Insertion Sort

- Insertion sort is a simple and efficient comparison sort.
- In this algorithm, each iteration removes an element from the input data and inserts it into the correct position in the list being sorted.
- The choice of the element being removed from the input is random and this process is repeated until all input elements have gone through.

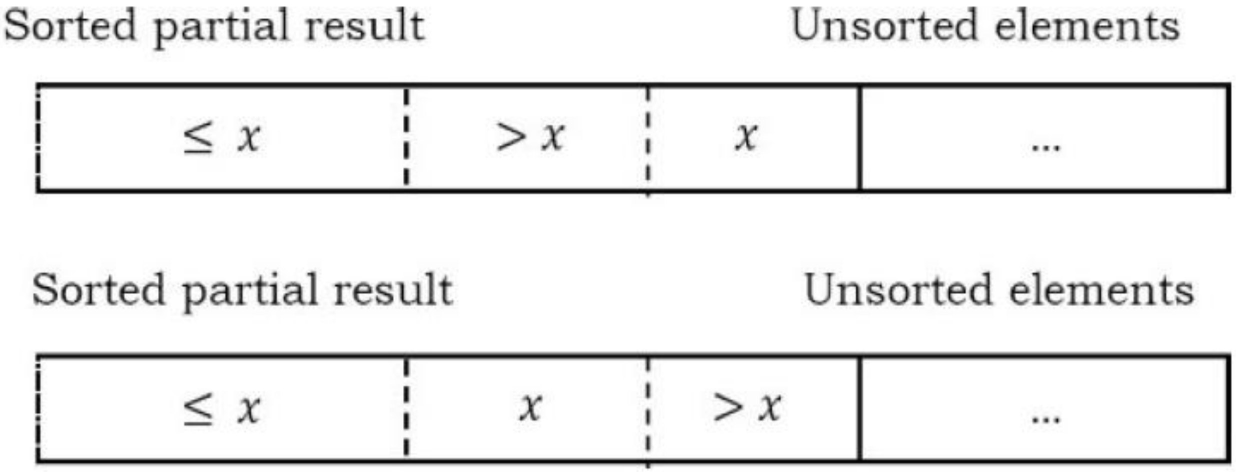
Advantages

- Simple implementation
- Efficient for small data
- **Adaptive**: If the input list is presorted [may not be completely] then insertions sort takes $O(n + d)$, where d is the number of inversions
- Practically more efficient than selection and bubble sorts, even though all of them have $O(n^2)$ worst case complexity
- **Stable**: Maintains relative order of input data if the keys(temp variable) are same
- **In-place**: It requires only a constant amount O(1) of additional memory space
- **Online**: Insertion sort can sort the list as it receives it

Algorithm

- Step 1 – If it is the first element, it is already sorted. return 1;
- Step 2 – Pick next element
- Step 3 – Compare with all elements in the sorted sub-list
- Step 4 – Shift all the elements in the sorted sub-list that is greater than the
- value to be sorted
- Step 5 – Insert the value
- Step 6 – Repeat until list is sorted

- **Algorithm**
- Every repetition of insertion sort removes an element from the input data, and inserts it into the correct position in the **already-sorted list until no input elements remain**.
- Sorting is typically **done in-place**.
- The resulting array after k iterations has the property where the first $k + 1$ entries are sorted.
- Each element greater than x is copied to the right as it is compared against x.



Sorted partial result

Unsorted elements

$\leq x$ | $> x$ | x | ...

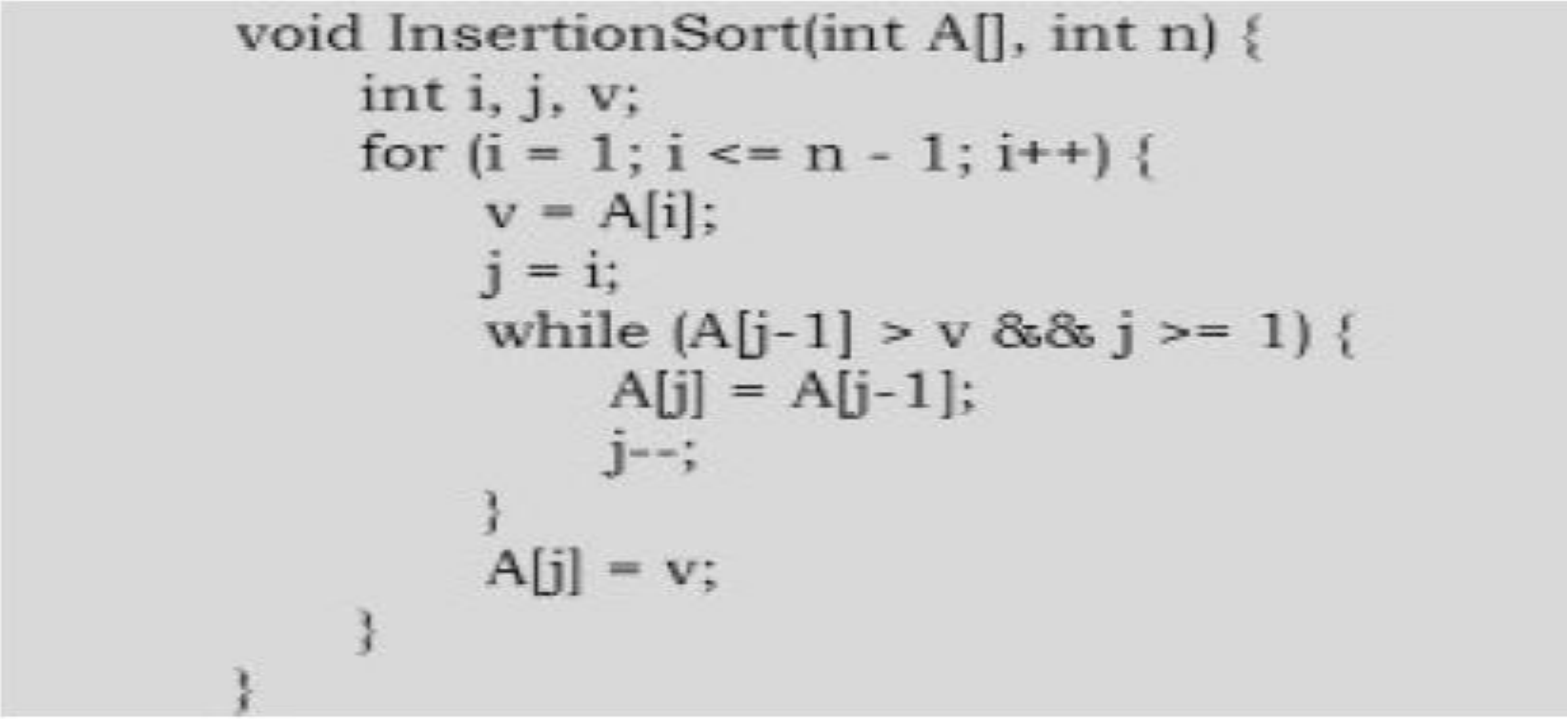
becomes

Sorted partial result

Unsorted elements

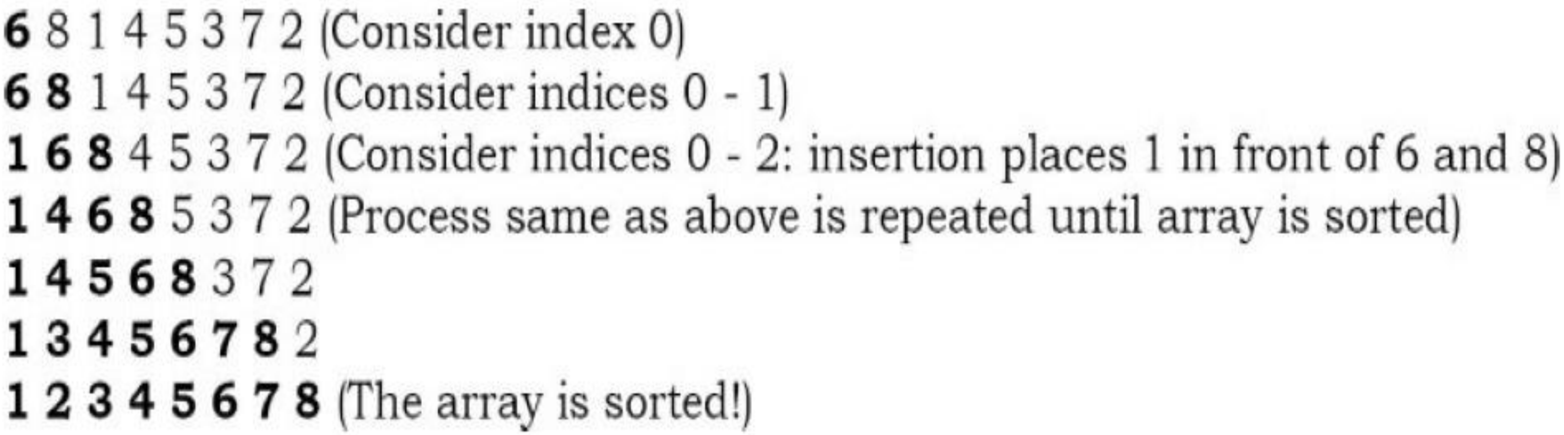
$\leq x$ | x | $> x$ | ...

Implementation



```
void InsertionSort(int A[], int n) {
    int i, j, v;
    for (i = 1; i <= n - 1; i++) {
        v = A[i];
        j = i;
        while (A[j-1] > v && j >= 1) {
            A[j] = A[j-1];
            j--;
        }
        A[j] = v;
    }
}
```

- **Example**
- Given an array: 6 8 1 4 5 3 7 2 and the goal is to put them in ascending order.



6 8 1 4 5 3 7 2 (Consider index 0)

6 8 1 4 5 3 7 2 (Consider indices 0 - 1)

1 6 8 4 5 3 7 2 (Consider indices 0 - 2: insertion places 1 in front of 6 and 8)

1 4 6 8 5 3 7 2 (Process same as above is repeated until array is sorted)

1 4 5 6 8 3 7 2

1 3 4 5 6 7 8 2

1 2 3 4 5 6 7 8 (The array is sorted!)

