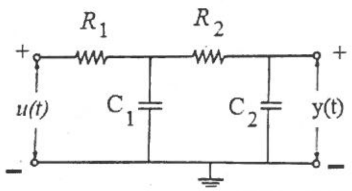
		UNIT-IV			
8	a)	The open loop transfer function of a unity feedback system is given by $G(s)=\frac{1}{s^2(1+s)(1+2s)}$. Sketch the polar plot.	L3	CO3	6 M
	b)	Define the various frequency response specifications.	L2	CO1	4 M
		OR			
9	a)	Draw the Bode plot and hence find the gain margin and phase margin for a unity feedback system whose transfer function is given by $$G(s)=\frac{10}{s(1+0.1s)(1+0.2s)}$$	L4	CO4	7 M
	b)	Explain the stability analysis from Nyquist stability criterion.	L2	CO3	3 M
		UNIT-V			
10	a)	State and prove the properties of State Transition Matrix.	L3	CO5	5 M
	b)	Evaluate the Observability of the system with, $A=\begin{bmatrix}0&1&0\\0&0&1\\0&-2&-3\end{bmatrix}$, $B=\begin{bmatrix}0\\0\\1\end{bmatrix}$ and $C=\begin{bmatrix}3&4&1\end{bmatrix}$.	L4	CO5	5 M
		OR			
11	a)	Obtain the state model for the electrical network shown in figure. 	L3	CO5	5 M
	b)	Obtain the transfer function of the system having the state model, $X'(t)=\begin{bmatrix}-5&-1\\3&-1\end{bmatrix}X(t)+\begin{bmatrix}2\\5\end{bmatrix}U(t)$, and $Y(t)=\begin{bmatrix}1&2\end{bmatrix}X(t)$.	L3	CO5	5 M

Code: 23ES1402

II B.Tech - II Semester – Regular Examinations - MAY 2025

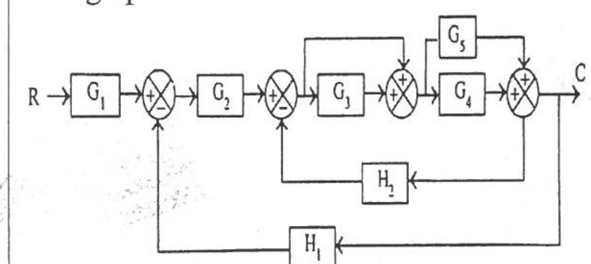
LINEAR CONTROL SYSTEMS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

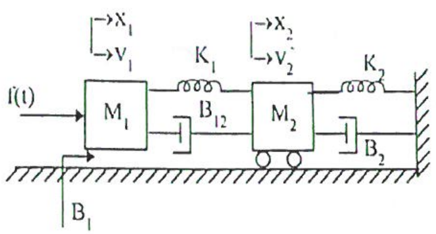
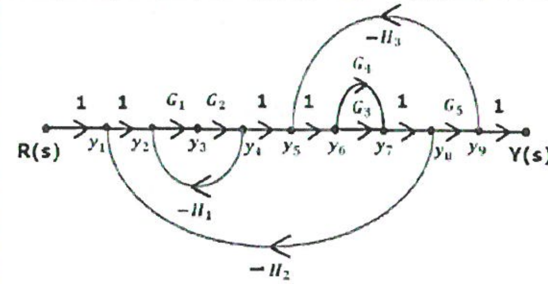
BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	List the merits of closed loop control system.	L2	CO1
1.b)	Why positive feedback is not preferred in control systems?	L2	CO3
1.c)	For the given block diagram, construct the signal flow graph. 	L3	CO4
1.d)	What is transient response and steady state response?	L2	CO2
1.e)	Write the statement of Routh's criteria for stability.	L2	CO1
1.f)	How many break away points exist for $G(s)H(s)=\frac{K(s+6)}{s(s+2)(s+4)}$.	L3	CO3
1.g)	Define resonant frequency.	L2	CO1
1.h)	Mention the requirement of gain margin and phase margin for a stable system.	L2	CO3

1.i)	Define state controllability.	L2	CO2
1.j)	What is the significance of State Transition Matrix?	L2	CO3

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Describe the concept of closed loop control system with one example.	L2	CO1	4 M
	b)	Write the differential equations governing the mechanical system shown in Fig., and hence draw the Force –voltage analogous circuit. 	L3	CO3	6 M
		OR			
3	a)	Compare open loop and closed loop control systems.	L3	CO3	6 M
	b)	Derive the transfer function of series RC circuit with voltage across capacitor as its output.	L3	CO3	4 M
		UNIT-II			
4	a)	Obtain the transfer function of below signal flow graph using Mason's Gain formula. 	L3	CO3	5 M
	b)	For a unity feedback control system, the open loop transfer function is $G(s)=\frac{10(s+2)}{s^2(s+1)}$. Find position, velocity and acceleration error constants.	L3	CO3	5 M
		OR			
5	a)	For a unity feedback system whose open loop transfer function is $G(s)=\frac{10}{s(s+1)(s+2)}$, find the steady state error when it is subjected to the input, *r(t)=1+2t.*	L4	CO4	5 M
	b)	List out various time domain specifications and define them.	L2	CO1	5 M
		UNIT-III			
6	a)	What are the difficulties in the formulation of the Routh table? Explain how they can be overcome.	L2	CO1	5 M
	b)	Use R-H criterion and comment on stability of a unity feedback system whose transfer function has poles at s = 0, s = -1, s = -3 and zeros at s = -5.	L3	CO3	5 M
		OR			
7		Sketch the root locus for unity feedback system $G(s)=\frac{K}{s(s+2)(s+4)}$. Find the value of K for the system to be stable.	L4	CO4	10 M

