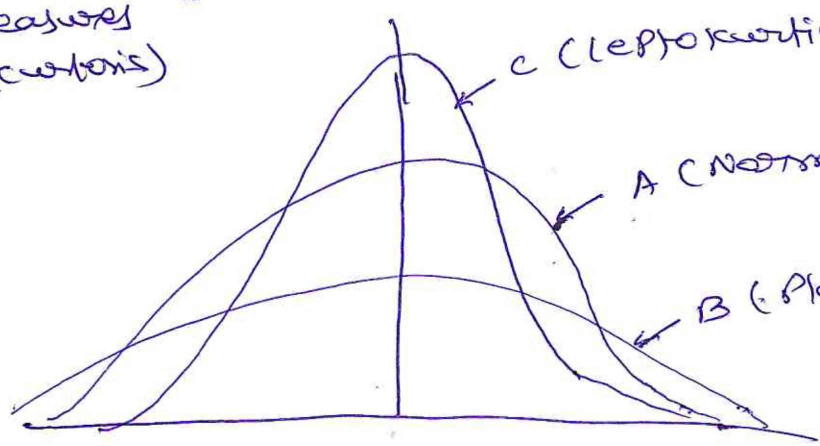
Kurtosis :-

Sometimes two (or more) distributions may have the same mean, the same S.D, & the same skewness but their shapes, in terms of height, may differ. To understand a distribution from the point of height, we calculate kurtosis (convexity of the frequency curve). Kurtosis enables us to have an idea about the 'flatness or peakedness' of the frequency curve. It is measured by the coefficient β_2 or its derivation γ_2 given by

$$\beta_2 = \frac{\mu_4}{\mu_2^2}, \quad \gamma_2 = \beta_2 - 3.$$

(Absolute measures of kurtosis) (Relative measure of kurtosis).



Curve of the type 'A' which is neither flat nor peaked is called the normal curve or mesokurtic curve & for such a curve $\boxed{\beta_2 = 3 \text{ i.e } \gamma_2 = 0}$.

Curve of the type 'B' which is ~~neither~~ flatter than the normal curve is known as platykurtic & for such a curve $\boxed{\beta_2 < 3 \text{ i.e } \gamma_2 < 0}$

curve of the type 'c' which is more peaked than the normal curve is called leptokurtic & for such a curve $\boxed{\beta_2 > 3 \text{ i.e } \gamma_2 > 0}$. for a platykurtic curve

For a distribution, the mean is 10, variance is 16, γ_1 is +1 & β_2 is 4. obtain the first 4 moments about the origin, i.e zero. comment upon the nature of distribution.

solution: Given that Mean = 10, $\mu_2 = 16 \Rightarrow \sigma^2 = 16$
$\Rightarrow \sigma = 4$

$\gamma_1 = +1$, $\beta_1 = 4$.

First 4 moments about origin are

$\mu_1' = \text{Mean} = 10$

$\mu_2 = \mu_2' - (\mu_1')^2$

$\Rightarrow \mu_2' = \mu_2 + (\mu_1')^2 = 16 + (10)^2 = 116$

Given $\gamma_1 = 1$

$$\gamma_1 = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\mu_3}{\sigma^3} = 1$$

$$\Rightarrow \mu_3 = \sigma^3 = 64$$

$$\mu_3' = \mu_3 + 3\mu_2'\mu_1' - 2(\mu_1')^3$$
$$= 64 + 3 \times 116 \times 10 - 2 \times 1000 = 1,544$$

Measures of skewness

1) Relationship between 3 M's of central tendency - commonly known as the Karl Pearson's measures of skewness.

2) Quartile measure of skewness - known as Bowley's measures of skewness.

3) Percentile measure of skewness also called the Kelley's measure of skewness

4) Measures of skewness based on moments.

All these measures tell us both the direction & the extent of the skewness.

