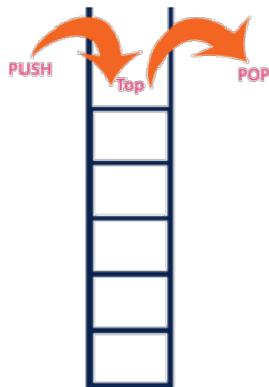


## UNIT-III

### Stack ADT

#### What is a Stack?

Stack is a linear data structure in which the insertion and deletion operations are performed at only one end. In a stack, adding and removing of elements are performed at single position which is known as "top". That means, new element is added at top of the stack and an element is removed from the top of the stack. In stack, the insertion and deletion operations are performed based on LIFO (Last In First Out) principle.



In a stack, the insertion operation is performed using a function called "push" and deletion operation is performed using a function called "pop".

In the figure, PUSH and POP operations are performed at top position in the stack. That means, both the insertion and deletion operations are performed at one end (i.e., at Top)

A stack data structure can be defined as follows...

Stack is a linear data structure in which the operations are performed based on LIFO principle.

Stack can also be defined as

"A Collection of similar data items in which both insertion and deletion operations are performed based on LIFO principle".

#### Example

If we want to create a stack by inserting 10,45,12,16,35 and 50. Then 10 becomes the bottom most element and 50 is the top most element. Top is at 50 as shown in the image below...



## Operations on a Stack

The following operations are performed on the stack...

Push (To insert an element on to the stack)

Pop (To delete an element from the stack)

Display (To display elements of the stack)

Stack data structure can be implement in two ways. They are as follows...

Using Array

Using Linked List

When stack is implemented using array, that stack can organize only limited number of elements.

When stack is implemented using linked list, that stack can organize unlimited number of elements.

A stack data structure can be implemented using one dimensional array. But stack implemented using array, can store only fixed number of data values. This implementation is very simple, just define a one dimensional array of specific size and insert or delete the values into that array by using LIFO principle with the help of a variable 'top'. Initially top is set to -1. Whenever we want to insert a value into the stack, increment the top value by one and then insert. Whenever we want to delete a value from the stack, then delete the top value and decrement the top value by one.

## Stack Operations using Array

A stack can be implemented using array as follows...

Before implementing actual operations, first follow the below steps to create an empty stack.

Step 1: Include all the header files which are used in the program and define a constant 'SIZE' with specific value.

Step 2: Declare all the functions used in stack implementation.

Step 3: Create a one dimensional array with fixed size (int stack[SIZE])

Step 4: Define a integer variable 'top' and initialize with '-1'. (int top = -1)

Step 5: In main method display menu with list of operations and make suitable function calls to perform operation selected by the user on the stack.

push(value) - Inserting value into the stack

In a stack, push() is a function used to insert an element into the stack. In a stack, the new element is always inserted at top position. Push function takes one integer value as parameter and inserts that value into the stack. We can use the following steps to push an element on to the stack...

Step 1: Check whether stack is FULL. (top == SIZE-1)

Step 2: If it is FULL, then display "Stack is FULL!!! Insertion is not possible!!!" and terminate the function.

Step 3: If it is NOT FULL, then increment top value by one (top++) and set stack[top] to value (stack[top] = value).

pop() - Delete a value from the Stack

In a stack, pop() is a function used to delete an element from the stack. In a stack, the element is always deleted from top position. Pop function does not take any value as parameter. We can use the following steps to pop an element from the stack...

Step 1: Check whether stack is EMPTY. (top == -1)

Step 2: If it is EMPTY, then display "Stack is EMPTY!!! Deletion is not possible!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then delete stack[top] and decrement top value by one (top--).

display() - Displays the elements of a Stack

We can use the following steps to display the elements of a stack...

Step 1: Check whether stack is EMPTY. (top == -1)

Step 2: If it is EMPTY, then display "Stack is EMPTY!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then define a variable 'i' and initialize with top. Display stack[i] value and decrement i value by one (i--).

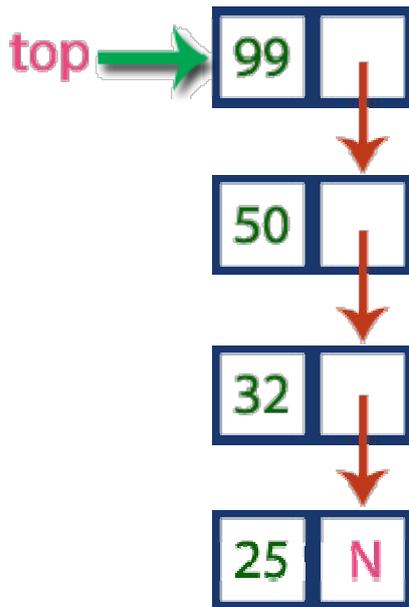
Step 3: Repeat above step until i value becomes '0'.