CODE: 23ES1402

PVP23

II B.Tech, II Semester Regular Examinations MAY 2025

LINEAR CONTROL SYSTEMS

(ECE)

Max: Marks: 70

SCHEME OF EVALUATION

PART-A

1 (a) Any Two merits, Each merit 1M.

1 (b) Any Two reasons, Each reason 1M.

1 (c) Signal flow graph diagram - 2M

1 (d) Transient response - 1M, steady state response - 1M

1 (e) Statement - 2M

1 (f) How to find break away point roots of $\left(\frac{dk}{ds} = 0\right)$ - 1M, Answer - 1M

1 (g) definition - 2M

1 (h) Requirement for stability - 2M

1 (i) Definition - 2M

1 (j) significance - 2M

PART-B

2 (a) Diagram - 1M, Concept - 2M, Any Example - 1M

2 (b) Free body diagrams - 2M, differential Equations - 3M
Final Electrical circuit - 1M

3 (a) Any six Comparisions, Each 1M

3 (b) Circuit - 1M, Equations - 2M, Answer - 1M

Formula – 1M, Solution – 3M, Answer – 1M

4 b Formulae – 3M, Answers – 2M

5 a Formula – 1M, Substitution – 3M, Answer – 1M

5 b Any five specifications, Each definition – 1M

6 a Difficulty 1 – 1M, difficulty 2 – 1M
How to overcome – 3M

6 b Transfer function – 1M, characteristic Equ – 1M,
Routh table – 2M, Answer – 1M

7 Finding poles, Zeros and branches of locus – 2M
Centroid – 1M, Asymptotes – 1M, break away
point – 2M, Value of 'K' – 1M,
Final root locus diagram – 3M

8 a calculations – 4M, final plot – 2M

8 b Any four specifications, Each definition – 1M

9 a Magnitude plot related table – 1M, phase plot
related calculations – 2M, Bode plot – 3M,
gain and phase Margins – 1M

9 b Explaination – 3M

10 a Properties – 2M, Proofs – 3M

10 b Matrix Equation – 1M, Calculations – 3M, Answer – 1M

11 a Equations – 3M, final state model form – 2M

11 b Transfer function formula – 1M, Calculations – 3M
final answer – 1M

KEY

PART-A

1 (a) → More accurate

→ More Reliable

→ Feedback is included

→ Less sensitive to Noise

→ output can be Controlled greatly.

1 (b)

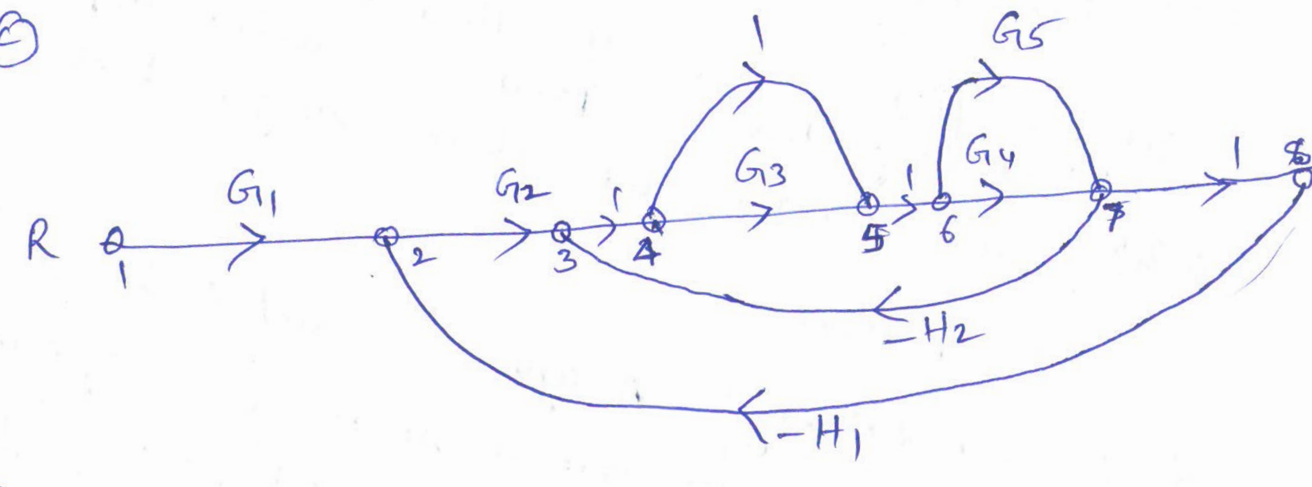
→ Positive feedback cause the system to be unstab[le]

→ output can not be controlled properly.