1) Karl Pearson's coefficient of skewness of a distribution is 0.32. Its S.D is 6.5 & mean is 29.6. Find the mode & median of the distribution.

Solution:- $\boxed{\text{coefficient of skewness} = \frac{\text{Mean} - \text{Mode}}{\sigma}}$

$$0.32 = \frac{29.6 - \text{Mode}}{6.5}$$

$$\Rightarrow 6.5 \times 0.32 = 29.6 - \text{Mode}$$

$$\Rightarrow \text{Mode} = 29.6 - 2.08$$

$$= 27.52$$

$\boxed{\text{coefficient of skewness} = \frac{3(\text{Mean} - \text{Median})}{\sigma}}$

$$0.32 = \frac{3(29.6 - \text{Median})}{6.5}$$

$$6.5 \times 0.32 = 88.8 - 3\,\text{Median}$$

$$\text{Median} = \frac{88.8 - 2.08}{3} = 28.91$$

Compute quartile deviation & coefficient of skewness from the following values.

Median = 18.8 cm

Q_1 = 14.6 cm , Q_3 = 25.2 cm

Solution :- quartile deviation = $\frac{Q_3 - Q_1}{2}$

$= \frac{25.2 - 14.6}{2} = \frac{10.6}{2} = 5.3$ cm

Coefficient of skewness = $\frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$

$= \frac{25.2 + 14.6 - 2(18.8)}{25.2 - 14.6}$

$= \frac{2.2}{10.6} = 0.207$

In a moderately skewed frequency distribution mean is 50 & median is 53. If the coefficient of variation is 20%. Find the pearsons coefficient of skewness.

Solution :- $C.V = \frac{\sigma}{\bar{x}} \times 100$

$20 = \frac{\sigma}{50} \times 100$

$\sigma = 10$

$$\boxed{\text{Coefficient of Skewness} = \frac{3(\text{Mean} - \text{Median})}{\sigma}}$$

$$= \frac{3(50-53)}{10}$$

$$= \frac{-9}{10} = -0.9$$

2) In a certain distribution the first 4 moments about the arbitrary origin (4) are $-1.5, 17, -30$ & 108. calculate β_2 & state whether the distribution is leptokurtic or platykurtic.

Solution:-

Given that $\mu_1' = -1.5$, $\mu_2' = 17$, $\mu_3' = -30$, $\mu_4' = 108$

$\mu_1 = \mu_1' - \mu_1' = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 17 - (-1.5)^2 = 17 - 2.25$
$= 14.75$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$
$= -30 - 3(-1.5)(17) + 2(-1.5)^3$
$= 39.75$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$= 142.313$

$$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}} = \frac{142.313}{(14.75)^2} = 0.65.$$

$\therefore$ $\boxed{\beta_2 < 3}$ i.e $\beta_2 = 0.65 < 3$, the distribution is Platykurtic

3) compute the pearsonian measure of skewness for the following distribution.

size in inches	No. of observations
30-33	2
33-36	4
36-39	26
39-42	47
42-45	15
45-48	6

solution:-

size(x)	f	Mid-value (x)	d'	fd'	fd'^2
30-33	2	31.5	-2	-4	8
33-36	4	34.5	-1	-4	4
36-39	26 - f_0	37.5	0	0	0
39-42	47 - f_1	40.5	1	47	47
42-45	15 - f_2	43.5	2	30	60
45-48	6	46.5	3	18	54.
	100			87	173.

$\boxed{d' = \frac{x-A}{h}}$ let $A = 37.5$, $h = 3$.

$\boxed{\text{skewness} = \frac{\bar{x} - \text{Mode}}{\sigma}}$

Highest frequency is 47 & corresponding class-interval is 39-42 (modal class).

calculation of Mean:-

$\boxed{\text{Mean} = A + \frac{\sum fd'}{N} \times h} = 37.5 + \frac{87}{100} \times 3 = 40.11$

Calculation of ~~Median~~ S.D.