Stack using Linked List

The major problem with the stack implemented using array is, it works only for fixed number of data values. That means the amount of data must be specified at the beginning of the implementation itself. Stack implemented using array is not suitable, when we don't know the size of data which we are going to use. A stack data structure can be implemented by using linked list data structure. The stack implemented using linked list can work for unlimited number of values. That means, stack implemented using linked list works for variable size of data. So, there is no need to fix the size at the beginning of the implementation. The Stack implemented using linked list can organize as many data values as we want.

In linked list implementation of a stack, every new element is inserted as 'top' element. That means every newly inserted element is pointed by 'top'. Whenever we want to remove an element from the stack, simply remove the node which is pointed by 'top' by moving 'top' to its next node in the list. The next field of the first element must be always NULL.

## Example



In above example, the last inserted node is 99 and the first inserted node is 25. The order of elements inserted is 25, 32, 50 and 99.

## Operations

To implement stack using linked list, we need to set the following things before implementing actual operations.

Step 1: Include all the header files which are used in the program. And declare all the user defined functions.

Step 2: Define a 'Node' structure with two members data and next.

Step 3: Define a Node pointer 'top' and set it to NULL.

Step 4: Implement the main method by displaying Menu with list of operations and make suitable function calls in the main method.

push(value) - Inserting an element into the Stack

We can use the following steps to insert a new node into the stack...

Step 1: Create a newNode with given value.

Step 2: Check whether stack is Empty (top == NULL)

Step 3: If it is Empty, then set newNode → next = NULL.

Step 4: If it is Not Empty, then set newNode → next = top.

Step 5: Finally, set top = newNode.

pop() - Deleting an Element from a Stack

We can use the following steps to delete a node from the stack...

Step 1: Check whether stack is Empty (top == NULL).

Step 2: If it is Empty, then display "Stack is Empty!!! Deletion is not possible!!!" and terminate the function

Step 3: If it is Not Empty, then define a Node pointer 'temp' and set it to 'top'.

Step 4: Then set 'top = top → next'.

Step 7: Finally, delete 'temp' (free(temp)).

display() - Displaying stack of elements

We can use the following steps to display the elements (nodes) of a stack...

Step 1: Check whether stack is Empty (top == NULL).

Step 2: If it is Empty, then display 'Stack is Empty!!!' and terminate the function.

Step 3: If it is Not Empty, then define a Node pointer 'temp' and initialize with top.

Step 4: Display 'temp → data --->' and move it to the next node. Repeat the same until temp reaches to the first node in the stack (temp → next != NULL).

Step 4: Finally! Display 'temp → data ---> NULL'.

Expressions

What is an Expression?

In any programming language, if we want to perform any calculation or to frame a condition etc., we use a set of symbols to perform the task. These set of symbols makes an expression.

An expression can be defined as follows...

An expression is a collection of operators and operands that represents a specific value.

In above definition, operator is a symbol which performs a particular task like arithmetic operation or logical operation or conditional operation etc.,

Operands are the values on which the operators can perform the task. Here operand can be a direct value or variable or address of memory location.

Expression Types

Based on the operator position, expressions are divided into THREE types. They are as follows...

Infix Expression

Postfix Expression

Prefix Expression

Infix Expression

In infix expression, operator is used in between operands.

The general structure of an Infix expression is as follows...

Operand1 Operator Operand2

Example



Postfix Expression

In postfix expression, operator is used after operands. We can say that "Operator follows the Operands".

The general structure of Postfix expression is as follows...

Operand1 Operand2 Operator

Example



Prefix Expression

In prefix expression, operator is used before operands. We can say that "Operands follows the Operator".

The general structure of Prefix expression is as follows...

Operator Operand1 Operand2

Example



Any expression can be represented using the above three different types of expressions. And we can convert an expression from one form to another form like Infix to Postfix, Infix to Prefix, Prefix to Postfix and vice versa.

### Expression Conversion

Any expression can be represented using three types of expressions (Infix, Postfix and Prefix). We can also convert one type of expression to another type of expression like Infix to Postfix, Infix to Prefix, Postfix to Prefix and vice versa.

