	b)	A Lossy cable which has R = 25.23 Ω/m, L =26H/m, C = 2pF/m, and G = 0 operates at f = 5MHz. Find the attenuation constant and phase constant of the line.	L3	CO4	5 M
		OR			
11	a)	Discuss the condition for lossless transmission lines.	L2	CO4	5 M
	b)	Derive the transmission line equations.	L3	CO4	5 M

Code: 23EC3401

II B.Tech - II Semester – Regular Examinations - MAY 2025

ELECTROMAGNETIC WAVES AND TRANSMISSION LINES

(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Define distance vector with an example.	L2	CO1
b)	Write a short notes on continuity equation.	L2	CO1
c)	Write any two Maxwell's equations in point form.	L2	CO2
d)	Define Magnetic Flux Density.	L2	CO2
e)	Distinguish conduction and displacement current densities.	L2	CO3
f)	Define Faraday's law.	L2	CO3
g)	Give the expressions for Wave equations.	L2	CO3
h)	Define skin depth.	L2	CO3
i)	A 100+j50Ω load is connected to a 75 Ω lossless line. Find Γ.	L2	CO4
j)	Write the relation between VSWR and Reflection Coefficient.	L2	CO4

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	With neat diagram, Explain the spherical system with coordinates (r, Θ, ϕ).	L2	CO1	5 M
	b)	A uniform line charge ρ_L = 25nC/m lies on the axis x=3m and y=4m in free space. Calculate the electric field intensity at a point (2,3,15)m.	L3	CO2	5 M
		OR			
3	a)	Define Gauss's Law? Derive an equation $\nabla . D = \rho_v$.	L3	CO1, CO2	5 M
	b)	Derive Laplace's and Poisson equation.	L3	CO2	5 M
		UNIT-II			
4	a)	Define and Explain Ampere's circuit law.	L2	CO2	5 M
	b)	Determine J at (2,0,0) in cylindrical coordinates if the magnetic field, H = 50 $\sin\phi\, a_z$ mA/m^2.	L3	CO2	5 M
		OR			
5	a)	In a magnetic field, H = $2a_x + 5a_y + 1a_z$ A/m exists at a point in a medium μ_r =4. Calculate the magnetic flux density at the point.	L3	CO2	5 M
	b)	State Biot-savart law and obtain the expression for magnetic field intensity at a point P due to an infinite current line element.	L3	CO2	5 M

		UNIT-III			
6		With necessary explanation, derive the Maxwell's equation in differential and integral forms for time varying fields.	L3	CO3	10 M
		OR			
7	a)	Given the conduction current density in a lossy dielectric as J_c = 0.02 sin 10^9t A/m^2. Calculate the displacement current density if σ =103 mho/m and ε_r= 6.5.	L3	CO3	5 M
	b)	What is inconsistency in Ampere's law? Develop an expression for the displacement current density.	L3	CO3	5 M
		UNIT-IV			
8		Define the terms Good conductor and loss less Dielectrics. Discuss about Wave propagation Characteristics in Good conductors.	L2	CO3	10 M
		OR			
9	a)	State and prove Poynting theorem.	L2	CO3	5 M
	b)	Explain about Circular and Elliptical polarization.	L2	CO3	5 M
		UNIT-V			
10	a)	Derive an equation of two-wire transmission line Input Impedance.	L3	CO4	5 M