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h.$$

$$= \sqrt{0.973} \times 3 = 2.96.$$

Calculation of Mode:

$$\boxed{\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h}$$

$$= 39 + \frac{47-26}{94-26-15} \times 3.$$

$$= 39 + \frac{21}{53} \times 3 = 40.19.$$

Coefficient of skewness $= \frac{40.11 - 40.19}{2.96} = \frac{-0.08}{2.96} = -0.027$

4) The first 4 central moments of a distribution are 0, 2.5, 0.7 & 18.75. Test the skewness & kurtosis of the distribution.

Solution :-

Skewness is tested by μ_3 which should be equal to zero in a symmetrical distribution.

∴ in the given problem it is $\mu_3 = 0.7$.

$\mu_1 = 0$, $\mu_2 = 2.5$, $\mu_3 = 0.7$, $\mu_4 = 18.75$.

∴ we can conclude that the distribution is not symmetrical.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad , \gamma_1 = \sqrt{\beta_1}$$

$$= \frac{\mu_3}{(\mu_2)^{3/2}} = \sqrt{0.031} = 0.18 .$$

$\therefore \gamma_1 = 0.18$, we conclude that the distribution is not symmetrical but has a +ve skewness $= 0.18$.

kurtosis is tested by β_2. β_2 should be equal to 3.

If $\beta_2 = 3$ - mesokurtic

If $\beta_2 < 3$ - platykurtic (flat peaked)

If $\beta_2 > 3$ - leptokurtic (more peaked)

$$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}} = \frac{18.75}{(2.5)^2} = \frac{18.75}{6.25} = 3.$$

$\therefore \beta_2 = 3$, we conclude that the curve is mesokurtic.

5) The S.D of a symmetrical distribution is 3. What must be the value of the 4th moment about the mean in order that the distribution be mesokurtic?

Solution:- Given that a normal curve is mesokurtic.

$\therefore \beta_2 = 3$. , Given that S.D $= 3 \Rightarrow \sigma = 3$

$\Rightarrow \sigma^2 = 9$.

$$\Rightarrow \beta_2 = \frac{\mu_4}{\mu_2^2}.$$

$$3 = \frac{\mu_4}{(\sigma^2)^2} = \frac{\mu_4}{\sigma^4} \quad (\because \mu_2 = \sigma^2).$$

$$3 = \frac{\mu_4}{81} \Rightarrow \mu_4 = 243.$$

6) Find the coefficient of skewness from the following information: Difference of two quartiles = 8, Mode = 11, sum of two quartiles = 22, Mean = 8.

Solution :-

$$\text{Median} = \frac{\text{Mode} + 2\bar{x}}{3} = \frac{11 + 2(8)}{3} = 9.$$

$$\boxed{\text{Coefficient of skewness} = \frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}}$$

$$= \frac{22 - 2(9)}{8} = 0.5.$$

7) In a frequency distribution the coefficient of skewness based on quartiles is 0.6. If the sum of the upper & the lower quartiles is 100 & the median is 38, find the value of the upper & the lower quartiles.

Solution :-

$$\text{Coefficient of skewness} = \frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$$

$$0.6 = \frac{100 - 2(38)}{Q_3 - Q_1}$$

$$\Rightarrow Q_3 - Q_1 = \frac{24}{0.6} = 40$$

Given that $Q_3 + Q_1 = 100 \rightarrow (1)$

$Q_3 - Q_1 = 40 \rightarrow (2)$.

By solving the two equations, we get

$Q_3 = 70$, $Q_1 = 30$.

8) The number of units sold by a firm during the last seven months of the current year is given below:

Months:	Jun.	Jul	Aug	Sep	Oct	Nov.	Dec
No. of units sold:	22	25	38	43	44	50	72

Compute:

a) The mean monthly sales

b) S.D & coefficient of variation.

c) Coefficient of skewness by Karl Pearson's method

d) Bowley's coefficient of skewness.

e) β_1 & β_2.