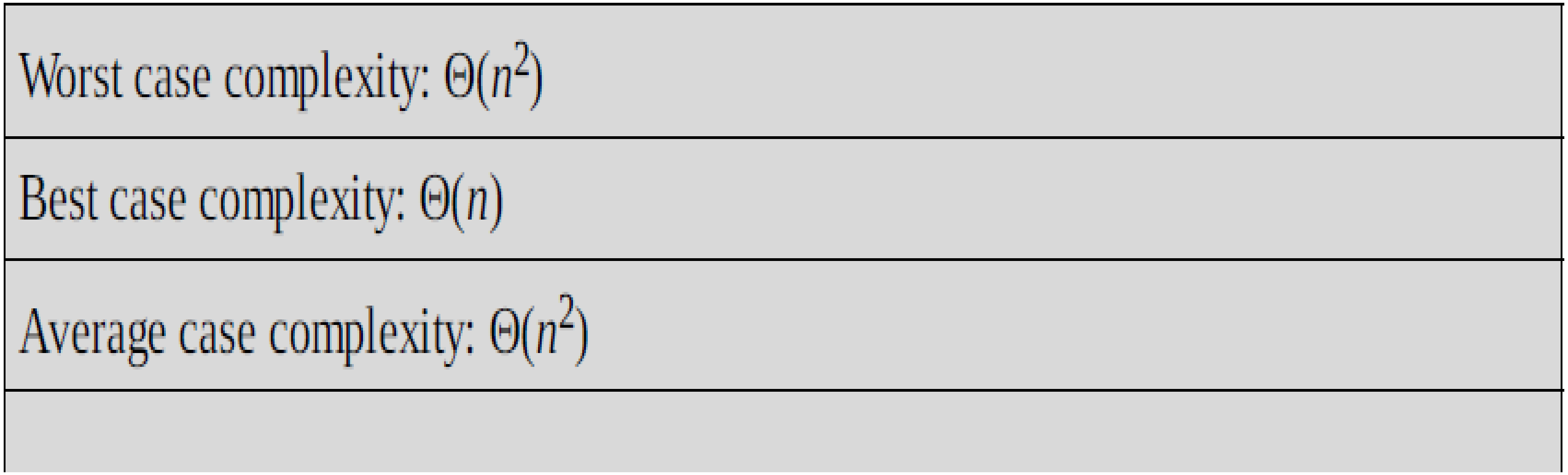
- **Analysis**
- **Worst case analysis**
- Worst case occurs when for every i the inner loop has to move all elements $A[1], \ldots, A[i-1]$ (which happens when $A[i]$ = key is smaller than all of them), that takes $\Theta(i-1)$ time.

$$
\begin{aligned}
T(n) &= \Theta(1) + \Theta(2) + \Theta(2) + \ \ldots\ldots + \Theta(n-1) \\
&= \Theta(1 + 2 + 3 + \ \ldots\ldots + n - 1) = \Theta(\frac{n(n-1)}{2}) \approx \Theta(n^2)
\end{aligned}
$$

- **Average case analysis**
- For the average case, the inner loop will insert $A[i]$ in the middle of $A[1], \ldots, A[i-1]$. This takes $\Theta(i/2)$ time.

$$T(n) = \sum_{i=1}^{n} \Theta(i/2) \approx \Theta(n^2)$$

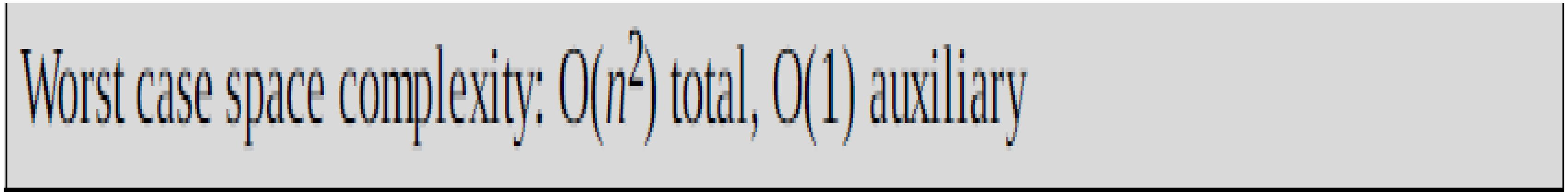
- **Performance**
- **If every element is greater than or equal to every element to its left**, the running time of insertion sort is **Θ(*n*)**.
- This situation occurs if the array starts out already sorted, and so an **already-sorted array** is the **best case** for insertion sort.



Worst case complexity: $\Theta(n^2)$

Best case complexity: $\Theta(n)$

Average case complexity: $\Theta(n^2)$



Worst case space complexity: $O(n^2)$ total, $O(1)$ auxiliary

- **Comparisons to Other Sorting Algorithms**
- **Insertion sort** is one of the elementary sorting algorithms with O(n^2) worst-case time.
- Insertion sort is used **when the data is nearly sorted** (due to its adaptiveness) or when the **input size is small** (due to its low overhead).
- For these reasons and **due to its stability**, **insertion sort is used** as the **recursive base case** (when the **problem size is small**) for **higher overhead divide-and-conquer sorting algorithms, such as merge sort or quick sort.**

Linear Search

- Let us assume we are given an array where the order of the elements is not known.
- Means the elements of the array are not sorted.
- Here we have to scan the complete array and see if the element is there in the given list or not

Algorithm

```
Int unORderedLinearSearch(int A[], int
data)
    For(int i=0; i<n;i++){
       If(A[i]==data)
          return i;
     }
       return -1;
}
```

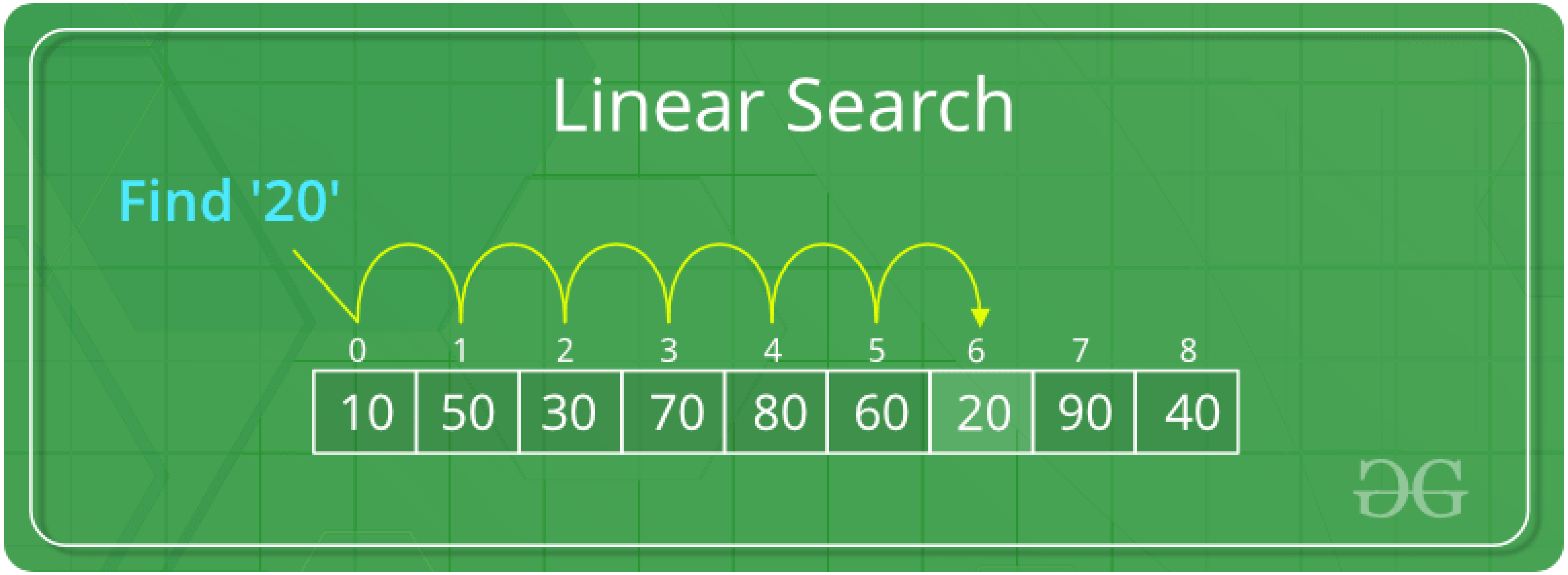
Complexity

- Time Complexity: O(n)
- In the worst case we need to scan the complete array.
- Space Complexity: O(1)

Algorithm

```
Int orderedLinearSearch(int A[], int n, int data){
        for(int i=0; i<n ; i++){
                if(A[i]==data)
                        return i;
                else if(A[i] > data)
                        return -1;
        }
        return -1;
}
```

Example



Complexity

- Time Complexity:O(n), in worst we scan the complete array.
- Space Complexity: O(1).

Merge Sort

- Merge sort is an example of the **divide and conquer strategy**.
- *Merging* is the **process of combining two sorted files** to make **one bigger sorted file.**

- *Selection* is the process of dividing a file into two parts: *k* smallest elements and *n* – *k* largest elements.
- Selection and merging are opposite operations
  - **selection splits a list into two lists**
  - **merging joins two files to make one file**

- Merge sort is Quick sort’s complement
- Merge sort accesses the data in a **sequential manner**
- **This algorithm is used for sorting a linked list**

- Merge sort is insensitive to the initial order of its input
- In Quick sort most of the work is done before the recursive calls.
- Quick sort starts with the largest sub file and finishes with the small ones and as a result it needs stack.
- This algorithm is not stable.

- Merge sort divides the list into two parts; then each part is conquered individually.
- Merge sort starts with the small subfiles and finishes with the largest one.
- As a result it doesn’t need stack.
- This algorithm is stable.