To convert any Infix expression into Postfix or Prefix expression we can use the following procedure...

Find all the operators in the given Infix Expression.

Find the order of operators evaluated according to their Operator precedence.

Convert each operator into required type of expression (Postfix or Prefix) in the same order.

Example

Consider the following Infix Expression to be converted into Postfix Expression...

$$D = A + B * C$$

Step 1: The Operators in the given Infix Expression : = , + , \*

Step 2: The Order of Operators according to their preference : \* , + , =

Step 3: Now, convert the first operator \* -----  $D = A + B C *$

Step 4: Convert the next operator + -----  $D = A B C * +$

Step 5: Convert the next operator = -----  $D A B C * + =$

Finally, given Infix Expression is converted into Postfix Expression as follows...

$$D A B C * + =$$

Infix to Postfix Conversion using Stack Data Structure

To convert Infix Expression into Postfix Expression using a stack data structure, We can use the following steps...

Read all the symbols one by one from left to right in the given Infix Expression.

If the reading symbol is operand, then directly print it to the result (Output).

If the reading symbol is left parenthesis '(', then Push it on to the Stack.

If the reading symbol is right parenthesis ')', then Pop all the contents of stack until respective left parenthesis is popped and print each popped symbol to the result.

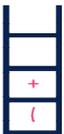
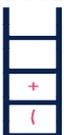
If the reading symbol is operator (+ , - , \* , / etc.), then Push it on to the Stack. However, first pop the operators which are already on the stack that have higher or equal precedence than current operator and print them to the result.

Example

Consider the following Infix Expression...

$$( A + B ) * ( C - D )$$

The given infix expression can be converted into postfix expression using Stack data Structure as follows...

Reading Character	STACK	Postfix Expression
Initially	Stack is EMPTY 	EMPTY
(	Push '(' 	EMPTY
A	No operation Since 'A' is OPERAND 	A
+	'+' has low priority than '(' so, PUSH '+' 	A +
B	No operation Since 'B' is OPERAND 	A B +
)	POP all elements till we reach '(' POP '+' POP '(' 	A B +
*	Stack is EMPTY & '*' is Operator PUSH '*' 	A B + *
(	PUSH '(' 	A B + *
C	No operation Since 'C' is OPERAND 	A B + C *
-	'-' has low priority than '(' so, PUSH '-' 	A B + C * -
D	No operation Since 'D' is OPERAND 	A B + C D * -
)	POP all elements till we reach '(' POP '-' POP '(' 	A B + C D * -
\$	POP all elements till Stack becomes Empty 	<b>A B + C D * -</b>

The final Postfix Expression is as follows...

A B + C D - \*

## Postfix Expression Evaluation

A postfix expression is a collection of operators and operands in which the operator is placed after the operands. That means, in a postfix expression the operator follows the operands.

Postfix Expression has following general structure...

Operand1 Operand2 Operator

Example



## Postfix Expression Evaluation using Stack Data Structure

A postfix expression can be evaluated using the Stack data structure. To evaluate a postfix expression using Stack data structure we can use the following steps...

Read all the symbols one by one from left to right in the given Postfix Expression

If the reading symbol is operand, then push it on to the Stack.

If the reading symbol is operator (+, -, \*, / etc.), then perform TWO pop operations and store the two popped operands in two different variables (operand1 and operand2). Then perform reading symbol operation using operand1 and operand2 and push result back on to the Stack.

Finally! perform a pop operation and display the popped value as final result.

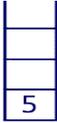
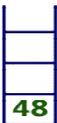
Example

Consider the following Expression...

Infix Expression  $(5 + 3) * (8 - 2)$

Postfix Expression  $5 3 + 8 2 - *$

Above Postfix Expression can be evaluated by using Stack Data Structure as follows...