Solution:

Month	Sales(x)	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^3$	$(x-\bar{x})^4$
~~Mo~~ Jun.	22	-20	400	-8 000	160 000
Jul	25	-17	289	-4,913	83,521
Aug	38	-4	16	-64	256
Sep.	43	+1	1	1	1
Oct.	44	2	4	8	16
Nov	50	8	64	512	4096
Dec.	72	30	900	27000	8,10,000

$n = 7$, $\Sigma x = 294$, $\Sigma(x-\bar{x}) = 0$, $\Sigma(x-\bar{x})^2 = 1674$

$\Sigma(x-\bar{x})^3 = 14544$, $\Sigma(x-\bar{x})^4 = 10,57,890$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{294}{7} = 42$$

$$\mu_1 = \frac{\Sigma(x-\bar{x})}{n} = 0.$$

$$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{1674}{7} = 239.14$$

$$\mu_3 = \frac{\Sigma(x-\bar{x})^3}{n} = \frac{14,544}{7} = 2,077.71$$

$$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{n} = \frac{10,57,890}{7} = 1,51,127.14$$

$\sigma = \sqrt{\mu_2} = \sqrt{239.14} = 15.46$

$$\boxed{\text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100}$$

$$= \frac{15.46}{42} \times 100 = 36.8\%.$$

Median $= \frac{(n+1)}{2}$ value $= 4^{th}$ value $= 43$

$Q_1 = \frac{(n+1)^{th}}{4}$ value $= 2^{nd}$ value $= 25$

$Q_3 = 3\frac{(n+1)}{4}$ value $=$ ~~8~~ 6^{th} value $= 50$

Karl Pearson's coefficient of skewness

$$= \frac{3(\bar{x} - \text{Median})}{\sigma} = \frac{3(42-43)}{15.46} = -0.19$$

Bowley's coefficient of skewness

$$= \frac{Q_3 + Q_1 - 2(\text{Median})}{Q_3 - Q_1}$$

$$= \frac{50 + 25 - 2 \times 43}{50 - 25} = -0.44$$

since coefficient of skewness is negative, it shows that the distribution is negatively skewed.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(2077.71)^2}{(239.14)^3} = \frac{43,16,896.65}{1,36,75,924} = 0.316$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{151,127.14}{(239.14)^2} = 2.64.$$

$\therefore \beta_2 < 3$ i.e $2.64 < 3$.

$\therefore$ the distribution is Platykurtic

7) A foreign bank is quite liberal in extending export credit to its customers provided the documentation is perfect. As on December 31, last year, the following information was available regarding export credit:

Amount of credit (in Rs. crores)	0-5	5-10	10-15	15-20	20-25
No. of accounts	6	11	28	9	1

From the above information calculate:

i) The four central moments.

ii) Moment coefficient of skewness.

iii) Moment coefficient of kurtosis

iv) Karl Pearson's coefficients of skewness

v) Bowley's coefficient of skewness.

vi) Moments about origin.

vii) coefficient of variation.

Solution:

Amount	f	cf	Mid Point (x)	d'	fd'	fd'^2	fd'^3	fd'^4
0-5	6	6	2.5	-2	-12	24	-48	96
5-10	11 (f_0)	17	7.5	-1	-11	11	-11	11
10-15	28 (f_1)	45	12.5	0	0	0	0	0
15-20	9 (f_2)	54	17.5	1	9	9	9	9
20-25	1	55	22.5	2	2	4	8	16