Code 23EC3401 PVP 23

II B.Tech-II Sem - Regular Examinations - May 2025

Electromagnetic waves and Transmission lines

ECE

Scheme of valuation

Part A

1\. a Definition (or) formula - 1M } 2M
Example - 1M

b formula — 1M } - 2M
Explanation — 1M

c Any two Maxwell's equations — 2M

d Definition — 2M

e Any two differences — 2M

f Definition - 2M

g Two Equations - 2M

h Formula (or) Definition - 2M

i Formula - 1M } 2M
Answer - 1M

j Formula — 2M

2 a Diagram — 2M } 5M
Explanation - 3M

2 b) Formula — 2M } 5M
substitution - 2M
Answer — 1M

3a Definition - 2M } 5M
Derivation - 3M

3b) Necessary equations – 2M
Derivation – 3M } 5M

4a) Definition – 2M
Explanation – 3M } 5M

4b) Formula – 2M
Substitution & Simplification 2M
Answer – 1M } 5M

5a) Formula – 2M
Substitution & Simplification 2M
Answer – 1M } 5M

5b) Statement – 2M
Derivation – 3M } 5M

6a) Maxwell's equation 4M
Derivation & explanation 6M } 10M

7a) Formulas – 2M
Substitution & Simplification 2M
Answer – 1M } 5M

7b) Necessary equation – 2M
Derivation – 3M } 5M

8) Definitions – 4M
Explanation – 6M } 10M

9a Statement – 2M
Proof – 3M } 5M

9b) Each type – 2.5M total 5M

10a) Derivation – 5M

10b) Formulas – 2M
Substitution & Answer – 3M } 5M

11a) Derivation & Explanation – 5M

11b) Diagram – 2M up to derivative form 2M, final form – 1M

Code: 23EC3401 PVP23

II B.Tech – II Semester – Regular Examinations May-2025

Electromagnetic waves and Transmission lines

Dept. of ECE

Scheme of valuation

1. a The distance vector is the displacement from one point to another.

Ex:- $A = a_x + 3a_z$ and $B = 5a_x + 2a_y - 6a_z$

$r_{AB} = r_B - r_A = 4a_x + 2a_y - 9a_z$

b. Due to the principle of charge conservation the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume.

$$I_{out} = \oint J \cdot ds = -\frac{dQ_{in}}{dt}$$

By simplifying this

$$\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$$

c. Maxwell's equations

$$\nabla \cdot D = \rho_v \qquad \text{(or)} \qquad \nabla \times E = 0 \qquad \text{(or)} \qquad \nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \cdot B = 0 \qquad\qquad \nabla \times H = J \qquad\qquad \nabla \times H = J + \frac{\partial D}{\partial t}$$

d. The no. of magnetic flux lines per unit area is called as magnetic flux density.

$$B = \mu H$$

e.

Conduction current density	Dinplacement current density
$J = \sigma E$	$J_d = \frac{\partial D}{\partial t}$
It in due to the movement of free chargen (like electronn) in a Conductor	It in introduced by Maxwell'n for the time varying electric field region with no free charge flow.
It exintn in Conductorn	It Can exint in both Conductorn & innulatorn.

f. Faraday'n Law :- It staten That The induced emf in any closed circuit in equal to The time rate of charge of magnetic flux linkagen by the Circuit.

$$V_{emf} = -\frac{d\lambda}{dt} = -N\frac{d\psi}{dt}$$

g. Wave equation

$$\nabla^2 E_s - \gamma^2 E_s = 0$$

$$\nabla^2 H_s - \gamma^2 H_s = 0$$

h. Skin depth:- The Skin depth in a meanure of The depth to which an EM wave Can Penetrate the medium

$$\delta = \frac{1}{\alpha}$$

i

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j50 - 75}{100 + j50 + 75} = \frac{25 + j50}{175 + j50}$$

(or)

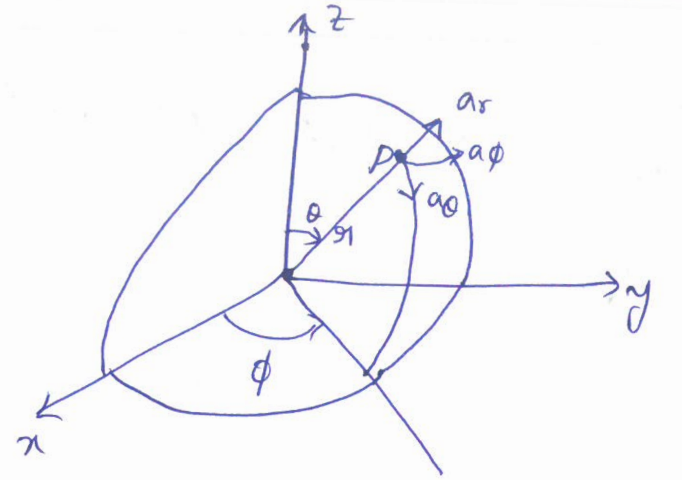
$$= 0.307 \angle 47.53^\circ$$

j

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Γ in the reflection coeffici

2 a Spherical co-ordinate system (5M)



point P and unit vectors in spherical co-ordinate system.

