7'$

$d(v_3) = v_3'$

$d(v_4) = v_4'$

AG_1	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	0	1	0	1	0	0	0	0.
v_2	1	0	1	0	0	1	0	0
v_3	0	1	0	1	0	0	0	0.
v_4	1	0	1	0	0	0	0.	1
v_5	0	0	0	0	0	1	0.	1
v_6	0	1	0	0	1	0	1	0
v_7	0	0	0	0	0	1	0	1
v_8	0	0	0	1	1	0	1	0.

AG_2	v_2'	v_1'	v_3'	v_4'	v_6'	v_5'	v_7'	v_8'
v_2'	0	1	1	0	0	0	0	0.
v_1'	1	0	0	1	0	1	0	0.
v_3'	1	0	0	1	0	0	0	0.
v_4'	0	1	1	0	0	0	0.	1
v_6'	0	0	0	0	0	1	1	0.
v_5'	0	1	0	0	1	0	0.	1
v_7'	0	0	0	0	1	0	0.	1
v_8'	0	0	0	1	0	1	1	0.

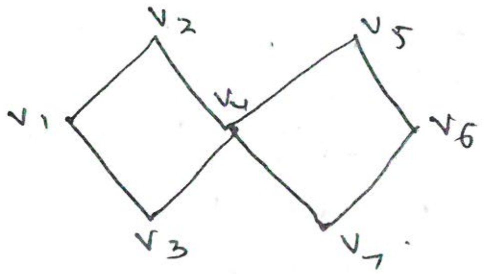
$A(G_1) \neq A(G_2)$.

$\therefore$ the graphs G_1 & G_2 are not isomorphic.

1M

10a) Explain Depth First search algorithm with an example.

Let us consider the graph.

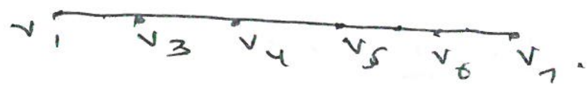


— 2M

Step-1 - choose the vertex. V_1 as a root.

V_1

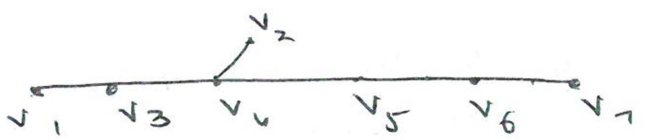
Step 2 - Form a path by successively adding edges incident with vertices which are not already in the path as long as possible. This ~~Produce~~ Procedure produces the path $(V_1, V_3, V_4, V_5, V_6, V_7)$



Step 3 - Now back track to V_6. There is no path begining at V_6 containing vertices not already visited.

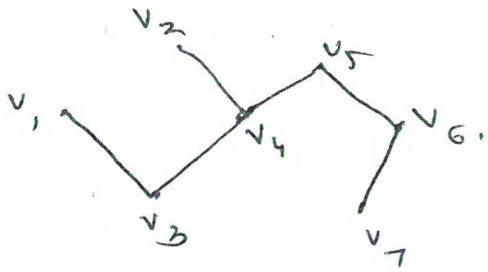
Step 4 - Similarly, on back tracking to V_5, we observe that there is no path. — 2M

Step 5 - Back track to V_4 & form the path (V_4, V_2). This produces the required.

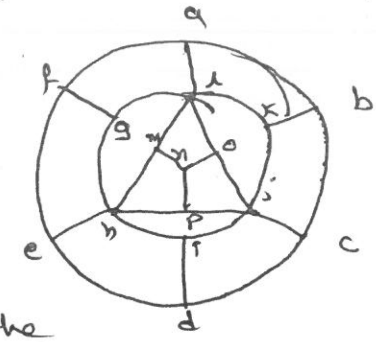


— 1M

Spanning tree is



10b). S.T the following graph is not Hamiltonian.



A Hamiltonian graph is a graph that contains a Hamiltonian cycle, which is a cycle that visits each vertex exactly once & returns to the starting vertex.

In this graph, the outer vertices {a, b, c, d, e, f} & inner vertices {g, h, i, j, k, l} are connected to the central vertices {m, n, o, p}.

The graph has a total of 16 vertices.

It is impossible to form a single continuous cycle that includes every vertex exactly once.

i.e a-b-c-d-e-f-g-h-i-j-k-l-m-n-o-j-p-h-g-l-a.

In this cycle g & h are repeated twice.

So, the given graph is not Hamiltonian. — 5M

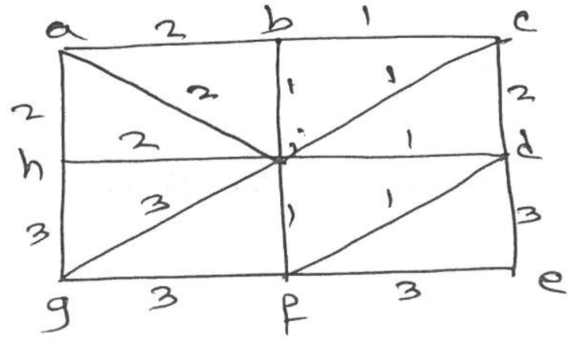
OR

11). minimal spanning tree is a subgraph of a connected weighted graph that connects all the vertices together, with out any cycles & with the minimum possible total weight.

Edge	(b,c)	(c,d)	(i,d)	(d,f)	(i,f)	(b,i)	(a,b)	(a,h)	(a,i)	(h,i)	(h,g)
weight	1	1	1	1	1	1	2	2	2	2	3

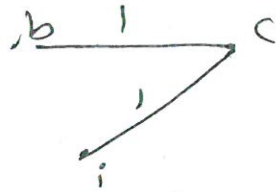
(c,d)	(g,i)	(g,f)	(f,e)	(d,e)
2	3	3	3	3

— 2M

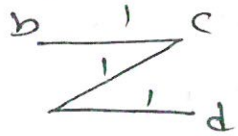


1) choose the edge (b,c) which has minimum weight.

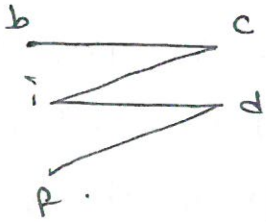
2) add the next edge (c,i)



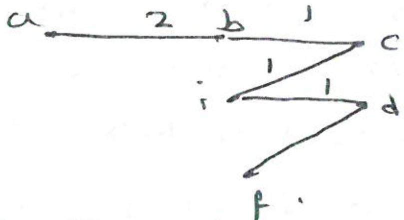
3) choose the next edge (i,d)



4) choose the next ~~edge~~ edge (d,f)

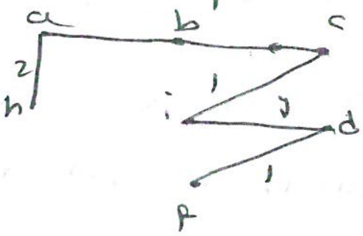


5) choose the edge (b,a)

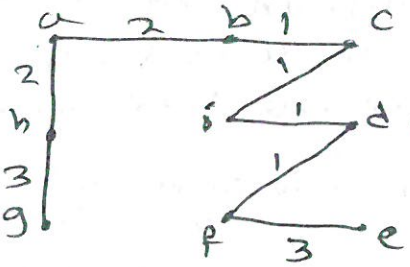


6 M

6) choose the next edge (a,h)



7) choose the next edge (h,g) & (f,e)



Minimum spanning tree = 3+2+2+1+1+1+1+3 = 14. → 2M

The graph has n vertices. A spanning tree must have (n-1) edges.

* please award marks for alternate methods also.