→ More sensitive to External disturbances

→ Reduced System Reliability.

1 (c)



1 (d)

Transient Response :- It is the portion of the total time response during which the output change[s] from one state to another state. It is the respons[e] before the output reaching the steady state value.

Steady state Response :- It is the response of the system for a given input after long time. In steady state the output settles to it's final steady state value.

(e) Routh's criteria for stability

It states that, for a system to be stable, it is necessary and sufficient that the first column of routh array are to be positive, if the first element of routh array is positive. If this condition is not met, then the system is unstable and the number of sign changes in the first column will determine the number of roots on the right hand side of s-plane.

1 (f) Number of break away points = 1
Break away point can be found by evaluating the roots of $\frac{dk}{ds} = 0$ from characteristic equation

1 (g) Resonant frequency: The frequency at which the resonant peak occurs is called as resonant frequency. It is indicative of speed of transient response.

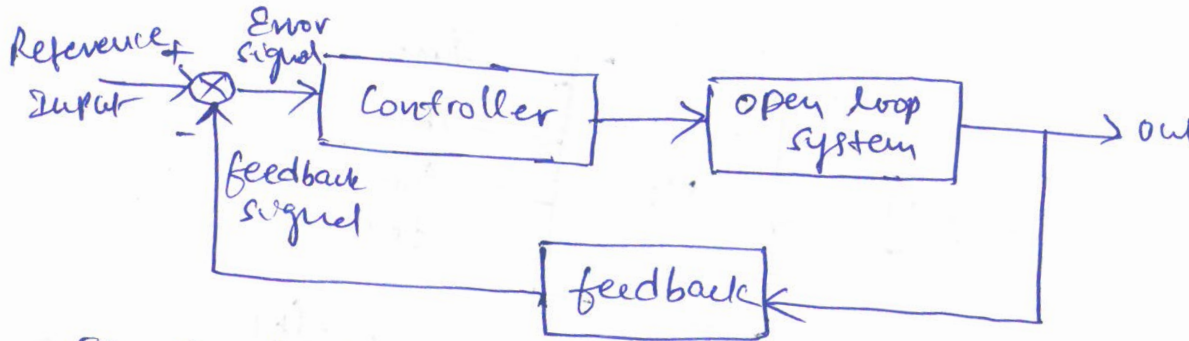
1 (h) Gain margin and phase margin both must be positive.

1 (i) Controllability: A system is said to be state controllable if the initial state of a system is changed to some other desired state by a controlled input in finite duration of time.

(j) State Transition matrix represents the free response of the system. It gives the response that is excited by some initial conditions. It describes the change of state from initial value at $t=0$ to any other time "t". It provides the solution to the state space equations.

PART-B

(2)(a) Concept :-



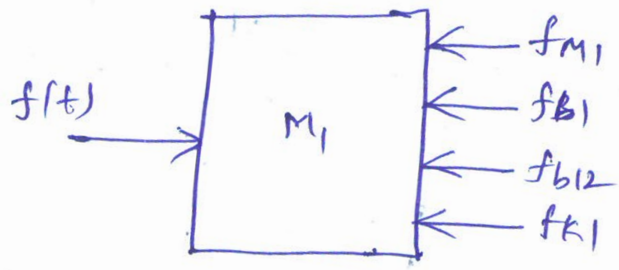
If the feedback is provided to open loop sy
they will become closed loop systems. In a
loop Control system, the Error signal which
difference between reference input signal and
signal is fed to the Controller so as to reduc
Error and bring the output of system to d
Value. A system that maintains a prescribed rel
between the output and reference input by co
them and using their difference as a means of
is called a feedback Control system or closed lo
control system. The term "closed loop" always i
the use of feedback action in order to reduce sy
Error.

Example :- Any other Example can also be considered

Traffic Control System :- Traffic Control by means o
traffic signals operated on a time basis is an ope
loop Control system. Here the time slots do not chang
according to traffic density. If the time slots of
the signals are based on density of traffic, then it
will become closed loop Control system. Here the dens
of traffic is measured on all sides and the timings of
Control signals are decided based on density of traffic.
Then the flow of vehicles will be better than open loop
system.

(2)(b)

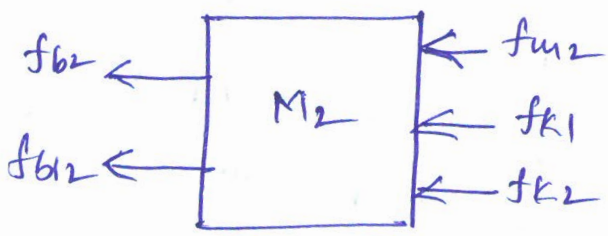
Free body diagram for M_1



$$f(t) = f_{M1} + f_{b1} + f_{b12} + f_{k1}$$

$$= M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_{12} \frac{d(x_1 - x_2)}{dt} + k_1 (x_1 - x_2) \rightarrow (1)$$

free body diagram for M_2



$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_{12} \frac{d}{dt}(x_2 - x_1) + k_2 x_2 + k_1 (x_2 - x_1) = 0 \rightarrow (2)$$