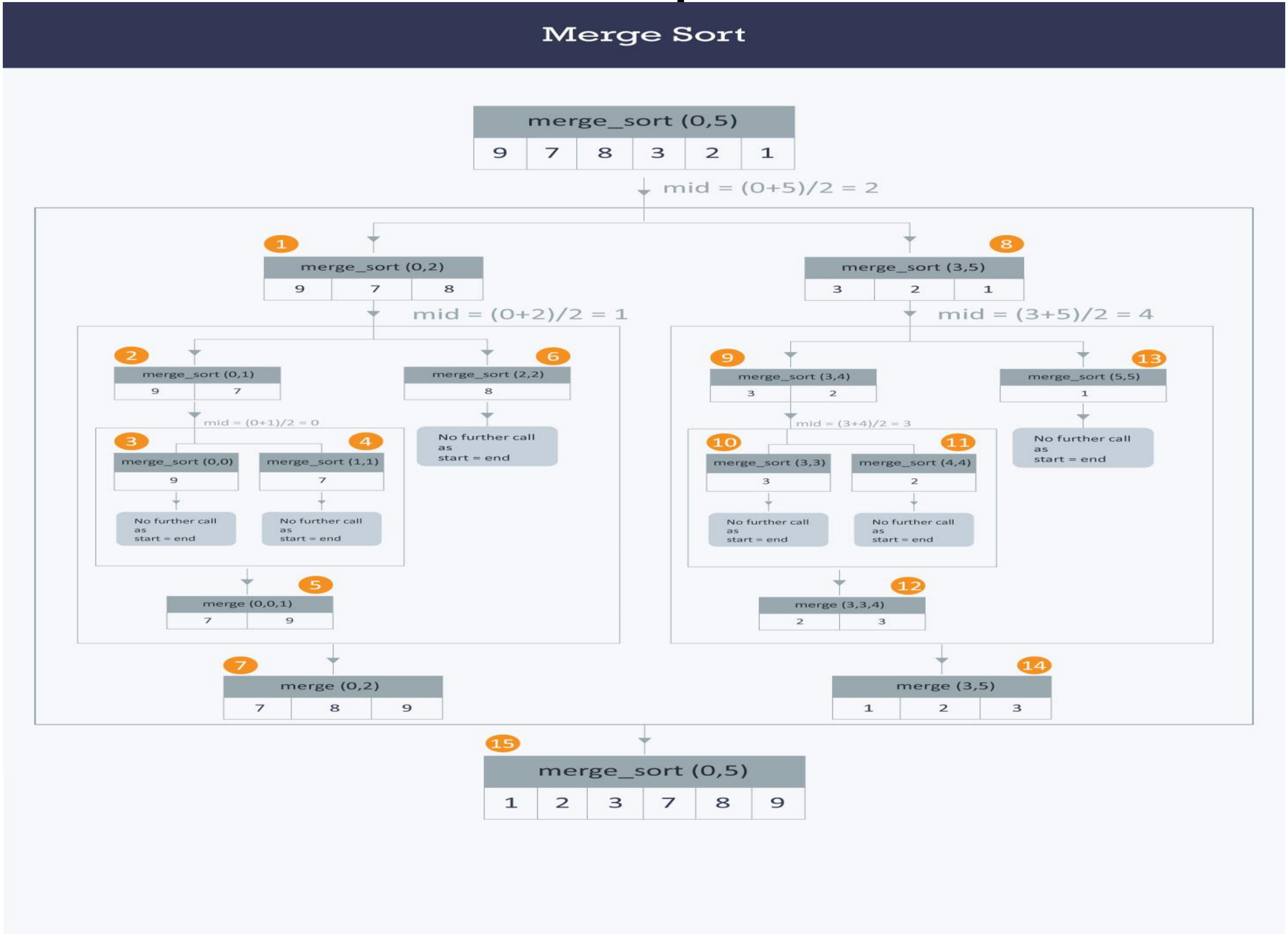
Algorithm

1. Divide the unsorted list into sub lists, each containing element.
2. Take adjacent pairs of two singleton lists and **merge** them to form a list of 2 elements. N. will now convert into lists of size 2.
3. Repeat the process till a single **sorted** list of obtained.

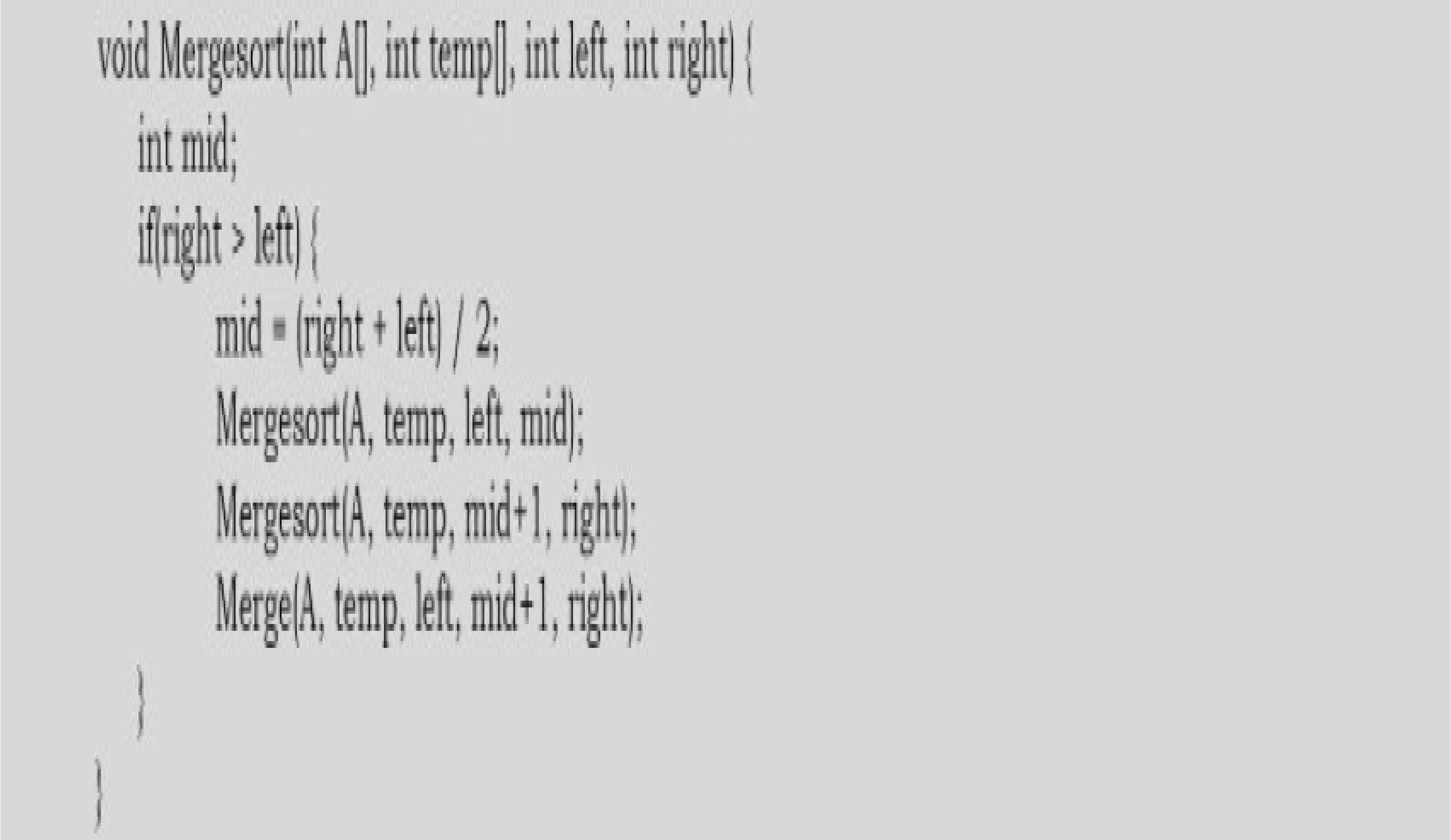
Algorithm

- The merge function works as follows:
- Create copies of the subarrays L ← A[p..q] and M ← A[q+1..r].
- Create three pointers i, j and k
    - i maintains current index of L, starting at 1
    - j maintains current index of M, starting at 1
    - k maintains the current index of A[p..q], starting at p.
- Until we reach the end of either L or M, pick the larger among the elements from L and M and place them in the correct position at A[p..q]
- When we run out of elements in either L or M, pick up the remaining elements and put in A[p..q]