Reading Symbol	Stack Operations	Stack	Evaluated Part of Expression
Initially	Stack is Empty		Nothing
5	push(5)		Nothing
3	push(3)		Nothing
+	value1 = pop() value2 = pop() result = value2 + value1 push(result)		<pre>value1 = pop(); // 3 value2 = pop(); // 5 result = 5 + 3; // 8 Push( 8 )</pre> <p><b>(5 + 3)</b></p>
8	push(8)		(5 + 3)
2	push(2)		(5 + 3)
-	value1 = pop() value2 = pop() result = value2 - value1 push(result)		<pre>value1 = pop(); // 2 value2 = pop(); // 8 result = 8 - 2; // 6 Push( 6 )</pre> <p><b>(8 - 2)</b></p> <p>(5 + 3), (8 - 2)</p>
*	value1 = pop() value2 = pop() result = value2 * value1 push(result)		<pre>value1 = pop(); // 6 value2 = pop(); // 8 result = 8 * 6; // 48 Push( 48 )</pre> <p><b>(6 * 8)</b></p> <p>(5 + 3) * (8 - 2)</p>
\$ End of Expression	result = pop()		<p>Display (result)</p> <p><b>48</b></p> <p>As final result</p>

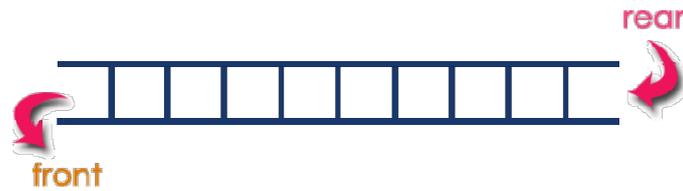
Infix Expression  $(5 + 3) * (8 - 2) = 48$

Postfix Expression  $5 3 + 8 2 - *$  value is **48**

## Queue ADT

Queue is a linear data structure in which the insertion and deletion operations are performed at two different ends. In a queue data structure, adding and removing of elements are performed at two different positions. The insertion is performed at one end and deletion is performed at other end. In a queue data structure, the insertion operation is performed at a position which is known as 'rear'

and the deletion operation is performed at a position which is known as 'front'. In queue data structure, the insertion and deletion operations are performed based on FIFO (First In First Out) principle.



In a queue data structure, the insertion operation is performed using a function called "enQueue()" and deletion operation is performed using a function called "deQueue()".

Queue data structure can be defined as follows...

Queue data structure is a linear data structure in which the operations are performed based on FIFO principle.

A queue can also be defined as

"Queue data structure is a collection of similar data items in which insertion and deletion operations are performed based on FIFO principle".

Example

Queue after inserting 25, 30, 51, 60 and 85.

After Inserting five elements...



Operations on a Queue

The following operations are performed on a queue data structure...

enQueue(value) - (To insert an element into the queue)

deQueue() - (To delete an element from the queue)

display() - (To display the elements of the queue)

Queue data structure can be implemented in two ways. They are as follows...

Using Array

Using Linked List

When a queue is implemented using array, that queue can organize only limited number of elements. When a queue is implemented using linked list, that queue can organize unlimited number of elements.

## Queue Using Array

A queue data structure can be implemented using one dimensional array. But, queue implemented using array can store only fixed number of data values. The implementation of queue data structure using array is very simple, just define a one dimensional array of specific size and insert or delete the values into that array by using FIFO (First In First Out) principle with the help of variables 'front' and 'rear'. Initially both 'front' and 'rear' are set to -1. Whenever, we want to insert a new value into the queue, increment 'rear' value by one and then insert at that position. Whenever we want to delete a value from the queue, then increment 'front' value by one and then display the value at 'front' position as deleted element.

### Queue Operations using Array

Queue data structure using array can be implemented as follows...

Before we implement actual operations, first follow the below steps to create an empty queue.

Step 1: Include all the header files which are used in the program and define a constant 'SIZE' with specific value.

Step 2: Declare all the user defined functions which are used in queue implementation.

Step 3: Create a one dimensional array with above defined SIZE (int queue[SIZE])

Step 4: Define two integer variables 'front' and 'rear' and initialize both with '-1'. (int front = -1, rear = -1)

Step 5: Then implement main method by displaying menu of operations list and make suitable function calls to perform operation selected by the user on queue.

enQueue(value) - Inserting value into the queue

In a queue data structure, enQueue() is a function used to insert a new element into the queue. In a queue, the new element is always inserted at rear position. The enQueue() function takes one integer value as parameter and inserts that value into the queue. We can use the following steps to insert an element into the queue...

Step 1: Check whether queue is FULL. (rear == SIZE-1)

Step 2: If it is FULL, then display "Queue is FULL!!! Insertion is not possible!!!" and terminate the function.

Step 3: If it is NOT FULL, then increment rear value by one (rear++) and set queue[rear] = value.

deQueue() - Deleting a value from the Queue

In a queue data structure, deQueue() is a function used to delete an element from the queue. In a queue, the element is always deleted from front position. The deQueue() function does not take any value as parameter. We can use the following steps to delete an element from the queue...