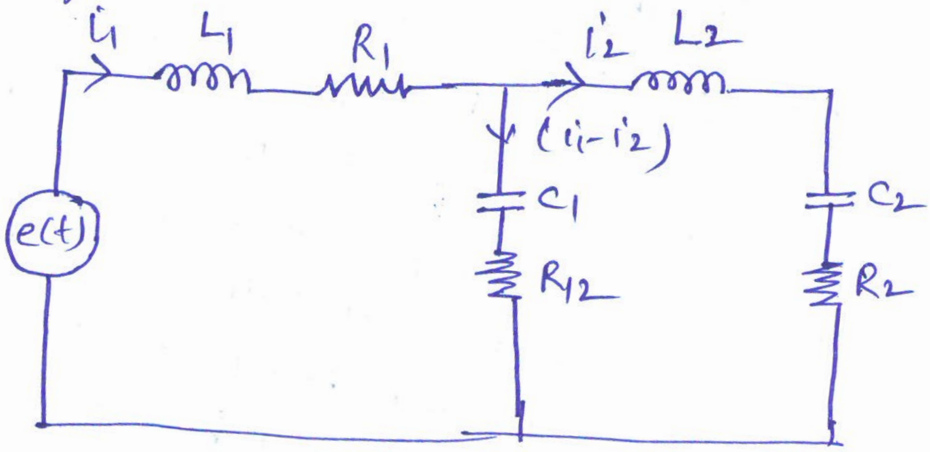
By using force voltage analogy the Equation (1) can be rewritten as

$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int (i_1 - i_2) dt + R_{12} (i_1 - i_2) \rightarrow (3)$$

Equation (2) can be rewritten as

$$L_2 \frac{di_2}{dt} + R_2 i_2 + R_{12} (i_2 - i_1) + \frac{1}{C_2} \int i_2 \, dt + \frac{1}{C_1} \int (i_2 - i_1) = 0 \rightarrow (4)$$

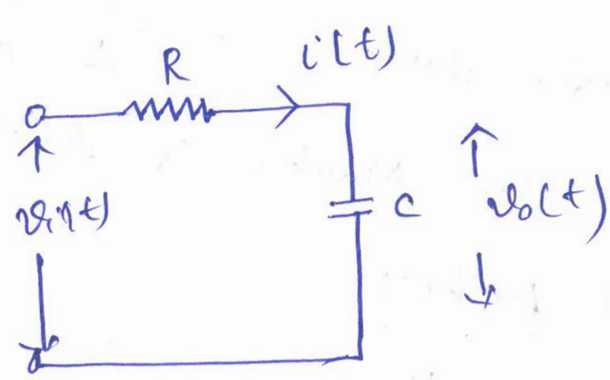
By using Equations (3) & (4), the force voltage analogous circuit can be drawn as follows.



3 a

Open Loop Control system	closed loop Control system
1) Simple and Economical	1) Complex and Costlier
2) Less accurate	2) More accurate
3) Less reliable	3) More reliable
4) changes in the output due to External disturbances are not corrected automatically	4) Can be corrected automatically
5) More sensitive to Noise	5) Less sensitive to Noise
6) feedback is not there	6) feedback is incorporated
7) Consumes less power	7) Consumes more power
8) Easier to construct	8) difficult to Construct
9) Control action is independent of desired output	9) Action is dependent of desired output
10) Less flexible	10) More flexible

3 b



$$\text{Transfer function} = \frac{V_o(s)}{V_{in}(s)}$$

From the circuit, we can write

$$V_i(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt \longrightarrow (1)$$

$$V_o(t) = \frac{1}{C}\int i(t)\,dt \longrightarrow (2)$$

Apply Laplace transform on both sides of above two Equations

$$V_i(s) = R\,I(s) + \frac{1}{Cs}\,I(s)$$

$$= I(s)\left[R + \frac{1}{Cs}\right] \longrightarrow (3)$$

$$V_o(s) = \frac{1}{Cs}\,I(s)$$

$$\Rightarrow I(s) = Cs\,V_o(s) \longrightarrow (4)$$

Substitute Equ (4) in Equ (3)

$$V_i(s) = Cs\,V_o(s)\left[R + 1/Cs\right]$$

$$\Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs}}$$

4 (a)

According to Mason's gain formula

$$\text{Transfer function} = \frac{1}{\Delta}\sum_{K=1}^{n} P_K \Delta_K$$

Here the Number of forward paths $n = 2$

Forward paths :

$$P_1 = G_1 G_2 G_3 G_5$$

$$P_2 = G_1 G_2 G_4 G_5$$

Single Loops:

$L_1 = -G_1G_2H_1$

$L_2 = -G_3G_5H_3$

$L_3 = -G_1G_2G_3H_2$

$L_4 = -G_1G_2G_4H_2$

$L_5 = -G_4G_5H_3$

Two Non touching loops:

$L_1L_2 = G_1G_2G_3G_5H_1H_3$

$L_1L_5 = G_1G_2G_4G_5H_1H_3$

Three non touching loops : NIL

$\Delta_1 = 1$

$\Delta_2 = 1$

$$\Delta = (1 + G_1G_2H_1 + G_3G_5H_3 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_4G_5H_3 + G_1G_2G_3G_5H_1H_3 + G_1G_2G_4G_5H_1H_3).$$

$$\text{Transfer function} = \frac{G_1G_2G_3G_5 + G_1G_2G_4G_5}{(1 + G_1G_2H_1 + G_5H_3(G_3+G_4) + G_1G_2H_2(G_3+G_4) + G_1G_2G_5H_1H_3(G_3+G_4)}$$

$$= \frac{G_1G_2G_5(G_3+G_4)}{1+G_1G_2H_1 + (G_3+G_4)(G_5H_3 + G_1G_2H_2 + G_1G_2G_5H_1H_3)}$$

4) b) Given Transfer function is

$$G(s) = \frac{10(s+2)}{s^2(s+1)}$$

different Error constants are

$$K_p = \underset{s \to 0}{Lt}\ G(s)\,H(s)$$

$$= \underset{s \to 0}{Lt}\ \frac{10(s+2)}{s^2(s+1)}$$

$$= \alpha$$

$$K_v = \underset{s \to 0}{Lt}\ s\,G(s)\,H(s)$$

$$= \underset{s \to 0}{Lt}\ s \times \frac{10(s+2)}{s^2(s+1)}$$

$$= \alpha$$

$$K_a = \underset{s \to 0}{Lt}\ s^2\,G(s)\,H(s)$$

$$= \underset{s \to 0}{Lt}\ \cancel{s^2}\,\frac{10(s+2)}{\cancel{s^2}(s+1)}$$

$$= 20$$

5 (a) Given that

$$G(s) = \frac{10}{s(s+1)(s+2)}$$

$$r(t) = 1 + 2t$$

$$R(s) = \frac{1}{s} + \frac{2}{s^2}$$

$$e_{ss} = \underset{s \to 0}{Lt}\ \frac{s\,R(s)}{1 + G(s)\,H(s)}$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s\left(\frac{1}{s}+\frac{2}{s^2}\right)}{1+\frac{10}{s(s+1)(s+2)}}$$

$$= \underset{s\to 0}{Lt} \frac{s\left(\frac{s+2}{s^2}\right)}{\frac{[s(s+1)(s+2)+10]}{s(s+1)(s+2)}}$$

$$= \underset{s\to 0}{Lt} \left(\frac{s+2}{s}\right) \times \frac{s(s+1)(s+2)}{[s(s+1)(s+2)+10]}$$

$$= \frac{2\times 1\times 2}{0+10} = \frac{4}{10} =$$

5 b

Peak Time (t_p) :- It is the time required for the response to reach the peak of the first over shoot.

Settling time (t_s) :- It is the time required for the response to reach the specified tolerance band of it's final value. The tolerance band is usually 2% to 5%.

Rise time (t_r) :- It is the time required for the response to rise from 10% to 90% of the final value ~~or~~ (desired value).

Delay time (t_d) :- It is the time required for the response to reach the 50% of the final value of desired value).

Peak Over Shoot (M_p):- It refers to the maximum amount by which the response of the system exceeds it's desired or final value.

Steady state Error (e_{ss}):- It is the deviation between actual output and desired output as time tends to infinity.

6) a)

Difficulty ①:- The first element of any row is zero while the rest of the elements of the same row consists of atleast one non zero element.

We can overcome this difficulty in the following methods.

Method ①:- To get modified characteristic equation, Put $S = 1/Z$ in original characteristic equation and adopt the same procedure for forming the Routh array in terms of 'Z'.

Method ②:- Substitute $0 = \varepsilon$ ($\varepsilon \to 0^+$) small positive value and then form the Routh table. Then to find out the stability, examine the first column of Routh table by substituting $\varepsilon \to 0^+$.

Difficulty ②:- All the elements of any row of Routh table consists of zeros.

Solution:- In such cases, we need to find out an equation called as auxiliary equation $A(S)$.

The auxiliary Equation can be written by the elements of row which is just above row consisting of all zeros.

Once the auxiliary Equation is find out, the first derivative of auxiliary Equation

Now by using the Co-efficients of $\frac{dA(s)}{ds}$, the elements in the row which consists of zeros. Then repeat the same procedure to complete Routh table.

6(b)

The characteristic Equation is

$$1+G(s)=0$$

$$\text{Here } G(s) = \frac{(s+5)}{s(s+1)(s+3)}$$ open lo

$$1+\frac{(s+5)}{s(s+1)(s+3)} = 0$$

$$\Rightarrow s(s+1)(s+3)+(s+5)=0$$

$$\Rightarrow s^3+4s^2+4s+5=0$$

Routh table

s^3	1	4
s^2	4	5
s^1	11/4	0
s^0	5	0

The system is <u>stable</u> as first column elements are all positive.