The spherical co-ordinates are (r, θ, ϕ) where r is defined as the distance from the origin to point P. θ is called as the angle b/w the z-axis and the position of vector of P, and ϕ is measured from the axis. The ranges of these variables are

$$0 \le r < \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi \le 2\pi$$

A vector in spherical co-ordinates may be represented as $A = A_r \cdot a_r + A_\theta a_\theta + A_\phi a_\phi$. where as A_r, A_θ and A_ϕ are called as components of the vector and a_r, a_θ, a_ϕ are called as unit vectors.

$$|A| = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$$

The unit vectors are mutually orthogonal. i.e

$$a_r \cdot a_r = a_\theta \cdot a_\theta = a_\phi \cdot a_\phi = 1$$
$$a_r \cdot a_\theta = a_\theta \cdot a_\phi = a_\phi \cdot a_r = 0$$

$$a_r \times a_\theta = a_\phi$$
$$a_\theta \times a_\phi = a_r$$
$$a_\phi \times a_r = a_\theta$$

2.b $\rho_L = 25\,nC/m$ (5M)

$x = 3m$

$y = 4m$

point (2, 3, 15)

The distance vector b/w point (2, 3, 15) and (3, 4, z) is

$$\vec{R} = -\vec{a_x} - \vec{a_y}$$

Since the line charge is along the z axis, so there is no a_z component

$$\therefore R = \sqrt{1+1} = \sqrt{2}$$

$$\vec{a_R} = \frac{-\vec{a_x} - \vec{a_y}}{\sqrt{2}}$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0 R}\vec{a_R} = \frac{25\times10^{-9}(-\vec{a_x} - \vec{a_y})}{2\times\pi\times8.854\times10^{-12}\times\sqrt{2}\times\sqrt{2}} = \frac{25\times10^{-9}(-\vec{a_x} - \vec{a_y})}{111.26\times10^{-12}}$$

$$= 0.224\times10^{3}(-\vec{a_x} - \vec{a_y})$$

$$= 224(-\vec{a_x} - \vec{a_y})\,V/m$$

3.a Gauss's Law (5M)

It states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by the surface.

Thus $\psi_{tot} = Q_{enc}$

i.e $\psi = \oint_s \bar{D}\cdot ds$ = total charge enclosed $Q = \int \rho_v\, dv$

$$\boxed{Q = \oint \bar{D}\cdot\bar{ds} = \int_v \rho_v \cdot dv}$$

By applying divergence theorem to the middle term

$$\oint_s \bar{D}\cdot\bar{ds} = \int_v \nabla\cdot D\, dv$$

Comparing the two volume integrals

$$\boxed{\therefore \nabla\cdot D = \rho_v}$$

This is the first maxwell's equation.

3. Laplace's & poisson's equations (5M)

from the gauss's Law

$$\nabla \cdot D = \nabla \cdot \varepsilon E = \rho_v \quad \& \quad \text{— (1)}$$

$$E = -\nabla V \quad \text{— (2)}$$

Substitute equa (2) in (1)

$$\nabla \cdot \varepsilon(-\nabla V) = \rho_v$$

for a homogeneous medium

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$$

This equation is known as poisson's equation.

if $\rho_v = 0$ (charge free (or) source free region) is considered $\boxed{\nabla^2 V = 0}$ → This equation is known as Laplace's equation.

4. a) Ampere's Circuital Law (5M)

It states that the line integral of the tangential component of H around a closed path is the same as the net current I_{enc} enclosed by the path.

i.e

$$\oint_l \vec{H} \cdot \vec{dl} = I_{enc} \quad \text{— (1)}$$

Ampere's Law is applied to determine H when the current distribution is symmetrical.

By applying stoke's theorem to the LHS of the above equa

$$I_{enc} = \oint_c \vec{H} \cdot \vec{dl} = \int_s (\nabla \times H)\, ds \quad \text{— (2)}$$

But $I_{enc} = \int_S \vec{J} \cdot \vec{ds}$ — (3)

Comparing The surface integrals (equa 2&3) it giv

$$\boxed{\nabla \times H = J}$$

This equation is known as Maxwell's third equation. Here $\nabla \times H = J \neq 0$ i.e magnetostatic field is not conservative.

4b) $H = 50 \sin\phi \, a_z \; mA/m^2$ (5M)

Point (2,0,0)

$J = ?$

$$\nabla \times H = J$$

$$\nabla \times H = \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & 0 & 50 \sin\phi \, a_z \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial\phi}(50\sin\phi) - 0\right]$$

$$= \frac{1}{\rho} 50 \cos\phi = \frac{1}{2} \cdot 50 \cdot \cos 0$$

$$= \boxed{25}$$

5a) $H = 2a_x + 5a_y + a_z$ A/m (5M)

$\mu_r = 4$

$$B = \mu H$$

$$= \mu_0 \cdot \mu_r \cdot H$$

$$= 4\pi \times 10^{-7} \times 4 \times (2a_x + 5a_y + a_z)$$

$$= (100.5 \, \vec{a_x} + 251.3 \, \vec{a_y} + 50.26 \, \vec{a_z}) \times 10^{-7} \text{ Tesla}$$