$N = 55$, $d' = \frac{x-A}{h}$. Let $A = 12.5$, $h = 5$

$\Sigma fd' = -12$, $\Sigma fd'^2 = 48$, $\Sigma fd'^3 = -42$, $\Sigma fd'^4 = 132$

i) $\mu_1' = \frac{\Sigma fd'}{N} \times h = \frac{-12}{55} \times 5 = -1.09$

$\mu_2' = \frac{\Sigma fd'^2}{N} \times h^2 = \frac{48}{55} \times 25 = 21.82$

$\mu_3' = \frac{\Sigma fd'^3}{N} \times h^3 = -\frac{42}{55} \times 125 = -95.45$

$$\mu_4' = \frac{\Sigma f d'^4}{N} \times h^4 = \frac{132}{55} \times 625 = 1500$$

$$\mu_1 = \mu_1' - \mu_1' = 0.$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = 21.82 - (-1.09)^2 = 20.63$$

$$\mu_3 = \mu_3' - 3\mu_1' \mu_2' + 2(\mu_1')^3$$

$$= -95.45 - 3(-1.09)(21.82) + 2(-1.09)^3$$

$$= -26.69$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2 \mu_2' - 3(\mu_1')^4$$

$$= 1500 - 4(-1.09)(-95.45) + 6(-1.09)^2(21.82) - 3(-1.09)^4$$

$$= 1235.16$$

ii) Moment coefficient of skewness $= \sqrt{\beta_1}$

$$\boxed{\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}} = \frac{-26.69}{\sqrt{(20.63)^3}} = -0.28$$

The distribution is negatively skewed.

iii) $\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}} = \frac{1235.16}{(20.63)^2} = 2.9$

Moment coefficient of kurtosis =

$(\beta_2 - 3) = 2.9 - 3 = -0.1$. $\because \beta_2 < 3$

$\therefore$ The distribution is Platykurtic

iii) Moments about origin

$v_1 = A + \mu_1' = 12.5 - 1.09 = 11.41 = \text{Mean}$

$v_2 = \mu_2 + (v_1)^2 = 20.63 + (11.41)^2 = 150.82$

$v_3 = \mu_3 + 3v_1v_2 - 2(v_1)^3$

$= -25.39 + 3(11.41)(150.82) - 2(11.41)^3$

$= 2,164.99$

$v_4 = \mu_4 + 4v_1v_3 - 6(v_1)^2 v_2 + 3(v_1)^4$

$= 123516 + 4(11.41)(1435.09) - 6(11.41)^2(150.82) + 3(11.41)^4 = 33,082.30$

iv) $\boxed{\text{Mode} = l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h}$

l = lower limit of the modal class

h = magnitude of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the classes preceding the modal class

f_2 = frequency of the class succeeding the modal class

$\text{Mode} = 10 + \left(\dfrac{28 - 11}{2 \times 28 - 11 - 9}\right)5 = 10 + \dfrac{17}{36} \times 5$

$= 12.36$

$\sigma = \sqrt{\mu_2} = \sqrt{20.63} = 4.54$

Karl Pearson's coefficient of skewness $= \dfrac{\bar{x} - \text{Mode}}{\sigma}$

$= \dfrac{11.41 - 12.36}{4.54} = -0.21$

v) Bowley's coefficient of skewness

$$= \frac{Q_3 + Q_1 - 2\,Median}{Q_3 - Q_1}$$

$$Median = l + \frac{\frac{n}{2} - C}{f} \times h.$$

l = lower limit of median class.

Median class is the class corresponding to cummulative frequency, just greater than $\frac{N}{2}$.

$\frac{N}{2} = \frac{55}{2} = 27.5.$

Median class - 10 - 15

frequency - 28 , $h = 5.$

C = cummulative frequency of the class preceding the median class

$$Median = 10 + \frac{27.5 - 17}{28} \times 5.$$

$$= 11.88$$

$$\boxed{Q_1 = l + \frac{\frac{n}{4} - cf}{f} \times h} = 5 + \frac{13.75 - 6}{11} \times 5 = 8.52$$

$$\boxed{Q_3 = l + \frac{\frac{3n}{4} - cf}{f} \times h} = 10 + \frac{41.25 - 17}{28} \times 5 = 14.33$$