Given that

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

No of poles $(P) = 3$ = No of root loci

No of zeros $(Z) = 0$

Poles are $s=0$, $s=-2$, $s=-4$

No of branches terminates at α = $P-Z = 3$

Centroid of asymptotes $= \frac{0-2-4-0}{3} = -2$

Angle of asymptotes $\phi = \frac{2q+1}{P-Z} \times 180°$, $q = 0, 1, \cdots (P-Z-1)$

$\phi_1 = 60°$, $\phi_2 = 180°$, $\phi_3 = 300°$

Break away point:

The characteristic Equation is

$$s(s+2)(s+4) + K = 0$$

$$\Rightarrow K = -(s^3 + 6s^2 + 8s)$$

$$\frac{dK}{ds} = -(3s^2 + 12s + 8) = 0$$

$$s = -0.85 \quad \text{and} \quad s = -3.15$$

Since -3.15 is not part of root locus therefore

The break away point is -0.85.

Point of intersection of root loci with Imaginary axis

Characteristic Equation is

$$s^3 + 6s^2 + 8s + K = 0$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	

Put $K = 48$

$$A(s) = 6s^2 + 48 = 0$$

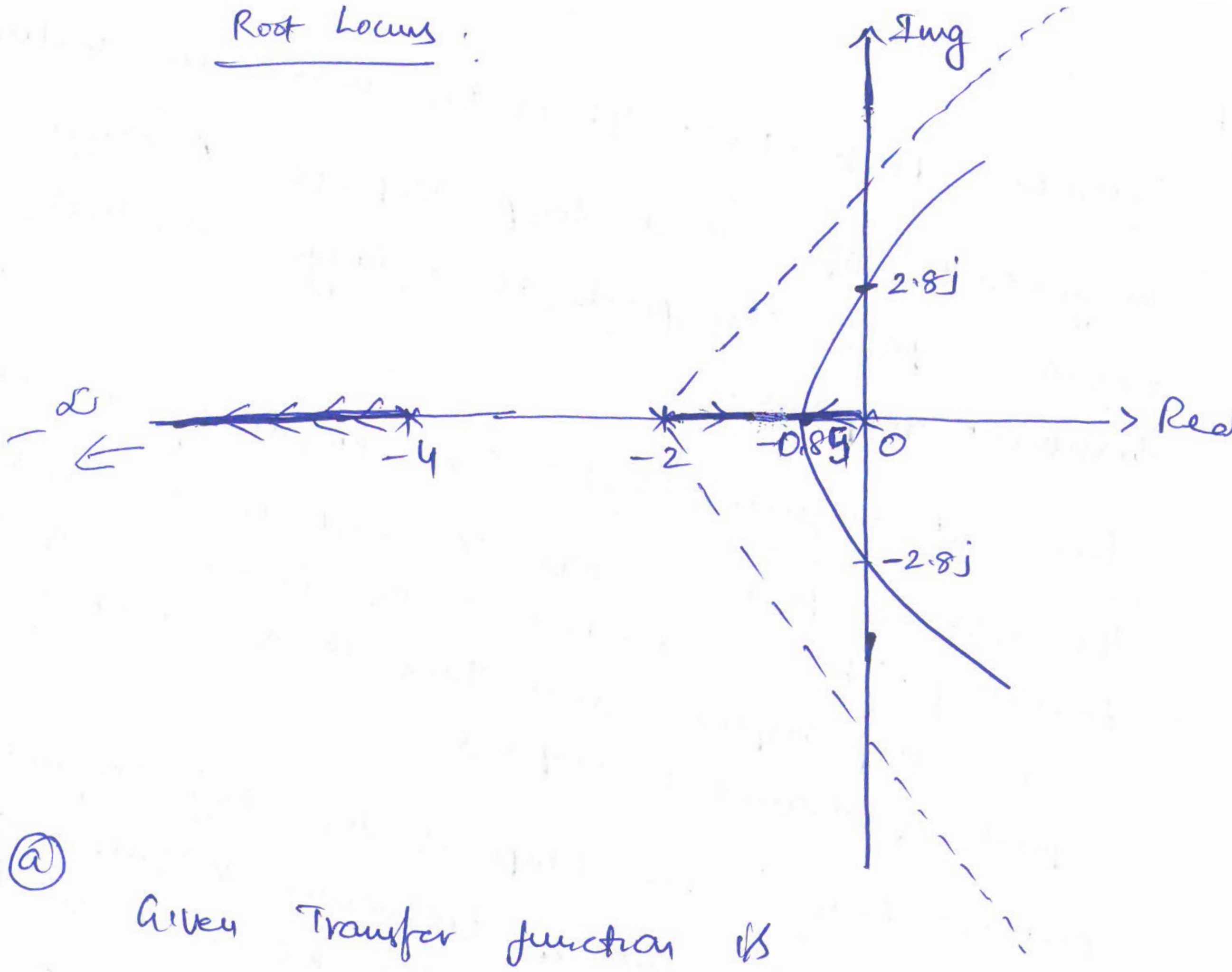
$$s = \pm j(2.8)$$

For system to be stable

$$K > 0, \quad \frac{48-K}{6} > 0$$

$$\Rightarrow \boxed{0 < K < 48}$$

Root Locus.