5b) Biot - Savart's Law and MFI due to line current (5M

It states that magnetic field intensity dH produced at a point p by the differential current element Idl is proportional to the product Idl and the sine of the angle α between the element and the line joining p to the element and is inversely proportional to the square of the distance R between p and the element.

$$dH \propto \frac{Idl \sin\alpha}{R^2}$$

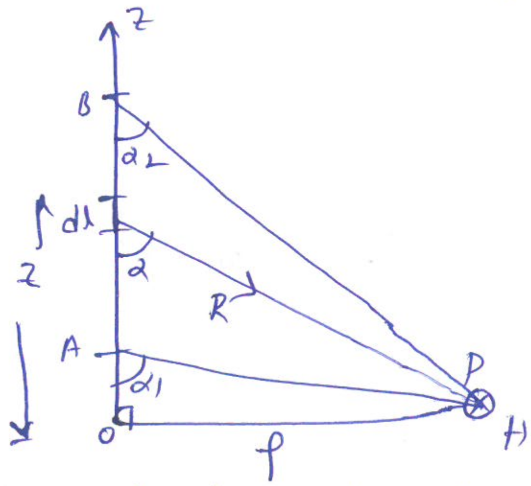
$$dH = \frac{K\, Idl \sin\alpha}{R^2}$$

in vector form

$$d\vec{H} = \frac{Idl \times \vec{a_R}}{4\pi R^2} = \frac{Idl \times \vec{R}}{4\pi R^3}$$

MFI due to line current

Assume a straight current carrying filamentary conductor of finite length AB along z-axis with its upper and lower ends respectively subtending angles α_2 and α_1 at point p.



Field at point P due to a straight filamentary

If we consider the contribution dH at P due to an element dl at $(0,0,z)$.

$$dH = \frac{I dl \times \vec{R}}{4\pi R^3}$$

But $dl = dz\, a_z$ and $\vec{R} = \rho a_\rho - z a_z$

$$\text{So } dl \times \vec{R} = \rho\, dz\, a_\phi$$

Hence $$H = \int \frac{I \rho\, dz}{4\pi \left[\rho^2 + z^2\right]^{3/2}} a_\phi$$

Let $z = \rho \cot\alpha$, $dz = -\rho \operatorname{cosec}^2\alpha\, d\alpha$ Then above equa becomes

$$H = -\frac{I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha\, d\alpha}{\rho^3 \operatorname{cosec}^3\alpha} a_\phi$$

$$= -\frac{I a_\phi}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \cdot d\alpha$$

$$\boxed{\therefore H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a_\phi}$$ finite length

If conductor is of semi infinite length $\alpha_1 = 90^\circ$, $\alpha_2 = 0^\circ$

$$\boxed{H = \frac{I}{4\pi\rho} a_\phi}$$

If conductor is of infinite length $\alpha_1 = 180^\circ$, $\alpha_2 = 0^\circ$

$$\boxed{\therefore H = \frac{I}{2\pi\rho} a_\phi}$$

unit vector $\underline{a_\phi} = a_l \times a_\rho$

6. Maxwell's equations [...] differential and integral form (10M)

Differential form	Integral form	Remarks
$\nabla \cdot D = \rho_v$	$\oint \bar{D} \cdot \bar{ds} = \int_v \rho_v \cdot dv$	Gauss's Law
$\nabla \cdot B = 0$	$\oint \bar{B} \cdot \vec{ds} = 0$	Non existance of [...] of magnetic charg[...]
$\nabla \times E = \frac{-\partial B}{\partial t}$	$\oint_L \vec{E} \cdot \bar{dl} = \frac{-\partial}{\partial t} \int \bar{B} \cdot \bar{ds}$	Faraday's Law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L \vec{H} \cdot \vec{dl} = \int_s \left(J + \frac{\partial D}{\partial t}\right) \cdot ds$	Ampere's Circu[...] Law

from Gauss's Law total flux through a close[...] surface is equal to the charge enclosed by [...] surf[...]

$$\Psi_{tot} = \oint \bar{D} \cdot \bar{ds} = Q_{enc}$$

where $Q_{enc} = \int_v \rho_v \cdot dv$

By applying divergence theorem to the term $\oint \bar{D}$[...]

$$\oint_s \bar{D} \cdot ds = \int_v \nabla \cdot D \, dv$$

Comparing the volume integrals

$$\boxed{\nabla \cdot D = \rho_v} \quad \text{— I equation}$$

unlike electric flux lines, magnetic flux [...] always close upon themselves. so it is no[...] to have an isolated magnetic charge. [...] Thus The total flux Through a closed su[...] a magnetic field must be zero i.e [...]