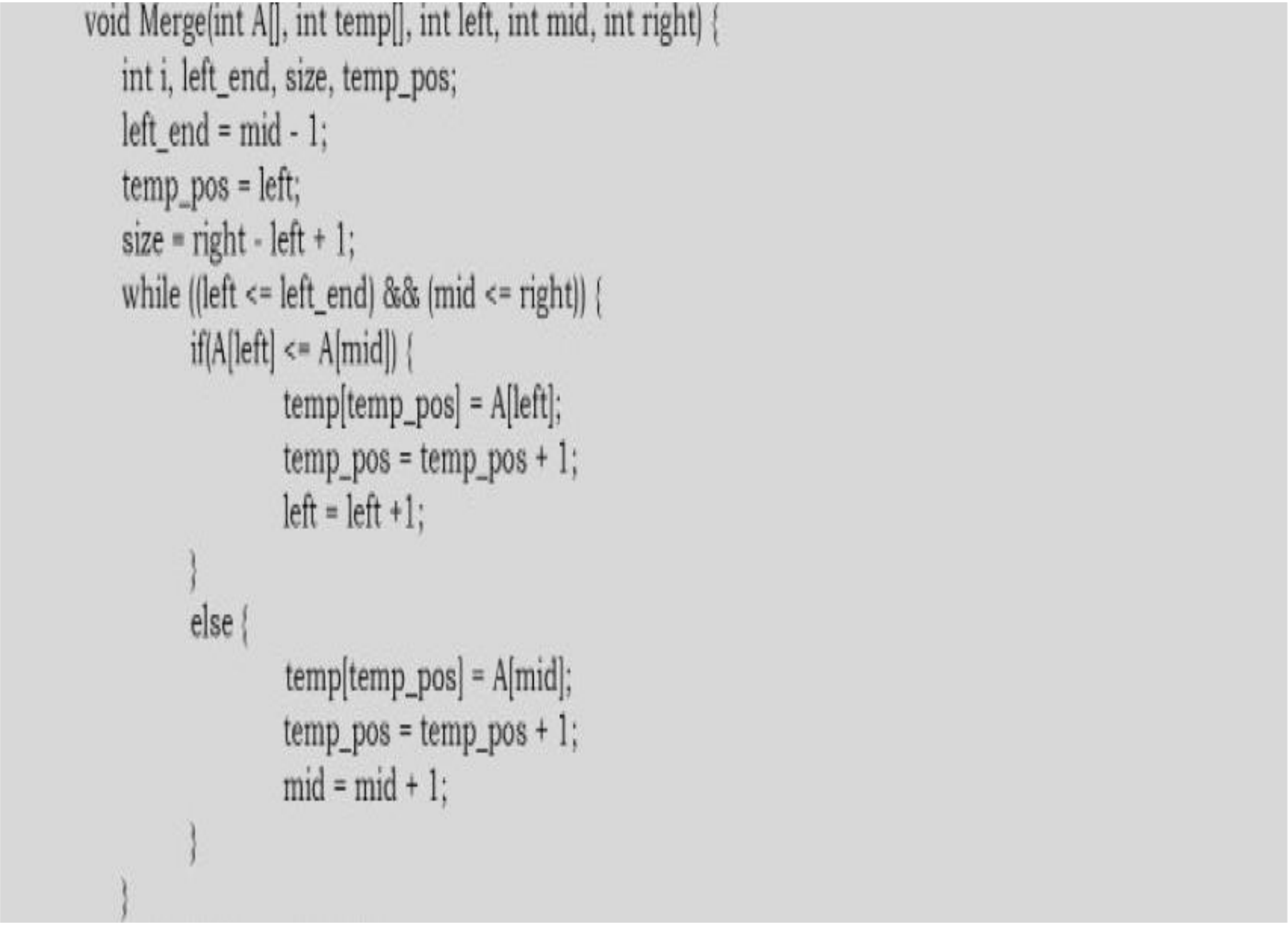
Example


Merge Sort
merge_sort (0,5)
9 7 8 3 2 1
mid = (0+5)/2 = 2
1
merge_sort (0,2)
9 7 8
mid = (0+2)/2 = 1
2
merge_sort (0,1)
9 7
mid = (0+1)/2 = 0
3
merge_sort (0,0)
9
No further call as start = end
4
merge_sort (1,1)
7
No further call as start = end
5
merge (0,0,1)
7 9
6
merge_sort (2,2)
8
No further call as start = end
7
merge (0,2)
7 8 9
8
merge_sort (3,5)
3 2 1
mid = (3+5)/2 = 4
9
merge_sort (3,4)
3 2
mid = (3+4)/2 = 3
10
merge_sort (3,3)
3
No further call as start = end
11
merge_sort (4,4)
2
No further call as start = end
12
merge (3,3,4)
2 3
13
merge_sort (5,5)
1
No further call as start = end
14
merge (3,5)
1 2 3
15
merge_sort (0,5)
1 2 3 7 8 9

Implementation



```
void Mergesort(int A[], int temp[], int left, int right) {
    int mid;
    if(right > left) {
            mid = (right + left) / 2;
            Mergesort(A, temp, left, mid);
            Mergesort(A, temp, mid+1, right);
            Merge(A, temp, left, mid+1, right);
    }
}
```



```
void Merge(int A[], int temp[], int left, int mid, int right) {
    int i, left_end, size, temp_pos;
    left_end = mid - 1;
    temp_pos = left;
    size = right - left + 1;
    while ((left <= left_end) && (mid <= right)) {
            if(A[left] <= A[mid]) {
                        temp[temp_pos] = A[left];
                        temp_pos = temp_pos + 1;
                        left = left +1;
            }
            else {
                        temp[temp_pos] = A[mid];
                        temp_pos = temp_pos + 1;
                        mid = mid + 1;
            }
    }
```

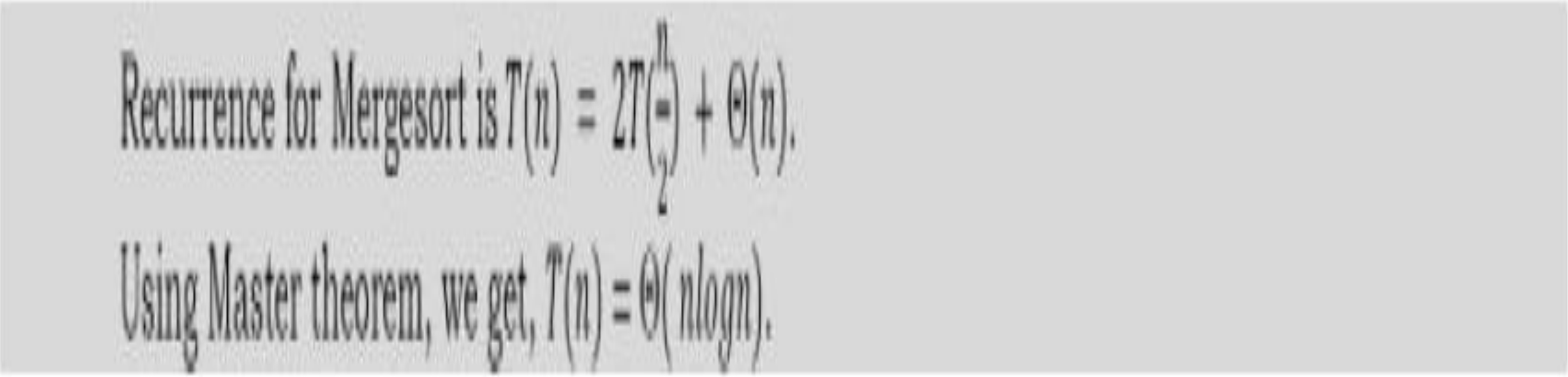
```
    while (left <= left_end) {
            temp[temp_pos] = A[left];
            left = left + 1;
            temp_pos = temp_pos + 1;
    }
    while (mid <= right) {
            temp[temp_pos] = A[mid];
            mid = mid + 1;
            temp_pos = temp_pos + 1;
    }
    for (i = 0; i <= size; i++) {
            A[right] = temp[right];
            right = right - 1;
    }
}
```

Time Complexity

- **Merge Sort** is a stable **sort** which means that the same element in an array maintain their original positions with respect to each other.
- Overall time complexity of **Merge sort** is O(nLogn).
- It is more **efficient** as it is in worst case also the runtime is O(nlogn) The space complexity of **Merge sort** is O(n).

Analysis

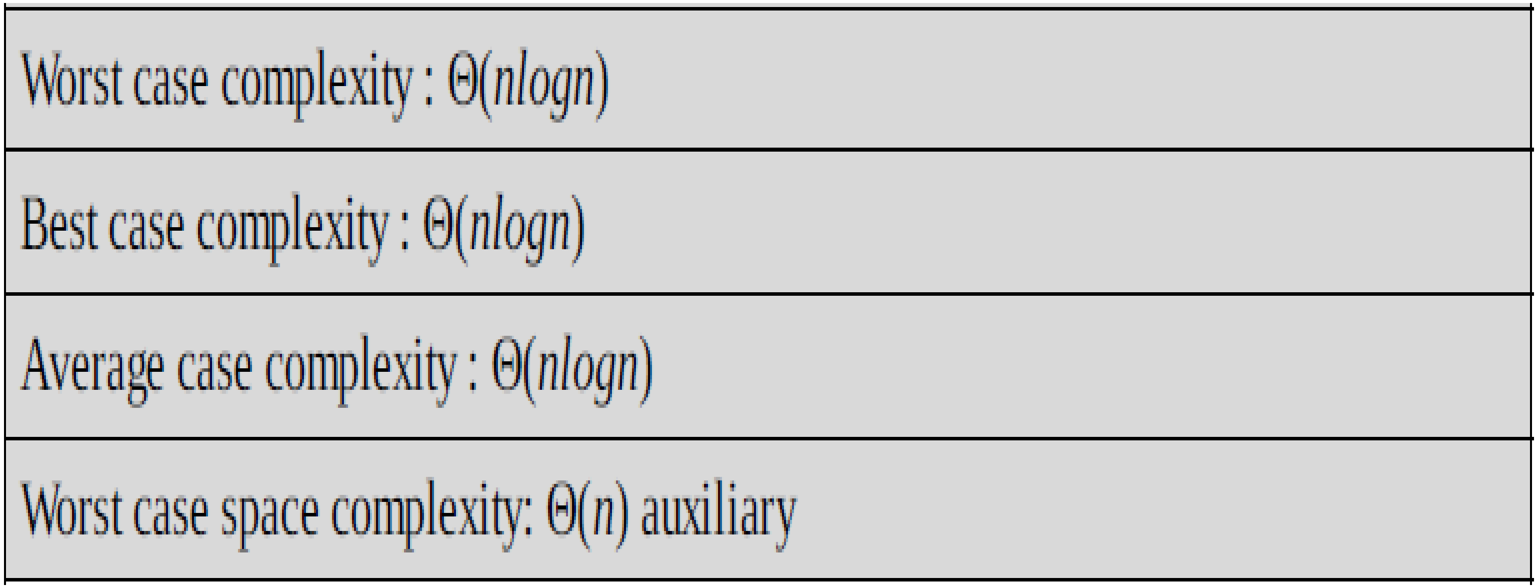
- In Merge sort the input list is divided into two parts and these are solved recursively.
- After solving the sub problems, they are merged by scanning the resultant sub problems.
- Let us assume $T(n)$ is the complexity of Merge sort with n elements.
- The recurrence for the Merge Sort can be defined as:



Recurrence for Mergesort is $T(n) = 2T(\frac{n}{2}) + \Theta(n)$.

Using Master theorem, we get, $T(n) = \Theta(n\log n)$.

Performance



Worst case complexity : $\Theta(nlogn)$

Best case complexity : $\Theta(nlogn)$

Average case complexity : $\Theta(nlogn)$

Worst case space complexity: $\Theta(n)$ auxiliary

Quicksort

- Quick sort is an example of a divide-and-conquer algorithmic technique. It is also called *partition exchange sort*.
- It uses recursive calls for sorting the elements, and it is one of the famous algorithms among comparison-based sorting algorithms.

- ***Divide:*** The array $A[low \ldots high]$ is partitioned into two non-empty sub arrays $A[low \ldots q]$ and $A[q + 1 \ldots high]$, such that each element of $A[low \ldots high]$ is less than or equal to each element of $A[q + 1 \ldots high]$.

- The index q is computed as part of this partitioning procedure.
- ***Conquer:*** The two sub arrays $A[low \ldots q]$ and $A[q + 1 \ldots high]$ are sorted by recursive calls to Quick sort.