(8) (a)

Given Transfer function is

$$G(s) = \frac{1}{s^2(s+1)(1+2s)}$$

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)} = \frac{-1}{\omega^2(1-2\omega^2 \ldots}$$

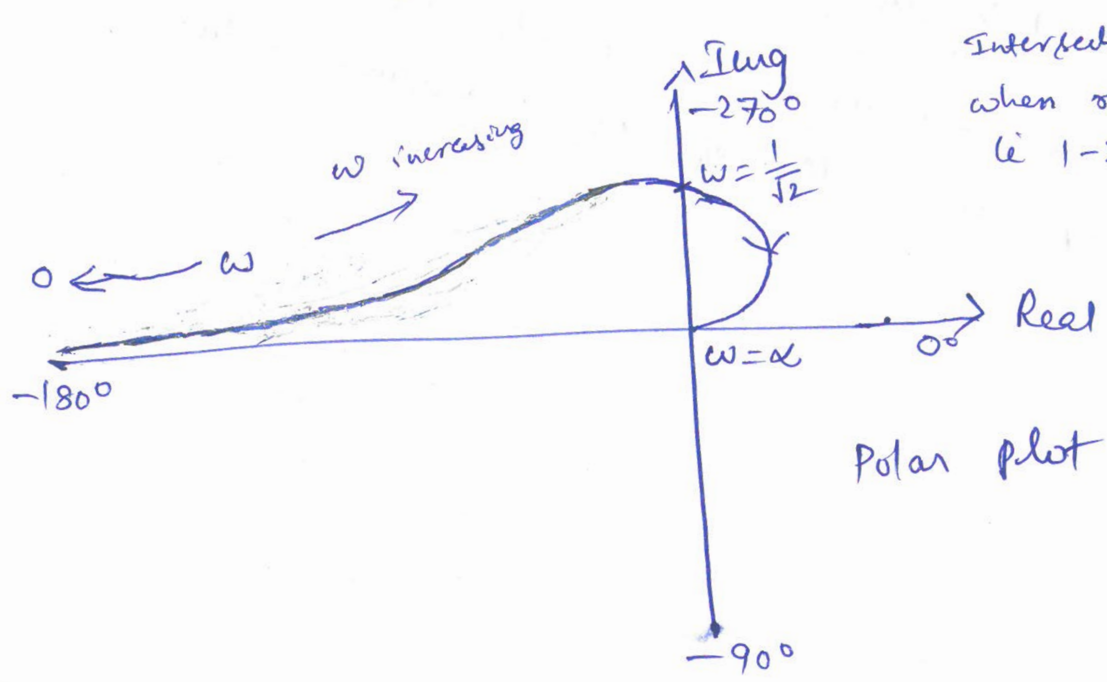
$$|G(j\omega)| = \frac{1}{\omega^2\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	α	-180^0
α	0	-360^0

$$\angle G(j\omega) = -180^0 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

Intersects imaginary axis when real part = 0

ie $1-2\omega^2 = 0$
$\Rightarrow \omega = \frac{1}{\sqrt{2}}$



Polar plot

8 (b)

Resonance Peak (M_r): It is the maximum value of magnitude of closed loop response. A large resonance peak corresponds to a large overshoot in transient response.

Resonant frequency (ω_r): The frequency at which the resonant peak occurs is called as resonant frequency. This is related to the frequency of oscillation in the step response and thus it is indicative of speed of transient response.

Cut-off Rate: The slope of the log magnitude curve near the cut-off frequency is called as cut-off rate. It is generally measured in dB/dec.

Gain Margin: It is the value of gain to be added to the system in order to bring back the system to the verge of instability.

Phase Margin: It is the phase lag to be added to the system in order to bring back system to the verge of instability.

Band width: It is defined as the frequency range between the points where the system's gain in dB drops to -3 dB of its maximum value.

9 (a)

Given Transfer function is

$$G(s) = \frac{10}{s(1+0.1s)(1+0.2s)}$$

$$G(j\omega) = \frac{10}{j\omega(1+0.1(j\omega))(1+0.2$$

$$|G(j\omega)| = \frac{10}{\omega\sqrt{1+0.01\omega^2}\sqrt{1+0.0$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}$$

S.NO	factor	Corner frequency	slope	Net slope
1	$20\log 10 = 20dB$	–	0 dB/dec	0 dB/dec
2	$1/s$	–	−20dB/dec	−20 dB/dec
3	$\frac{1}{(1+0.2s)}$	5	−20 dB/dec	−40 dB/de
4	$\frac{1}{(1+0.1s)}$	10	−20dB/dec	−60 dB/dec

ω	$\phi = \angle G(j\omega)$
0.1	-91.7°
0.5	-98.57°
1	-107.02°
5	-161.56°
10	-198.43°
10^2	-261.42°
10^3	-269.14°
10^4	-269.91°

Gain margin $= 0 - 4 = -4\,dB$

phase margin $= 180 + (-194^\circ) = -14^\circ$

Both values have been measured from Bode plot drawn on semi log sheet.

9 (b)

The Nyquist Stability Criteria helps to determine the stability of closed loop system using open loop transfer function. The Nyquist stability criteria is defined by

$$\boxed{N = P - Z}$$

where

$N \rightarrow$ No. of Encirclements of $(-1+j0)$ point

If is positive for anti clock wise Encirclements

Negative for clock wise Encirclements.