Bowley's coefficient of skewness $= \frac{Q_3 + Q_1 - 2\,Median}{Q_3 - Q_1}$

$$= \frac{14.33 + 8.52 - 2 \times 11.88}{14.33 - 8.52} = -0.16.$$

vii) coefficient of variation : $\frac{\sigma}{\bar{x}} \times 100$

$$= \frac{4.54}{11.41} \times 100 = 39.79\%$$

10) Following are data of daily earning (in Rs) of employees in a company:

Earnings:	50-70	70-90	90-110	110-130	130-150	150-170	170-190
No. of workers:	4	8	12	20	6	7	3

Calculate the first 4 moments about the point 120. Convert the results into moments about the mean. Compute the value of γ_1 & γ_2 & comment on the result.

Solution:

class	Mid point (x)	f	d'	fd'	fd'^{2}	fd'^{3}	fd'^{4}
5070	60	4	-3	-12	36	-108	324
70-90	80	8	-2	-16	32	-64	128
90-110	100	12	-1	-12	12	-12	12
110-130	120	20	0	0	0	0	0
130-150	140	6	1	6	6	6	6
150-170	160	7	2	14	28	56	112
170-190	180	3	3	9	27	81	243

$N = 60$, $\Sigma fd' = -11$, $\Sigma fd'^2 = 141$, $\Sigma fd'^3 = -41$.

$\Sigma fd'^4 = 825$

$d' = \frac{x-A}{h}$ $A = 120$, $h = 20$.

$\mu_1' = \frac{\Sigma fd'}{N} \times h = \frac{-11}{60} \times 20 = -3.66$

$\mu_2' = \frac{\Sigma fd'^2}{N} \times h^2 = \frac{141}{60} \times 400 = 940$

$\mu_3' = \frac{\Sigma fd'^3}{N} \times h^3 = -\frac{41}{60} \times 8000 = -5,466.66$

$\mu_4' = \frac{\Sigma fd'^4}{N} \times h^4 = \frac{825}{60} \times 1,60,000 = 22,00,000$

$\mu_1 = \mu_1' - \mu_1' = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 940 - (-3.66)^2 = 926.6$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$

$= -5466.66 - 3(-3.66)(940) + 2(-3.66)^3$

$= 4,756.49$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$= 22,00,000 - 4(-3.66)(-5,466.66) + 6(-3.66)^2(940) - 3(-3.66)^4$

$= 21,13,789.78$

$\alpha_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{4,756.49}{\sqrt{(926.6)^3}} = \frac{4,756.49}{28,205.80} = 0.169$

$\Rightarrow$ The distribution is positively skewed.

$\gamma_2 = \beta_2 - 3 = \frac{\mu_4}{\mu_2^2} - 3 = \frac{21,13,789.78}{8,58,587.56} - 3$

$= -0.54$

$\therefore \gamma_2 < 0.$

The distribution is platykurtic $\because$ this value is -ve.

1) Given the following information:

$N = 100$, $\Sigma(x-98) = 50$, $\Sigma(x-98)^2 = 1970$,

$\Sigma(x-98)^3 = 2948$, $\Sigma(x-98)^4 = 86752$.

Do you think that the distribution is platykurtic?

solution:

$\because d = x - A$ where $A = 98$.

$\mu_1' = \frac{\Sigma d}{N} = \frac{50}{100} = 0.5$

$\mu_2' = \frac{\Sigma d^2}{N} = \frac{1970}{100} = 19.7$

$\mu_3' = \frac{\Sigma d^3}{N} = \frac{2948}{100} = 29.48$

$\mu_4' = \frac{\Sigma d^4}{N} = \frac{86752}{100} = 867.52$

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 19.70 - (0.5)^2 = 19.45$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$

$= 29.48 - 3(19.7)(0.5) + 2(0.5)^3$

$= 29.48 - 29.55 + 0.25 = 0.18$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$= 867.52 - 4(29.48)(0.5) + 6(19.7)(0.5)^2 - 3(0.5)^4$

$= 867.52 - 58.96 + 29.55 - 0.1875$

$= 837.9225$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{837.9225}{(19.45)^2} = \frac{837.9225}{378.3025} = 2.21$

$\because \beta_2 < 3$, the distribution is platykurtic.