$$\oint \bar{B} \cdot \bar{ds} = 0$$

By applying divergence Theorem $\oint \bar{B} \cdot ds = \int_v \nabla \cdot B = 0$

$$\boxed{\therefore \nabla \cdot B = 0} \quad — \text{II equation}$$

from Faraday's Law induced emf is equal to the rate of change of flux linkages

$$V_{emf} = -\frac{d\lambda}{dt} = -\frac{d\psi}{dt} \quad (N=1)$$

from electric potential $\qquad \psi = \int \bar{B} \cdot \bar{ds}$

$$V_{emf} = \oint_L \vec{E} \cdot \bar{dl} = -\frac{d}{dt} \int_S \bar{B} \cdot ds$$

Suppose we are Considering stationary loop in a moving B field

$$V_{emf} = \oint_L \vec{E} \cdot \bar{dl} = -\int_S \frac{\partial B}{\partial t}$$

By applying stoke's theorem to the middle term

$$\oint_L \vec{E} \cdot \bar{dl} = \int_S \nabla \times E \, ds = -\int_S \frac{\partial B}{\partial t}$$

By removing surface integrals

$$\boxed{\therefore \nabla \times E = \frac{-\partial B}{\partial t}} \quad — \text{III equa}$$

from Ampere's Law

$$\nabla \times H = J \quad — ①$$

But the divergence of curl of any vector field is zero

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 \quad — ②$$

from Continuity of current equation

$$\nabla \cdot J = \frac{-\partial}{\partial t} \rho_v \neq 0 \quad — ③$$

Equa 2 & 3 are in Compatible. So modify equa ① to agree with equa 2 & 3 add the term Jd

$\nabla \times H = J + J_d$

$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot J_d = 0$

$\therefore \nabla \cdot J_d = -\nabla \cdot J$

$= \frac{\partial}{\partial t} \rho_v$ (where as $\nabla \cdot D = \rho_v$)

$\nabla \cdot J_d = \frac{\partial}{\partial t} \nabla \cdot D$

$\boxed{\therefore J_d = \frac{\partial D}{\partial t}}$ substitute in equa ①

$\boxed{\therefore \nabla \times H = J + J_d}$

7a $J_c = 0.02 \sin 10^9 t \ A/m^2$ (5M)

$\sigma = 103$ mho/m

$\varepsilon_r = 6.5$

$J_d = ?$

$J_d = \frac{\partial D}{\partial t}$

$J_c = \sigma E$

$0.02 \sin 10^9 t = 103 E$

$E = \frac{0.02 \sin 10^9 t}{103}$

$= 1.94 \times 10^{-4} \sin 10^9 t$ V/m

$D = \varepsilon E = \varepsilon_0 \cdot \varepsilon_r \cdot E = 8.854 \times 10^{-12} \times 6.5 \times 1.94 \times 10^{-4} \sin 10^9 t$

$= 0.011 \sin 10^9 t \times 10^{-12} C/m$

$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t} (0.011 \sin 10^9 t \times 10^{-12})$

$= 0.011 \times 10^9 \times 10^{-12} \cos 10^9 t$

$= 0.011 \times 10^{-3} \cos 10^9 t \ A/m^2$

7b Inconsistency of Ampere's Law, Displacement current density (5M)

Consider the Ampere's Law for static fields

$$\nabla \times H = J \quad \text{—— (1)}$$

But the divergence of curl of any vector field is identically zero

$$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J \quad \text{— (2)}$$

Consider the continuity of current equa

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad \text{— (3)}$$

Equations 2 & 3 are incompatible. There is a inconsistency in ampere's Law i.e equa (1). To avoid this inconsistency add the term J_d to equa (1)

$$\nabla \times H = J + J_d.$$

Apply divergence on both sides & vector identity

$$\nabla \cdot (\nabla \times H) = (\nabla \cdot J + \nabla \cdot J_d) = 0$$

$$\therefore \nabla \cdot J_d = -\nabla \cdot J$$

$$= -\left(-\frac{\partial}{\partial t}\rho_v\right)$$

$$= \frac{\partial}{\partial t}(\rho_v)$$

$$\nabla \cdot J_d = \frac{\partial}{\partial t}(\nabla \cdot D) \qquad (\because \nabla \cdot D = \rho_v)$$

Gauss's Law

$$\boxed{\therefore J_d = \frac{\partial D}{\partial t}}$$

∴ Therefore displacement current density $J_d = \frac{\partial D}{\partial t}$ and displacement current is $I_d = \int_S J_d \cdot ds$. Therefore the Maxwell's equa is modified as

$$\boxed{\nabla \times H = J + J_d}$$

8. Good Conductors and lossless Dielectrics, wave propaga[tion] in Good Conductors (10M)

For a lossless Dielectric $\sigma = 0$ & $\alpha = 0$, $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \cdot \mu_0$ (or) $\sigma << \omega\varepsilon$.