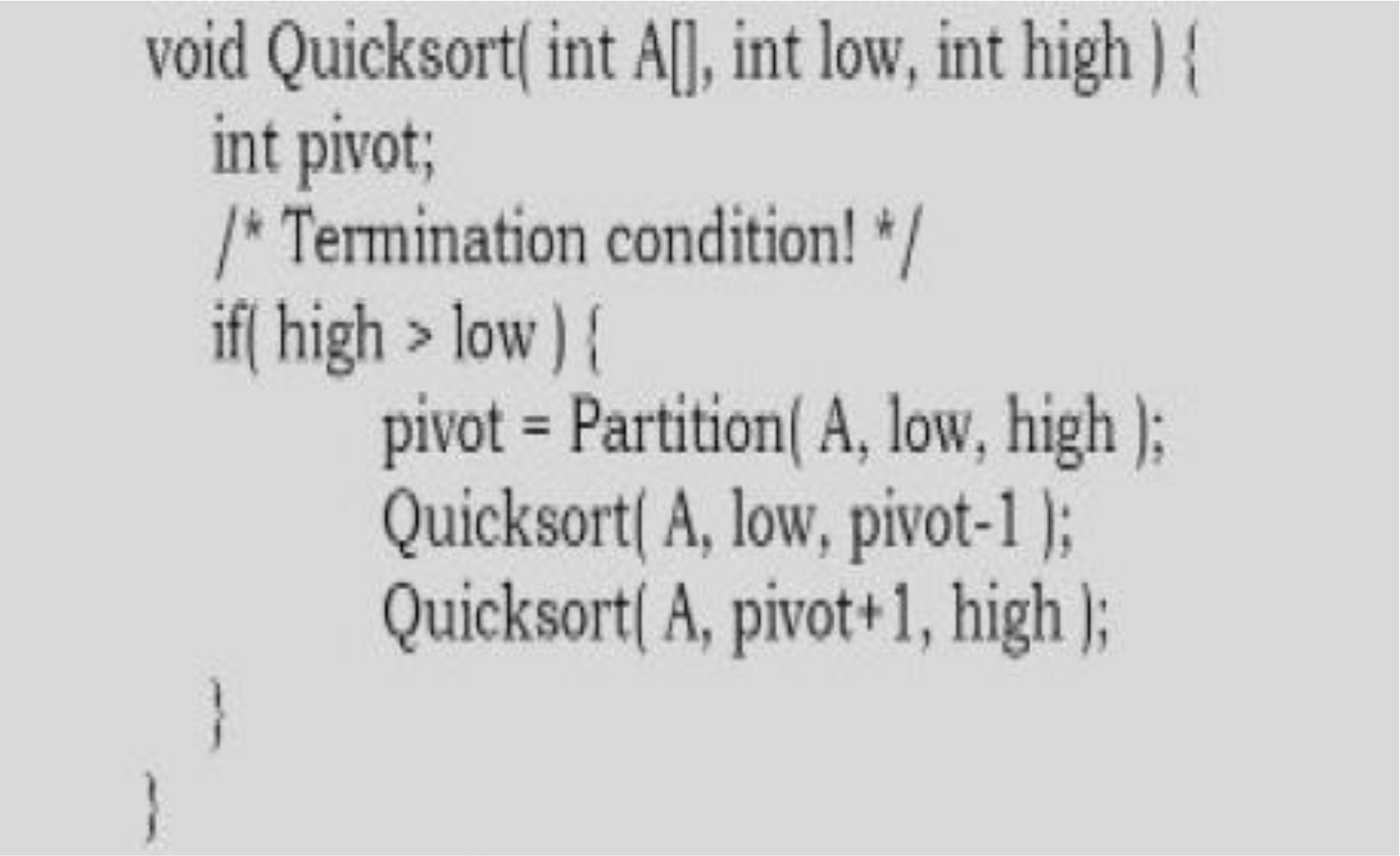
Algorithm

- The recursive algorithm consists of four steps:
- 1) If there are one or no elements in the array to be sorted, return.
- 2) Pick an element in the array to serve as the *“pivot”* point. (Usually the left-most element in the array is used.)

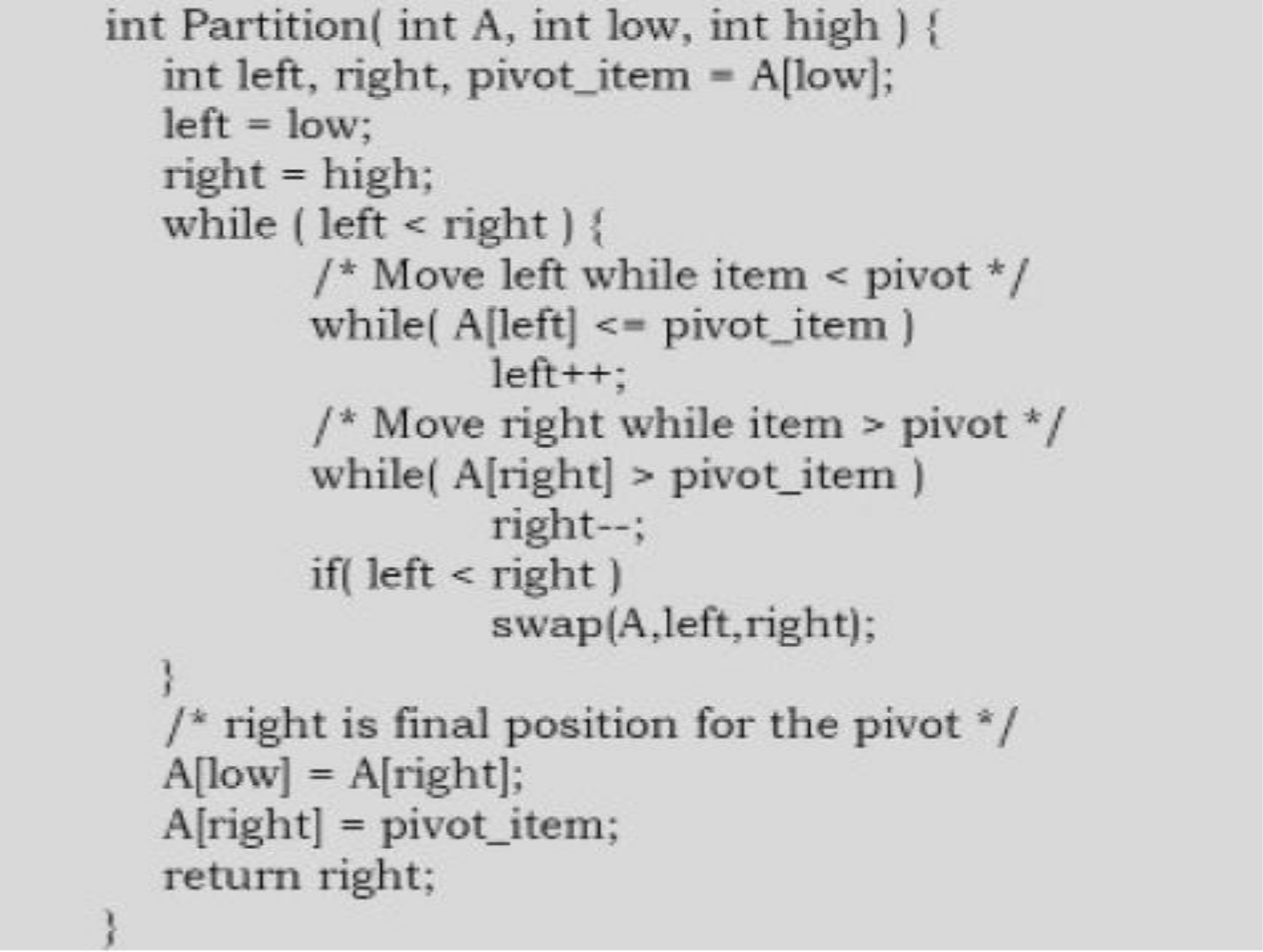
Algorithm

- 3) Split the array into two parts – one with elements larger than the pivot and the other with elements smaller than the pivot.
- 4) Recursively repeat the algorithm for both halves of the original array.

Implementation



```
void Quicksort( int A[], int low, int high ) {
    int pivot;
    /* Termination condition! */
    if( high > low ) {
            pivot = Partition( A, low, high );
            Quicksort( A, low, pivot-1 );
            Quicksort( A, pivot+1, high );
    }
}
```



```
int Partition( int A, int low, int high ) {
   int left, right, pivot_item = A[low];
   left = low;
   right = high;
   while ( left < right ) {
           /* Move left while item < pivot */
           while( A[left] <= pivot_item )
                   left++;
           /* Move right while item > pivot */
           while( A[right] > pivot_item )
                   right--;
           if( left < right )
                   swap(A,left,right);
   }
   /* right is final position for the pivot */
   A[low] = A[right];
   A[right] = pivot_item;
   return right;
}
```

Analysis

- Let us assume that $T(\mathrm{n})$ be the complexity of Quick sort and also assume that all elements are distinct.
- Recurrence for $T(n)$ depends on two sub problem sizes which depend on partition element.
- If pivot is i^{th} smallest element then exactly $(i - 1)$ items will be in left part and $(\mathrm{n} - i)$ in right part.
- Let us call it as i –split.
- Since each element has equal probability of selecting it as pivot the probability of selecting *ith* element is 1/n

- **Best Case:** Each partition splits array in halves and gives
- $T(n) = 2T(n/2) + \Theta(n) = \Theta(n\log n)$, [using *Divide and Conquer* master theorem]

- **Worst Case:** Each partition gives unbalanced splits and we get
- $T(n) = T(n - 1) + \Theta(n) = \Theta(n2)$[*using Subtraction and Conquer master theorem*]
- The worst-case occurs when the list is already sorted and last element chosen as pivot.

- **Average Case:** In the average case of Quick sort, we do not know where the split happens.
- For this reason, we take all possible values of split locations, add all their complexities and divide with *n* to get the average case complexity.