Step 1: Check whether queue is EMPTY. (front == rear)

Step 2: If it is EMPTY, then display "Queue is EMPTY!!! Deletion is not possible!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then increment the front value by one (front ++). Then display queue[front] as deleted element. Then check whether both front and rear are equal (front == rear), if it TRUE, then set both front and rear to '-1' (front = rear = -1).

display() - Displays the elements of a Queue

We can use the following steps to display the elements of a queue...

Step 1: Check whether queue is EMPTY. (front == rear)

Step 2: If it is EMPTY, then display "Queue is EMPTY!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then define an integer variable 'i' and set 'i = front+1'.

Step 3: Display 'queue[i]' value and increment 'i' value by one (i++). Repeat the same until 'i' value is equal to rear (i <= rear)

### Queue using Linked List

The major problem with the queue implemented using array is, It will work for only fixed number of data. That means, the amount of data must be specified in the beginning itself. Queue using array is not suitable when we don't know the size of data which we are going to use. A queue data structure can be implemented using linked list data structure. The queue which is implemented using linked list can work for unlimited number of values. That means, queue using linked list can work for variable size of data (No need to fix the size at beginning of the implementation). The Queue implemented using linked list can organize as many data values as we want.

In linked list implementation of a queue, the last inserted node is always pointed by 'rear' and the first node is always pointed by 'front'.

Example



In above example, the last inserted node is 50 and it is pointed by 'rear' and the first inserted node is 10 and it is pointed by 'front'. The order of elements inserted is 10, 15, 22 and 50.

Operations

To implement queue using linked list, we need to set the following things before implementing actual operations.

Step 1: Include all the header files which are used in the program. And declare all the user defined functions.

Step 2: Define a 'Node' structure with two members data and next.

Step 3: Define two Node pointers 'front' and 'rear' and set both to NULL.

Step 4: Implement the main method by displaying Menu of list of operations and make suitable function calls in the main method to perform user selected operation.

enqueue(value) - Inserting an element into the Queue

We can use the following steps to insert a new node into the queue...

Step 1: Create a newNode with given value and set 'newNode → next' to NULL.

Step 2: Check whether queue is Empty (rear == NULL)

Step 3: If it is Empty then, set front = newNode and rear = newNode.

Step 4: If it is Not Empty then, set rear → next = newNode and rear = newNode.

dequeue() - Deleting an Element from Queue

We can use the following steps to delete a node from the queue...

Step 1: Check whether queue is Empty (front == NULL).

Step 2: If it is Empty, then display "Queue is Empty!!! Deletion is not possible!!!" and terminate from the function

Step 3: If it is Not Empty then, define a Node pointer 'temp' and set it to 'front'.

Step 4: Then set 'front = front → next' and delete 'temp' (free(temp)).

display() - Displaying the elements of Queue

We can use the following steps to display the elements (nodes) of a queue...

Step 1: Check whether queue is Empty (front == NULL).

Step 2: If it is Empty then, display 'Queue is Empty!!!' and terminate the function.

Step 3: If it is Not Empty then, define a Node pointer 'temp' and initialize with front.

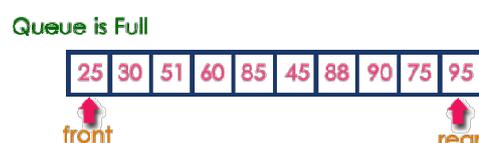
Step 4: Display 'temp → data --->' and move it to the next node. Repeat the same until 'temp' reaches to 'rear' (temp → next != NULL).

Step 4: Finally! Display 'temp → data ---> NULL'.

## Circular Queue

In a normal Queue Data Structure, we can insert elements until queue becomes full. But once if queue becomes full, we can not insert the next element until all the elements are deleted from the queue. For example consider the queue below...

After inserting all the elements into the queue.



Now consider the following situation after deleting three elements from the queue...

Queue is Full (Even three elements are deleted)



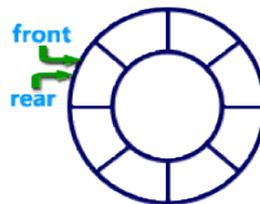
This situation also says that Queue is Full and we can not insert the new element because, 'rear' is still at last position. In above situation, even though we have empty positions in the queue we can not make use of them to insert new element. This is the major problem in normal queue data structure. To overcome this problem we use circular queue data structure.

What is Circular Queue?

A Circular Queue can be defined as follows...