For Good conductors the conductivity is very high compared to the wave frequency i.e $\sigma >> \omega\varepsilon$, $\sigma \simeq \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_r \cdot \mu_0$.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Since $\sigma >> \omega\varepsilon$, so we neglect the imaginary part

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\,\angle 45^\circ$$

$$\sqrt{\omega\mu\sigma}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right] = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\therefore \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

and $$\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Thus for good conductors α and β are equal

In a highly conducting medium α is proportional to $\sqrt{\sigma}$, the losses are very high and the wave attenuates rapidly.

Intrinsic impedance η

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

for a good conductor $\sigma >> \omega\varepsilon$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

(or)

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

phase velocity $v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega\mu\sigma}{2}}}$

$$\therefore v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$$

wave length $\lambda = \frac{2\pi}{\beta}$

So from the value of η we observed that E field leads H by 45° If

$$\vec{E} = E_0 \cdot e^{-\alpha z} \cos(\omega t - \beta z) a_x$$

$$\vec{H} = \frac{E_0}{\sqrt{\frac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ) a_y$$

$\therefore$ E (or) H wave travells in a conducting medium its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance through which the wave amplitude is decreases by a factor e^{-1} (about 37%) is called skin depth (or) depth of penetration of the medium

i.e $E_0 \cdot e^{-\alpha\delta} = E_0 \cdot e^{-1}$

(or) $\delta = \frac{1}{\alpha}$ → It is valied for any med

The skin depth is a measure of depth to whi an EM wave can penetrate into the medium.

For good conductors $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}}$

9.a <u>Poynting Theorem</u> (5M)

Energy can be transported from one point to another point by means of EM waves. The rate of charge of such energy transportation can be obtained from Maxwell <u>equa</u>

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow -\mu \cdot \frac{\partial H}{\partial t} \quad —(1)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \Rightarrow \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad -(2)$$

Dotting both sides of equa (2) with E gives

$$E \cdot (\nabla \times H) = \sigma E^2 + E \cdot \varepsilon \frac{\partial E}{\partial t} \quad -(3)$$

Consider the vector identity

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

Apply this vector identity to equa (3) Let $A = H$ & $B = E$ gives

$$H \cdot (\nabla \times E) + \nabla \cdot (H \times E) = \sigma E^2 + \varepsilon E \frac{\partial E}{\partial t} \quad —(4)$$

$$= \sigma E^2 + \frac{1}{2} \varepsilon \cdot \frac{\partial}{\partial t} E^2$$

Dotting both sides of equa (1) with H gives

$$H \cdot (\nabla \times E) = -\mu H \frac{\partial H}{\partial t} = -\frac{1}{2} \mu \frac{\partial}{\partial t} H^2 \quad —(5)$$

Substitute the $H \cdot (\nabla \times E)$ in equa (4) gives

$$\nabla \cdot (H \times E) - \frac{1}{2} \mu \frac{\partial}{\partial t} H^2 = \sigma E^2 + \frac{1}{2} \varepsilon \frac{\partial E^2}{\partial t}$$

$$\Rightarrow -\nabla \cdot (E \times H) = \sigma E^2 + \frac{1}{2} \varepsilon \frac{\partial E^2}{\partial t} + \frac{1}{2} \mu \frac{\partial H^2}{\partial t}$$

$$\Rightarrow \nabla \cdot (E \times H) = -\sigma E^2 - \frac{1}{2} \varepsilon \frac{\partial E^2}{\partial t} - \frac{1}{2} \mu \frac{\partial H^2}{\partial t}$$

Taking volume integral on both sides

$$\int_V \nabla \cdot (E \times H)\, dv = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right] dv - \int_V \sigma E^2 dv$$

Apply divergence theorem to the LHS

$$\oint_S (E \times H)\, ds = -\frac{\partial}{\partial t} \int_V \left[\frac{1}{2} \varepsilon E^2 + \frac{1}{2} \mu H^2\right] dv - \int_V \sigma E^2 dv \quad —(6)$$

poynting theorem states that the net power flowing out of the given volume 'v' is equal to the time rate of decrease in the energy stored within v minus the conduction losses.