Nested Dependent Loops

```
for i = 1 to n do
  for j = i to n do
    sum = sum + 1
```

$$\sum_{i=1}^{n} \sum_{j=i}^{n} 1 = \sum_{i=}^{n} (n-i+1) = \sum_{i=1}^{n} (n+1) - \sum_{i=1}^{n} i =$$

$$n(n+1) - \frac{n(n+1)}{2} = \frac{n(n+1)}{2} \approx n^{2}$$

Recursion

- A recursive procedure can often be analyzed by solving a recursive equation
- Basic form:

```
T(n)  =  if (base case) then some constant
         else ( time to solve subproblems +
                time to combine solutions )
```

- Result depends upon
  - how many subproblems
  - how much smaller are subproblems
  - how costly to combine solutions (coefficients)

Example: Sum of Integer Queue

```
sum_queue(Q){
  if (Q.length == 0 ) return 0;
  else return Q.dequeue() +
              sum_queue(Q);   }
```

- One subproblem
- Linear reduction in size (decrease by 1)
- Combining: constant c (+), 1×subproblem

Equation:

$$T(0) \leq b$$

$$T(n) \leq c + T(n-1) \quad \text{for } n>0$$

Sum, Continued

Equation: $T(0) \leq b$

$T(n) \leq c + T(n-1)$ for $n>0$

Solution:

$$\begin{aligned} T(n) &\leq c + c + T(n-2) \\ &\leq c + c + c + T(n-3) \\ &\leq kc + T(n-k) \quad \text{for all } k \\ &\leq nc + T(0) \quad \text{for } k=n \\ &\leq cn + b \quad = O(n) \end{aligned}$$

Example: Recursive Fibonacci

- Recursive Fibonacci:

```
int Fib(n){
  if (n == 0 or n == 1) return 1 ;
  else return Fib(n - 1) + Fib(n - 2); }
```

- Running time: *Lower* bound analysis

$$T(0),\ T(1) \geq 1$$

$$T(n) \geq T(n - 1) + T(n - 2) + c \quad \textit{if} \quad n > 1$$

- Note: $T(n) \geq Fib(n)$
- Fact: $Fib(n) \geq (3/2)^n$

$O(\ (3/2)^n\ )$ *Why?*

Direct Proof of Recursive Fibonacci

- Recursive Fibonacci:

```
int Fib(n)
   if (n == 0 or n == 1) return 1
  else return Fib(n - 1) + Fib(n - 2)
```

- *Lower* bound analysis
- $T(0), T(1) \geq b$

  $T(n) \geq T(n - 1) + T(n - 2) + c \quad \text{if } n > 1$
- Analysis

  let ϕ be $(1 + \sqrt{5})/2$ which satisfies $\phi^2 = \phi + 1$

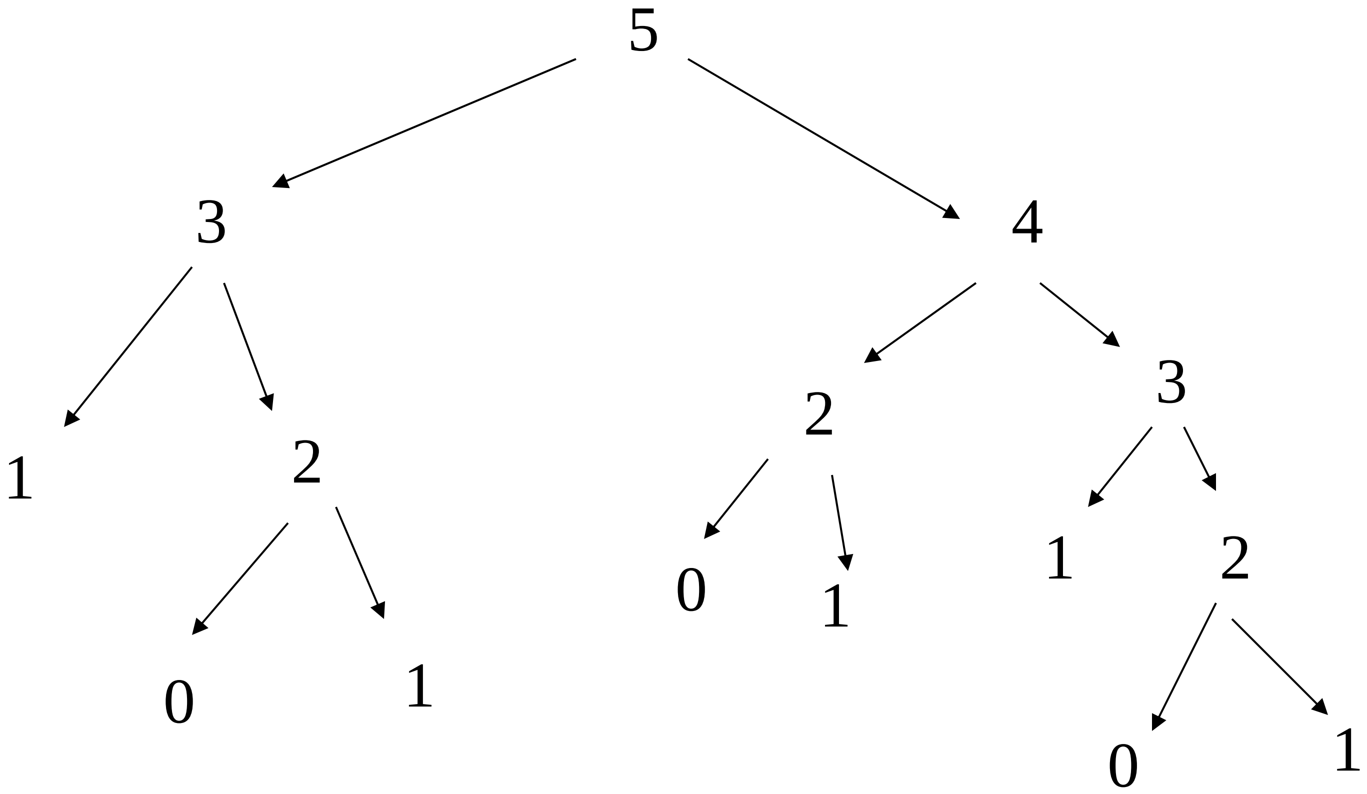
  show by induction on n that $T(n) \geq b\phi^{n - 1}$

Direct Proof Continued

- Basis: $T(0) \geq b > b\phi^{-1}$ and $T(1) \geq b = b\phi^{0}$
- Inductive step: Assume $T(m) \geq b\phi^{m-1}$ for all $m < n$

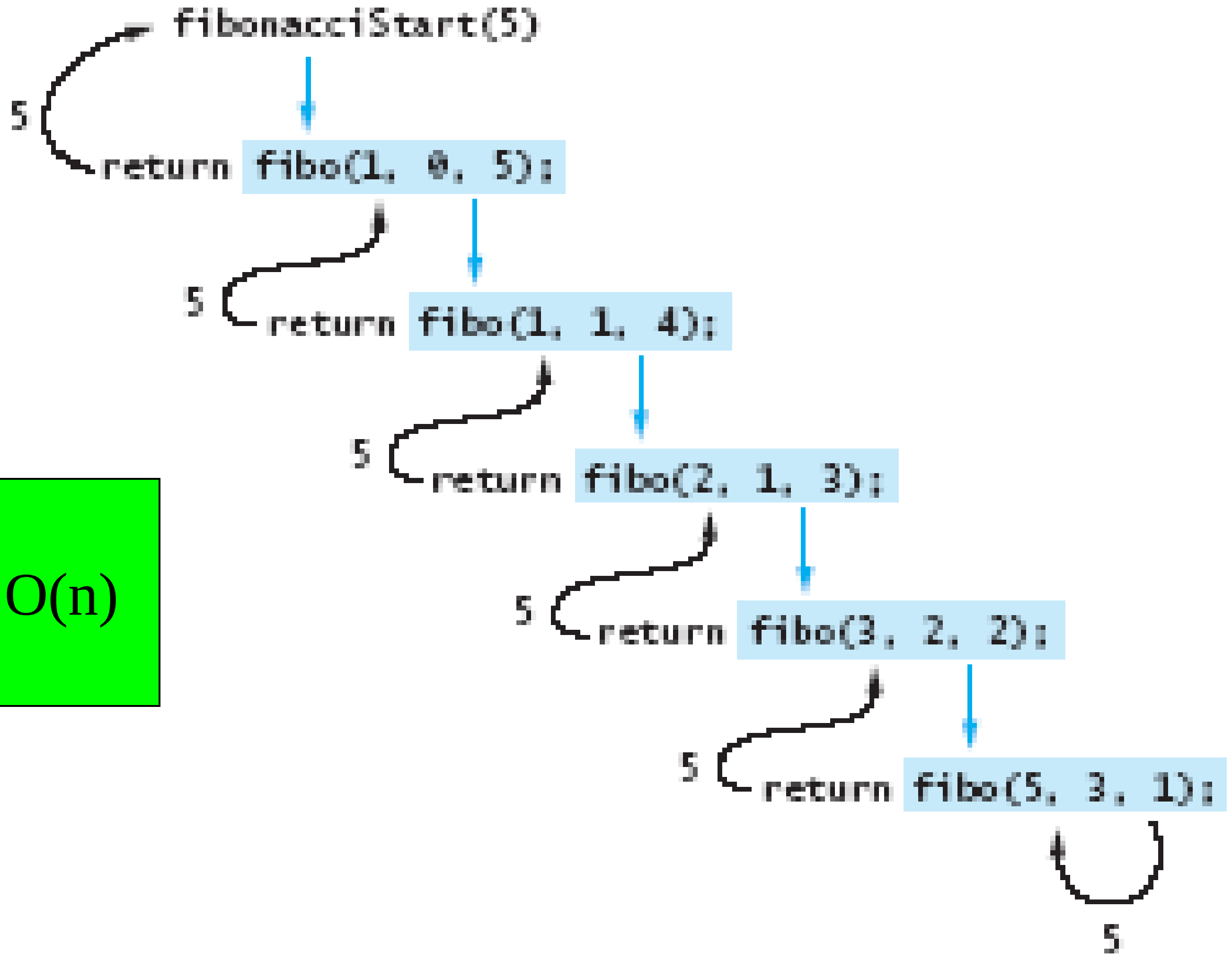
$$
\begin{aligned}
T(n) &\geq T(n-1) + T(n-2) + c \\
&\geq b\phi^{n-2} + b\phi^{n-3} + c \\
&\geq b\phi^{n-3}(\phi + 1) + c \\
&= b\phi^{n-3}\phi^{2} + c \\
&\geq b\phi^{n-1}
\end{aligned}
$$

Fibonacci Call Tree



Efficient Fibonacci: A Trace

FIGURE 7.7
Trace of
`fibonacciStart(5)`



```
fibonacciStart(5)
return fibo(1, 0, 5);
return fibo(1, 1, 4);
return fibo(2, 1, 3);
return fibo(3, 2, 2);
return fibo(5, 3, 1);
```

Performance is O(n)

Recursive Definitions: Power

- $x^0 = 1$
- $x^n = x \times x^{n-1}$

```
public static double power
    (double x, int n) {
  if (n <= 0)  // or: throw exc. if < 0
    return 1;
  else
    return x * power(x, n-1);
}
```

Recursive Definitions: Factorial Code

```
public static int factorial (int n) {
  if (n == 0) // or: throw exc. if < 0
    return 1;
  else
    return n * factorial(n-1);
}
```

Another example

- The factorial function: multiply together all numbers from 1 to n.
- denoted n!

$$n!=n*(n-1)*(n-2)*\ldots 2*1$$

$$n!=\begin{cases} n*(n-1)! & \text{if } n>0 \\ 1 & \text{if } n==0 \end{cases}$$

←General case: Uses a solution to a simpler sub-problem

←Base case: Solution is given directly

4! Walk-through

4!=

$$n! = \begin{cases} n*(n-1)! & \text{if } n>0 \\ 1 & \text{if } n==0 \end{cases}$$

Java implementation of n!