Circular Queue is a linear data structure in which the operations are performed based on FIFO (First In First Out) principle and the last position is connected back to the first position to make a circle.

Graphical representation of a circular queue is as follows...



Implementation of Circular Queue

To implement a circular queue data structure using array, we first perform the following steps before we implement actual operations.

Step 1: Include all the header files which are used in the program and define a constant 'SIZE' with specific value.

Step 2: Declare all user defined functions used in circular queue implementation.

Step 3: Create a one dimensional array with above defined SIZE (int cQueue[SIZE])

Step 4: Define two integer variables 'front' and 'rear' and initialize both with '-1'. (int front = -1, rear = -1)

Step 5: Implement main method by displaying menu of operations list and make suitable function calls to perform operation selected by the user on circular queue.

enQueue(value) - Inserting value into the Circular Queue

In a circular queue, enQueue() is a function which is used to insert an element into the circular queue. In a circular queue, the new element is always inserted at rear position. The enQueue() function takes one integer value as parameter and inserts that value into the circular queue. We can use the following steps to insert an element into the circular queue...

Step 1: Check whether queue is FULL. ((rear == SIZE-1 && front == 0) || (front == rear+1))

Step 2: If it is FULL, then display "Queue is FULL!!! Insertion is not possible!!!" and terminate the function.

Step 3: If it is NOT FULL, then check  $rear == SIZE - 1 \ \&\& \ front != 0$  if it is TRUE, then set  $rear = -1$ .

Step 4: Increment rear value by one ( $rear++$ ), set  $queue[rear] = value$  and check ' $front == -1$ ' if it is TRUE, then set  $front = 0$ .

**deQueue() - Deleting a value from the Circular Queue**

In a circular queue, **deQueue()** is a function used to delete an element from the circular queue. In a circular queue, the element is always deleted from front position. The **deQueue()** function doesn't take any value as parameter. We can use the following steps to delete an element from the circular queue...

Step 1: Check whether queue is EMPTY. ( $front == -1 \ \&\& \ rear == -1$ )

Step 2: If it is EMPTY, then display "Queue is EMPTY!!! Deletion is not possible!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then display  $queue[front]$  as deleted element and increment the front value by one ( $front++$ ). Then check whether  $front == SIZE$ , if it is TRUE, then set  $front = 0$ . Then check whether both front - 1 and rear are equal ( $front - 1 == rear$ ), if it TRUE, then set both front and rear to '-1' ( $front = rear = -1$ ).

**display() - Displays the elements of a Circular Queue**

We can use the following steps to display the elements of a circular queue...

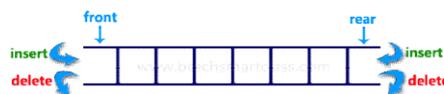
Step 1: Check whether queue is EMPTY. ( $front == -1$ )

Step 2: If it is EMPTY, then display "Queue is EMPTY!!!" and terminate the function.

Step 3: If it is NOT EMPTY, then define an integer variable 'i' and set ' $i = front$ '.

### Double Ended Queue (Deque)

Double Ended Queue is also a Queue data structure in which the insertion and deletion operations are performed at both the ends (front and rear). That means, we can insert at both front and rear positions and can delete from both front and rear positions.



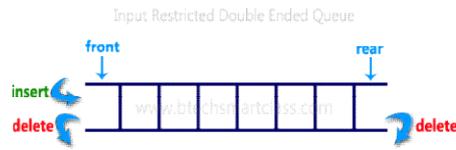
Double Ended Queue can be represented in TWO ways, those are as follows...

Input Restricted Double Ended Queue

Output Restricted Double Ended Queue

Input Restricted Double Ended Queue

In input restricted double ended queue, the insertion operation is performed at only one end and deletion operation is performed at both the ends.



### Output Restricted Double Ended Queue

In output restricted double ended queue, the deletion operation is performed at only one end and insertion operation is performed at both the ends.



Step 4: Check whether 'front <= rear', if it is TRUE, then display 'queue[i]' value and increment 'i' value by one (i++). Repeat the same until 'i <= rear' becomes FALSE.

Step 5: If 'front <= rear' is FALSE, then display 'queue[i]' value and increment 'i' value by one (i++). Repeat the same until 'i <= SIZE - 1' becomes FALSE.

Step 6: Set i to 0.

Step 7: Again display 'cQueue[i]' value and increment i value by one (i++). Repeat the same until 'i <= rear' becomes FALSE.