12) The first 4 moments of a distribution about $x=4$ are 1, 4, 10 & 45. obtain the various characteristics of the distribution on the basis of information given. comment on the nature of the distribution.

Solution:

Given $A = 4$, $\mu_1' = 1$, $\mu_2' = 4$, $\mu_3' = 10$, $\mu_4' = 45$

$\mu_1 = \mu_1' - \mu_1' = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 4 - 1 = 3$

$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3 = 10 - 3(1)(4) + 2(1)^3$
$= 0$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4$

$= 45 - 4(1)(10) + 6(1)^2(4) - 3(1)^4 = 26$

Mean $= \bar{x} = A + \mu_1' = 4 + 1 = 5$

$\mu_2 = \sigma^2$

$\Rightarrow \sigma = \sqrt{\mu_2} = \sqrt{3} = 1.73$

coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{1.73}{5} \times 100 = 34.6\%$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0 \quad \Rightarrow \gamma_1 = \sqrt{\beta_1} = 0.$

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{(3)^2} = 2.89.$

$\because \beta_2 < 3.$

$\therefore$ The distribution is platykurtic.

$\therefore$ the distribution is symmetrical.

$\because \gamma_1 = 0$, the distribution is symmetrical.

13) For a distribution, the mean is 10, variance is 16, γ_1 is 1 & β_2 is 4. obtain the first 4 moments about the origin, i.e zero. comment upon the nature of distribution.

solution :- Given $\mu_1' = 10 =$ Mean

$\mu_2 = \sigma^2 = 16.$

$\gamma_1 = 1 \quad , \beta_2 = 4.$

$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$

$\mu_3 = \gamma_1 \times \sqrt{\mu_2^3} = 1 \times \sqrt{(16)^3} = 64.$

$\beta_2 = \frac{\mu_4}{\mu_2^2} \Rightarrow 4 = \frac{\mu_4}{16 \times 16} \Rightarrow \mu_4 = 1024$

Moments about origin

ν_1 = Mean = 10.

$\nu_2 = \mu_2 + (\nu_1)^2 = 16 + 100 = 116$

$\nu_3 = \mu_3 + 3\nu_1\nu_2 - 2(\nu_1)^3$

$= 64 + 3(10)(116) - 2(1000)$

$= 1544$

$\nu_4 = \mu_4 + 4\nu_1\nu_3 - 6(\nu_1)^2\nu_2 + 3(\nu_1)^4$

$= 1024 + 4(10)(1544) - 6(100)(116) + 3(10000)$

$= 1024 + 61,760 - 69,600 + 30,000$

$= 23,184$

$\because \gamma_1 = 1$, the distribution is +vely skewed.

$\because \beta_2 = 4$ i.e $\beta_2 > 3$ $\therefore$ the distribution is leptokurtic.

4) For the two distributions the following moments about mean are given:

For the first distribution : $\mu_2 = 25$, $\mu_3 = -10$, $\mu_4 = 1560$

For the second " : $\mu_2 = 36$, $\mu_3 = -28.8$, $\mu_4 = 4000$

i) which of the two distributions is more skewed to the left?

ii) From the point of view of peakedness which distribution is more nearly approximates normal distribution?

Solution:

i) Moment coefficient of skewness

$$\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

For the first distribution

$$\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-10}{\sqrt{(25)^3}} = \frac{-10}{125} = -0.08$$

For the second distribution.

$$\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{-28.8}{\sqrt{(36)^3}} = \frac{-28.8}{216} = -0.133$$

=> The second distribution is more skewed to the left.

ii) For the first distribution.

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1560}{(25)^2} = 2.496.$$

For the second distribution

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{4000}{(36)^2} = 3.086.$$

∴ the second distribution approximates normal distribution.