9b) Circular & Elliptical polarization (5M)

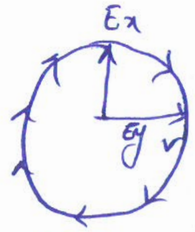
Consider a uniform plane wave travelling in the z-direction with the electric field components $\vec{E_x}$ and $\vec{E_y}$ not in phase, i.e they reach their maximum values at different instants of time. Then the direction of the resultant electric vector $\vec{E}$ will vary with time.

If $\vec{E_x}$ and $\vec{E_y}$ have equal magnitude with a 90° phase difference Then the locus of the resultant vector $\vec{E}$ is a circle. This wave is called a circularly polarized wave.

$E_x^2 + E_y^2 = E_a^2$. This equa represents the locus of a circle, E_a is the amplitude at $z=0$

In circular polarization if $\vec{E_y}$ leads $\vec{E_x}$ by 90° Then it is called as Left hand circular polarization. If $\vec{E_y}$ lags behind $\vec{E_x}$ by 90° the direction of rotation is a right handed screw advancing in the z-direction is called right hand circularly polarized wave

LHC polarization



RHC polarization

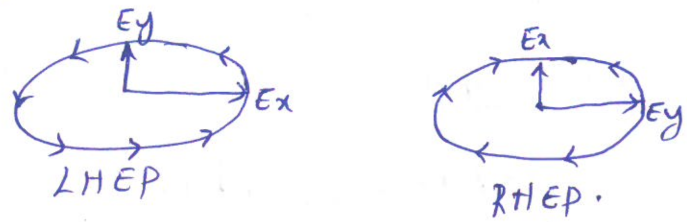
Elliptical polarization

If $\vec{E_x}$ and $\vec{E_y}$ have different magnitudes with 90° phase difference then the locus of the resultant vector $\vec{E}$ is an ellipse. The wave is called an elliptical polarized wave.

$$\frac{E_x^2}{E_1^2} + \frac{E_y^2}{E_2^2} = 1$$ → this equa represents the locus of an ellipse.

If $\vec{E_y}$ leads $\vec{E_x}$ by 90° the direction of rotation is a left handed screw advancing in the z-direction is called left hand elliptically polarized wave.

Similarly if $\vec{E_y}$ lags $\vec{E_x}$ by 90°, the direction of rotation is a right handed screw advancing in the z-direction then the wave is called a right hand elliptically polarized wave.



10a Input impedance of a transmission line (5M)

It is the impedance at the i/p (or) generator side of the line. It is represented as Z_{in}

Consider the voltage and current equas of a transmission line at point z are

$$V(z) = V^+ e^{\gamma z} + V^- e^{-\gamma z}$$

$$I(z) = \frac{V^+}{Z_0} e^{\gamma z} + \frac{V^-}{Z_0} e^{-\gamma z}$$

Z_0 is the characteristic impedance.

The line impedance at point z is

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \frac{e^{\gamma z} + \Gamma_L e^{-\gamma z}}{e^{\gamma z} - \Gamma_L \cdot e^{-\gamma z}}$$

where on $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$ and Γ_L

is the reflection co-efficient at load. substitute the value of Γ_L in the above equa

$$Z(z) = Z_0 \frac{((Z_L+Z_0)e^{\gamma z} + (Z_L - Z_0)e^{-\gamma z})}{((Z_L+Z_0)e^{\gamma z} - (Z_L - Z_0)e^{-\gamma z})}$$

$$= Z_0 \frac{Z_L(e^{\gamma z} + e^{-\gamma z}) + Z_0(e^{\gamma z} - e^{-\gamma z})}{Z_0(e^{\gamma z} + e^{-\gamma z}) + Z_L(e^{\gamma z} - e^{-\gamma z})}$$

Substitute $\cosh \gamma z = \frac{e^{\gamma z} + e^{-\gamma z}}{2}$ and $\sinh \gamma z = \frac{e^{\gamma z} - e^{-\gamma z}}{2}$

in the above equa

$$Z(z) = Z_0 \frac{Z_L \cosh \gamma z + Z_0 \sinh \gamma z}{Z_0 \cosh \gamma z + Z_L \sinh \gamma z} = Z_0 \cdot \frac{Z_L + Z_0 \tanh \gamma z}{Z_0 + Z_L \tanh \gamma z}$$

Suppose if z in at i/p side $Z(z) = Z_{in}$

$$\therefore Z_{in} = Z_0 \cdot \left[\frac{Z_L + Z_0 \tanh \gamma z}{Z_0 + Z_L \tanh \gamma z} \right]$$