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

$$n! = \begin{cases} n*(n-1)! & \text{if } n>0 \\ 1 & \text{if } n==0 \end{cases}$$

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

factorial(4)

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

factorial(4);

```
public int factorial(int
n){
        if (n==0)
                return  1;
        else
                return
            n*factorial(n-
            1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

n=2

Returns 2*factorial(1)

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

n=2

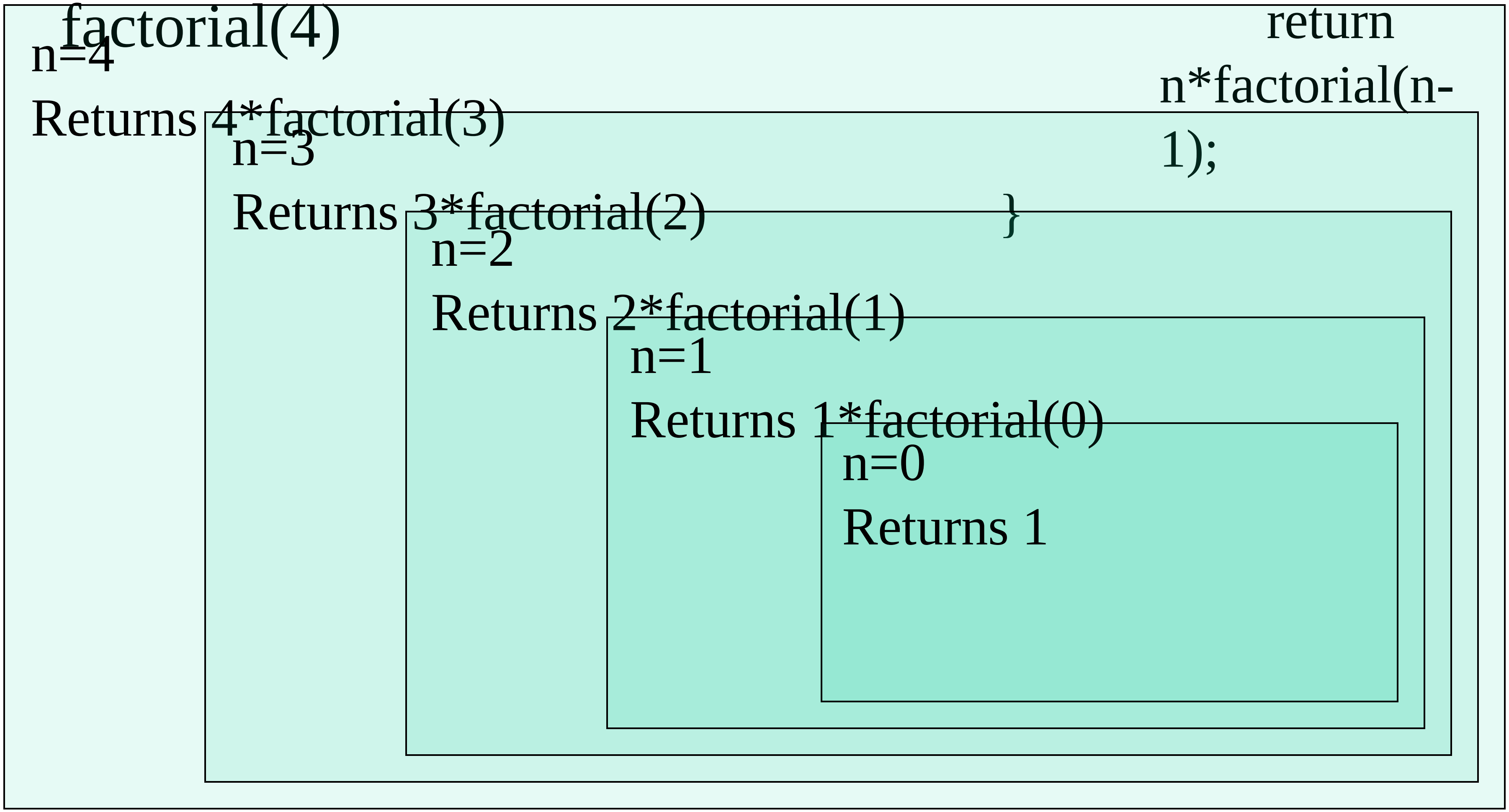
Returns 2*factorial(1)

n=1

Returns 1*factorial(0)

factorial(4);

```
public int factorial(int
n){
        if (n==0)
                return  1;
        else
                return
            n*factorial(n-
            1);
      }
```



factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

n=2

Returns 2*factorial(1)

n=1

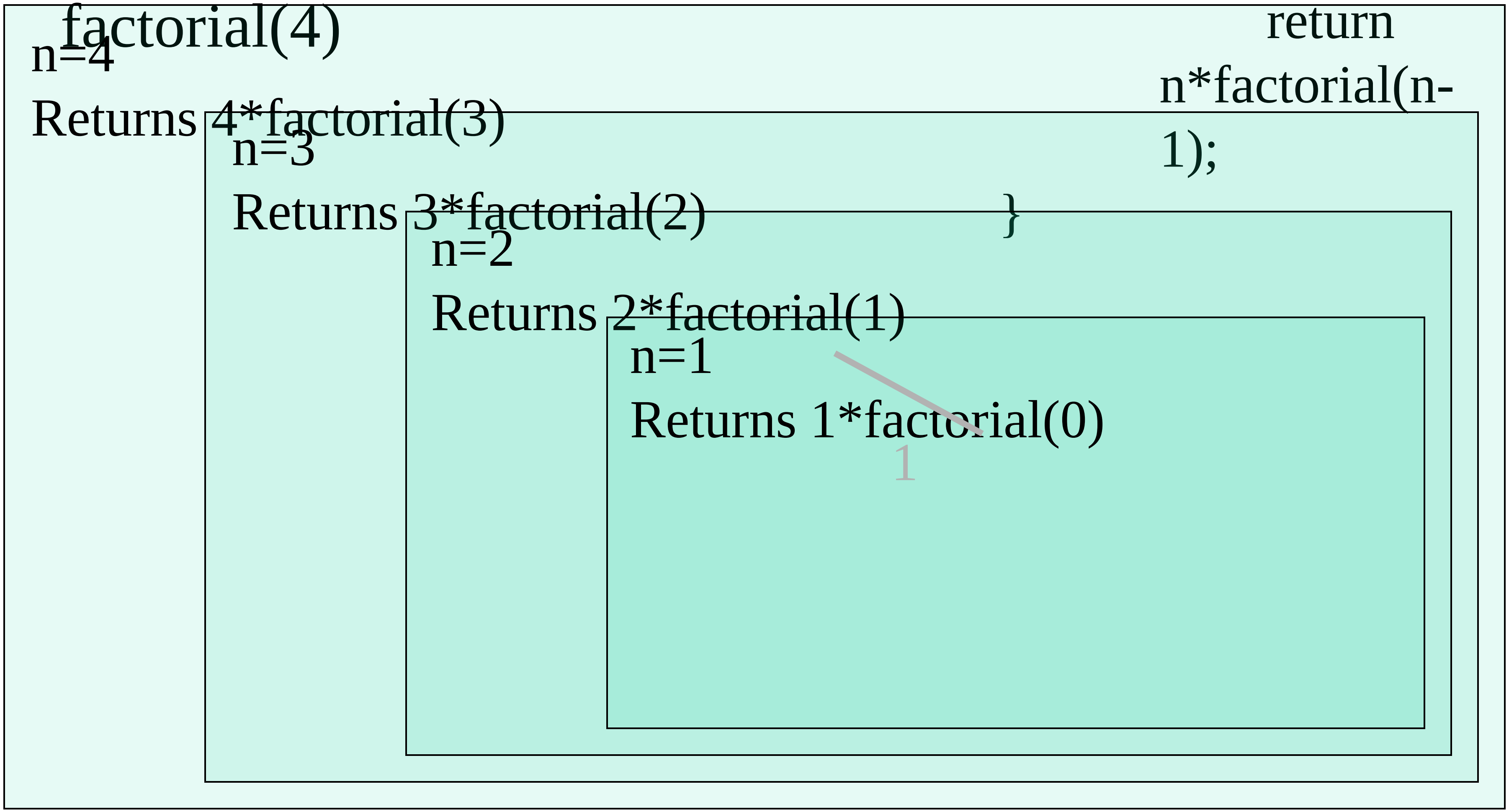
Returns 1*factorial(0)

n=0

Returns 1

factorial(4);