15) There are nine machines in a job shop. In a particula week, work was done on these machines for the hours specified against them. Calculate Bowley's & Kelly's skewness & coefficient of skewness of the hours worked on these machines.

Machine No.	1	2	3	4	5	6	7	8	9
Hours worked :	3	7	8	11	14	19	23	24	30

Solution:

$Q_1 = \left(\frac{n+1}{4}\right)$th item $= 2.5$th item $= 7 + 0.5(8-7) = 7.5$.

$Q_3 = 3\left(\frac{n+1}{4}\right)$th item $= 7.5$th item $= 23 + 0.5(24-23) = 23.5$

Median $= \left(\frac{n+1}{2}\right)$th item $=$ 5th item $= 14$.

Bowley's coefficient of skewness

$$= \frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$$

Bowley's skewness $= Q_3 + Q_1 - 2\,\text{Median}$.

$= 23.5 + 7.5 - 28 = 3$.

Bowley's coefficient of skewness $= \frac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$

$$= \frac{3}{16} = 0.19.$$

$P_{90} = 90\left(\frac{n+1}{100}\right)^{th}$ item $= 9^{th}$ item $= 30$.

$P_{10} = 10\left(\frac{n+1}{100}\right)^{th}$ item $= 1^{st}$ item $= 3$

Kelley's skewness $= P_{90} + P_{10} - 2\,\text{Median}$

$$= 30 + 3 - 2(14) = 5.$$

Kelley's coefficient of skewness

$$= \frac{P_{90} + P_{10} - 2\,\text{Median}}{P_{90} - P_{10}}$$

$$= \frac{5}{27} = 0.19.$$

16) The storekeeper of a firm furnished the following details of inventory. Calculate Bowley's & Kelly's coefficient of skewness from these inventory data.

Inventory Balance (RS. lakhs) :	5	10	15	20	25	30	35
No. of days :	120	38	30	25	16	12	8.

Solution:

x	f	c.f
5	120	120
10	38	158
15	30	188
20	25	213
25	16	229
30	12	241
35	8	249

$N = \Sigma f = 249 = n$

Median $\left(\frac{n+1}{2}\right)$th value $= \left(\frac{249+1}{2}\right)$th value $= 125$th value $= 10$

$Q_1 = \left(\frac{n+1}{4}\right)$th value $= \left(\frac{249+1}{4}\right)$th value $= 62.5$th value $= 5$

$Q_3 = 3\left(\frac{n+1}{4}\right)$th value $= 3\left(\frac{249+1}{4}\right)$th value $= 187.5$th value $= 15$

$P_{10} = 10\left(\frac{n+1}{100}\right)$th value $= 10\left(\frac{249+1}{100}\right)$th value $= 25$th value $= 5$.

$P_{90} = 90\left(\frac{n+1}{100}\right)$th value $= 90\left(\frac{249+1}{100}\right)$th value $= 225$th value $= 25$

Bowley's coefficient of skewness $= \dfrac{Q_3 + Q_1 - 2\,\text{Median}}{Q_3 - Q_1}$

$$= \frac{15+5-2\times 10}{15-5} = 0.$$

Kelly's coefficient of skewness = $\frac{P_{90} + P_{10} - 2\text{Median}}{P_{90} - P_{10}}$

$$= \frac{25+5-2(10)}{25-5} = 0.5.$$

17) From a moderately skewed distribution of retail prices for men's shaving kitts, it is found that the mean price is Rs.300 & the median price is Rs 255, if the coefficient of variation is 20%. find the pearsonian coefficient of skewness of the distribution.

Solution:-

$$\text{coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$20 = \frac{\sigma}{300} \times 100$$

$$\sigma = 60$$

$$\text{coefficient of skewness} = \frac{3(\bar{x} - \text{Median})}{\sigma}$$

$$= \frac{3(300-255)}{60}$$

$$= 2.25$$