$L = 26 H/m$
$C = 2 PF/m$
$G = 0$ $\qquad\qquad \omega = 2\pi f = 31.4\times10^{6}$ rad/sec
$f = 5 MHz$

$$Z = R + j\omega L$$
$$= 25.23 + j\, 31.4\times10^{6}\times26 = 25.23 + 816400000\, j$$
$$= 816400000 \angle 89.99$$

$$Y = G + j\omega C$$
$$= j\,31.4\times10^{6}\times2\times10^{-12} = 6.28\times10^{-5}\, j$$
$$= 6.28\times10^{-5}\angle 90^{\circ}$$

$$\gamma = \sqrt{Z\times Y}$$
$$= \sqrt{816400000\angle 89.99 \times 6.28\times10^{-5}\angle 90}$$
$$= \sqrt{51269.92\angle 180}$$
$$= 226.4\angle 90^{\circ}$$
$$0 + j226.4$$
$\alpha = 0$; $\beta = 226.4$ rad/m

11 a For a lossless transmission line $R=0$, $G=0$. So the line is made of perfect conductors and perfect dielectrics (5M)

Conditions for lossless transmissions line

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

By substituting $R=0; G=0$

$$\alpha + j\beta = \sqrt{j\omega L \times j\omega C} = j\omega\sqrt{LC}$$

$$Z_0 = \sqrt{j\omega L / j\omega C} = \sqrt{L/C}$$

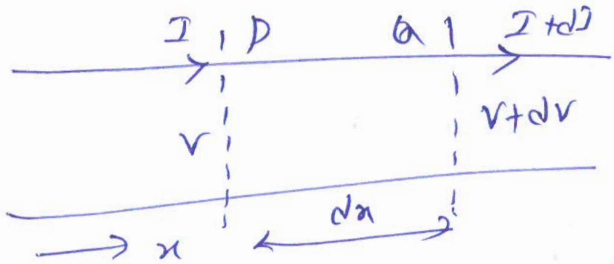
$$\therefore \alpha = 0; \quad \beta = \omega\sqrt{LC} \quad \& \quad Z_0 = \sqrt{L/C}$$

phase velocity $v_p = \omega/\beta = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{\mu\varepsilon}}$

11 b Transmission line Equations (5M)

Consider a transmission line with two parallel conductors. Let R, L, G and C be the primary constants. Assume that these values do not vary with frequency. Consider a point P on the line at a distance x from the source as shown in fig.

Let another point Q be at a small distance dx from point P.



Let V and I be the voltage and current at point P. Since the voltage and currents are uniformly distributed along line, at point Q let the voltage be $V+dV$ and the current be $I+dI$.

For a small length dx of the line the series impedance is $(R+j\omega L)\,dx$ and the shunt admittance is $(G+j\omega C)\,dx$

The pottential difference b/w P and Q is

$V-(V+dV) = I(R+jωL)dx$

and the current difference b/w p and Q in

$I-(I+dI) = V(G+jωC)dx$

$\therefore -\frac{dV}{dx} = (R+jωL)I$ & $-\frac{dI}{dx} = (G+jωC)V$

Taking derivatives of above equa in

$$-\frac{d^2V}{dx^2} = (R+jωL)\frac{dI}{dx}$$

Substitute the value of $\frac{dI}{dx}$

$$+\frac{d^2V}{dx^2} = (R+jωL)(G+jωC)V$$

Similarly $\frac{d^2I}{dx^2} = (R+jωL)(G+jωC)I$

Let $\gamma^2 = (R+jωL)(G+jωC)$; $\gamma = \alpha + j\beta$

α is attenuation constant & β is phase constan[t]

$$\boxed{\therefore \frac{d^2V}{dx^2} = \gamma^2 V \; ; \quad \frac{d^2I}{dx^2} = \gamma^2 I}$$

These are the differential equa of the trans[-]mission lines. The standard solutions for these equa are

$V = ae^{\gamma x} + be^{-\gamma x}$ where a, b, c, d are the

& $I = ce^{\gamma x} + de^{-\gamma x}$ constants

By simplifying the above equations

$V = a\cdot e^{\gamma x} + be^{-\gamma x}$ &

$I = \frac{1}{Z_0}(be^{-\gamma x} - ae^{\gamma x})$

In terms of hyperbolic function

substitute $e^{\gamma x} = \cosh\gamma x + \sinh\gamma x$ &

$e^{-\gamma x} = \cosh\gamma x - \sinh\gamma x$ Then

$$V = A\cosh\gamma x + B\sinh\gamma x$$

$$I = \frac{-1}{Z_0}(A\sinh\gamma x + B\cosh\gamma x)$$