$P \rightarrow$ No. of open loop poles in RHS of s-plane

$Z \rightarrow$ No. of closed loop zeros in RHS of s-plane

The closed loop system is absolutely stable if and only if $Z=0$, means that the number of Encirclements of $-1+j0$ point in counter clock wise direction is Equal to the number of open loop poles in RHS of s-plane that is $P=N$.

(10) (a) Properties of state transition Matrix

$$\phi(t) = e^{At}$$

(i) $\phi(0) = I$

At $t=0$, $\phi(0) = \cancel{\phi} e^{0} = 1$

(or) $x(t) = e^{At} x(0)$, at $t=0$, $x(0) = \phi(0) x(0) \Rightarrow \phi(0) = I$

(ii) $\dfrac{d\phi(t)}{dt} = A\phi(t)$

$$\phi(t) = e^{At}$$

$$\frac{d\phi(t)}{dt} = A e^{At}$$

$$= A\phi(t)$$

(iii) $\phi(t_2-t_1)\,\phi(t_1-t_0) = \phi(t_2-t_0)$

$$\phi(t_2-t_1) = e^{A(t_2-t_1)}$$

$$\phi(t_1-t_0) = e^{A(t_1-t_0)}$$

$$\phi(t_2-t_1)\,\phi(t_1-t_0) = e^{A(t_2-\cancel{t_1}+\cancel{t_1}-t_0)}$$

$$= e^{A(t_2-t_0)}$$

$$= \phi(t_2-t_0)$$

(iv) $[\phi(t)]^K = \phi(Kt)$

$$[\phi(t)]^k = [e^{At}]^k$$

$$= e^{A(kt)}$$

$$= \phi(kt)$$

(v) $\phi^{-1}(t) = \phi(-t)$

$$\phi^{-1}(t) = (e^{At})^{-1} = e^{A(-t)}$$

$$= \phi(-t)$$

10 b

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = [3 \ 4 \ 1]$$

Here $n = 3$

$$Q_0 = [C^T \quad A^T C^T \quad (A^T)^2 C^T \ \ldots]$$

$$C^T = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 6 \\ 1 & -3 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$$

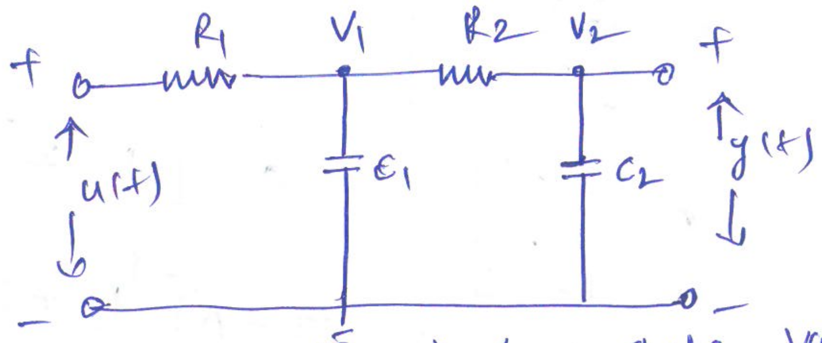
$$\therefore \phi_0 = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 1 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$

$|\phi_0| = 0$

Rank is less than 3

So Not observable.

(1)(a)



chose 'V_1' and 'V_2' as state variables. $x_1 = V_1$, $x_2 = V_2$

write KCL at node V_1

$$\Rightarrow \frac{V_1 - u(t)}{R_1} + C_1 \frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} = 0 \longrightarrow (1)$$

write KCL at node V_2

$$\frac{V_2 - V_1}{R_2} + C_2 \frac{dV_2}{dt} = 0 \longrightarrow (2)$$

from Equation (1),

$$\frac{dV_1}{dt} = V_1' = \frac{-1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_1 + \frac{1}{C_1 R_2} V_2 + \frac{1}{C_1 R_1} u(t) \rightarrow (3)$$

from Equation (2)

$$\frac{dV_2}{dt} = V_2' = \left(\frac{1}{C_2 R_2}\right) V_1 + \left(\frac{-1}{C_2 R_2}\right) V_2 + (0) u(t) \longrightarrow (4)$$

we can put these Equations in state space model as follows

$$\begin{bmatrix} V_1' \\ V_2' \end{bmatrix} = \begin{bmatrix} -\frac{(R_1 + R_2)}{C_1 R_1 R_2} & \frac{1}{C_1 R_2} \\ \frac{1}{C_2 R_2} & \frac{-1}{C_2 R_2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1 R_1} \\ 0 \end{bmatrix} u(t)$$

from the circuit we can write

$$y(t) = V_2$$

$$\therefore \; y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Here $A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 2 \end{bmatrix}$

Transfer function $= C[SI - A]^{-1}B + D$

$$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$[SI-A]^{-1} = \frac{1}{\det[SI-A]} \times \operatorname{adj}[SI-A]$$

$$= \frac{1}{s^2+6s+8} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

$$C[SI-A]^{-1} = \frac{1}{s^2+6s+8} \begin{bmatrix} s+7 & 2s+9 \end{bmatrix}$$

$$T.F = C[SI-A]^{-1}B = \boxed{\frac{12s+59}{s^2+6s+8}}$$