```
public int factorial(int
n){
        if (n==0)
                return  1;
        else
                return
        n*factorial(n-
        1);
}
```



factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

n=2

Returns 2*factorial(1)

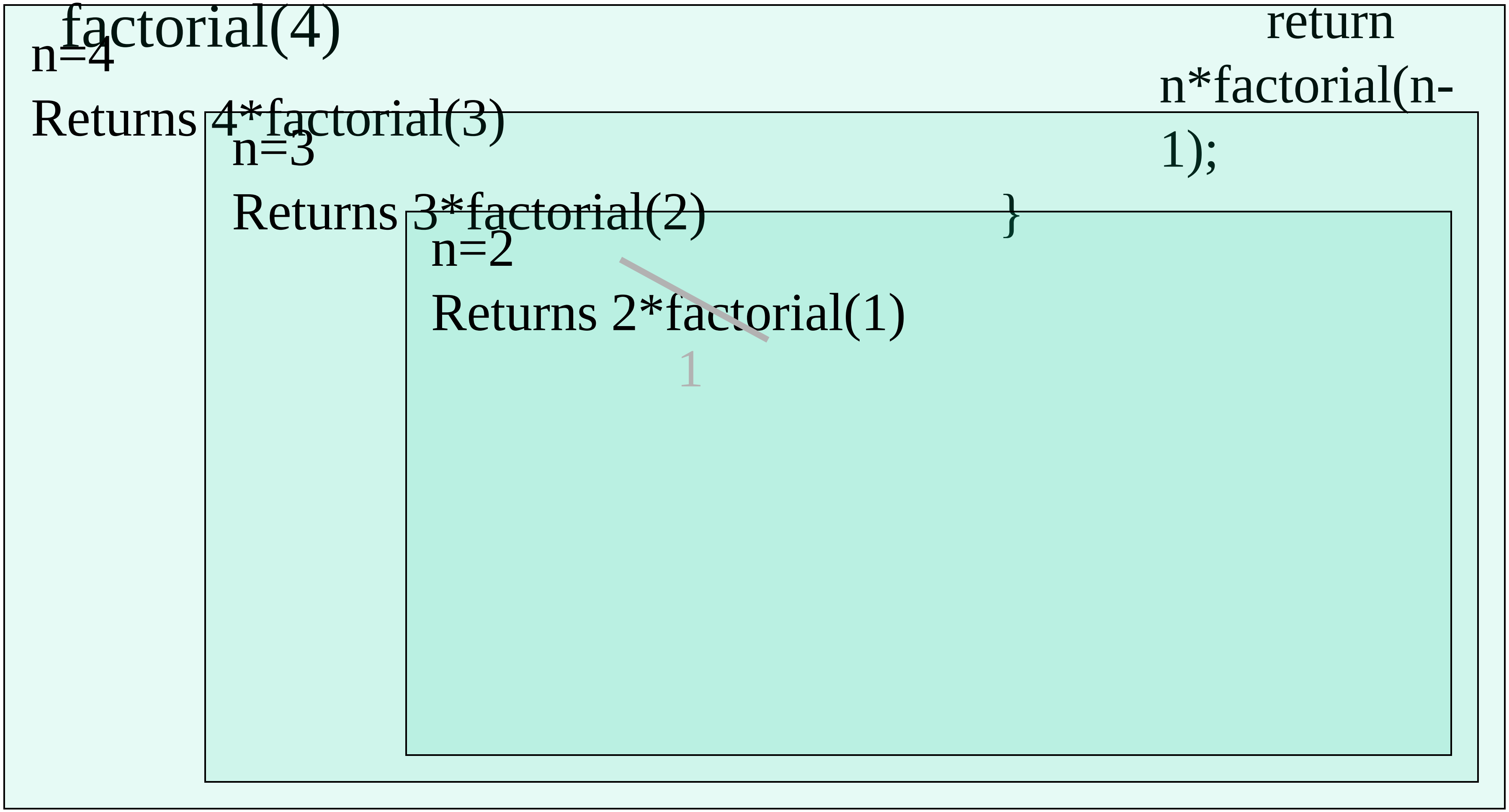
n=1

Returns 1*factorial(0)

1

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```



factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

n=2

Returns 2*~~factorial(1)~~

1

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return
            n*factorial(n-1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

n=3

Returns 3*factorial(2)

2

factorial(4);

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

factorial(4)

n=4

Returns 4*factorial(3)

6

factorial(4);

factorial(4)

24

```
public int factorial(int n){
        if (n==0)
                return  1;
        else
                return n*factorial(n-1);
}
```

Recursive Definitions: Greatest Common Divisor

Definition of gcd(m, n), for integers m > n > 0:

- gcd(m, n) = n, if n divides m evenly
- gcd(m, n) = gcd(n, m % n), otherwise

```
public static int gcd (int m, int n) {
  if (m < n)
    return gcd(n, m);
  else if (m % n == 0) // could check n>0
    return n;
  else
    return gcd(n, m % n);
}
```

Example: Binary Search

7	12	30	35	75	83	87	90	97	99

One subproblem, half as large

Equation: $T(1) \leq b$

$T(n) \leq T(n/2) + c$ for $n>1$

Solution:

$T(n) \leq T(n/2) + c$

$\leq T(n/4) + c + c$

$\leq T(n/8) + c + c + c$

$\leq T(n/2^k) + kc$

$\leq T(1) + c \log n$ where $k = \log n$

$\leq b + c \log n = O(\log n)$

Example: MergeSort

Split array in half, sort each half, merge together

- 2 subproblems, each half as large
- linear amount of work to combine

$$T(1) \le b$$

$$T(n) \le 2T(n/2) + cn \qquad \text{for } n>1$$

$T(n) \le 2T(n/2)+cn \quad \le \quad 2(2(T(n/4)+cn/2)+cn$

$= 4T(n/4) +cn +cn \quad \le \quad 4(2(T(n/8)+c(n/4))+cn+cn$

$= 8T(n/8)+cn+cn+cn \quad \le \quad 2kT(n/2k)+kcn$

$\le 2kT(1) + cn \log n \quad$ where $k = \log n$

$= O(n \log n)$

Recursion Versus Iteration

- Recursion and iteration are *similar*
- **Iteration:**
  - Loop repetition test determines whether to exit
- **Recursion:**
  - Condition tests for a base case
- Can always write iterative solution to a problem solved recursively, *but:*
- Recursive code often simpler than iterative
  - Thus easier to write, read, and debug

Searching

Definition

- Searching is the process of finding an item with specified properties from a collection of items.

- The items may be stored as
  - Records in a database
  - Simple data elements in arrays
  - Text in files
  - Nodes in trees

  Etc

Purpose of Searching

- Computers store a lot of information.
- To retrieve information proficiently searching algorithms are used.

Types of searching

- Unordered Linear Serarch
- Sorted/Ordered Linear Search
- Binary Search

Unordered Linear Search

- Let us assume we are given an array where the order of the elements is not known.
- Means the elements of the array are not sorted.
- Here we have to scan the complete array and see if the element is there in the given list or not

Algorithm

```
Int unORderedLinearSearch(int A[], int data)
    For(int i=0; i<n;i++){
       If(A[i]==data)
          return i;
     }
       return -1;
}
```

Example

Input : A[] = {10, 20, 80, 30, 60, 50,
110, 100, 130, 170}
x = 110;

Output : 6

Element x is present at index 6

Input : arr[] = {10, 20, 80, 30, 60, 50,
110, 100, 130, 170}
x = 175;

Output : -1

Element x is not present in A[].

Complexity

- Time Complexity: O(n)
- In the worst case we need to scan the complete array.
- Space Complexity: O(1)

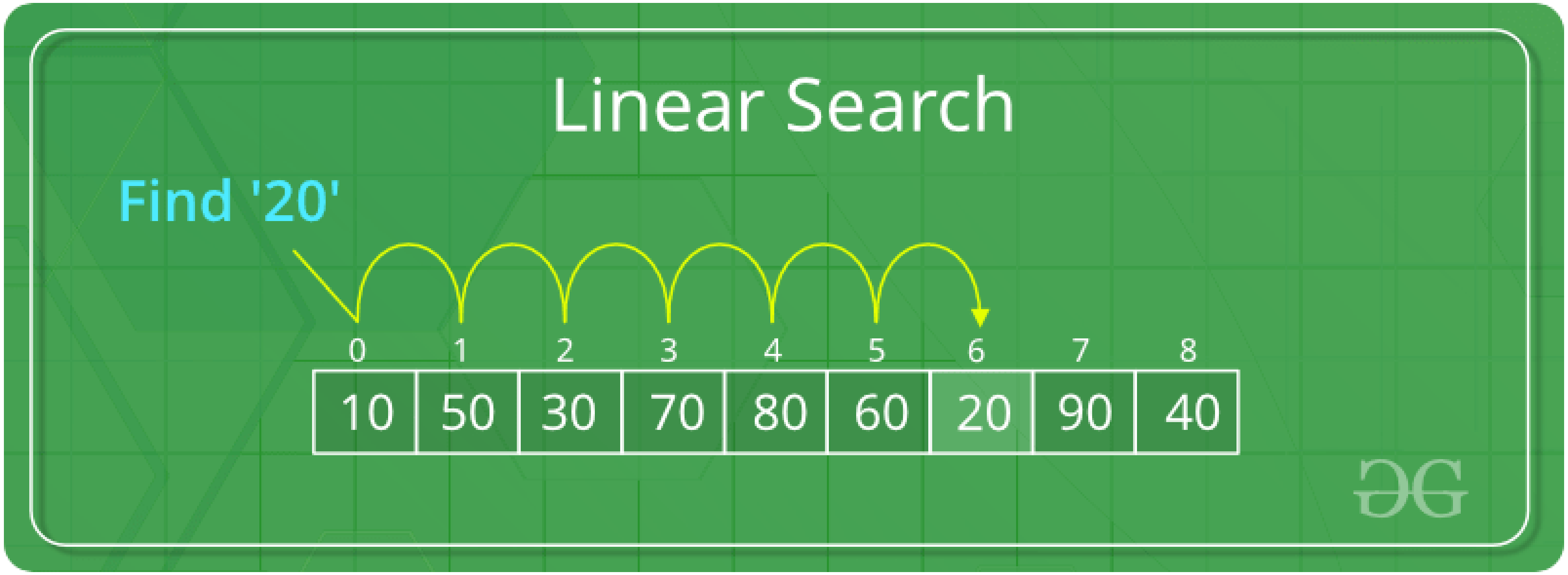
Sorted/Ordered Linear Search

- If the elements of the array are already sorted, we don’t have to scan the complete array to see if the element is there in the given array or not.
- In the algorithm below, if the value at A[i] is greater than the data to be searched, then we just return -1 without searching the remaining array.

Algorithm

```
Int orderedLinearSearch(int A[], int n, int data){
        for(int i=0; i<n ; i++){
                if(A[i]==data)
                        return i;
                else if(A[i] > data)
                        return -1;
        }
        return -1;
}
```

Example



Complexity

- Time Complexity:O(n), in worst we scan the complete array.
- Space Complexity: O(1).

Binary Search

- Let us consider the problem of searching a word in a dictionary.
- It works on the principle of divide and conquer technique.
- We go to some approximate page(say, middle page) and start searching from that point.
- If the name that we are searching is the same then the search is complete.
- If the page is before the selected pages then apply the same process for the first half; otherwise apply the same to the second half.

- Binary search also works in the same way.
- The algorithm applying such a strategy is referred to as binary search algorithm

Mid = low + (high-low)/2

or

Mid= (low+high)/2

Algorithm Method 1

- **//Iterative Binary Search Algorithm**

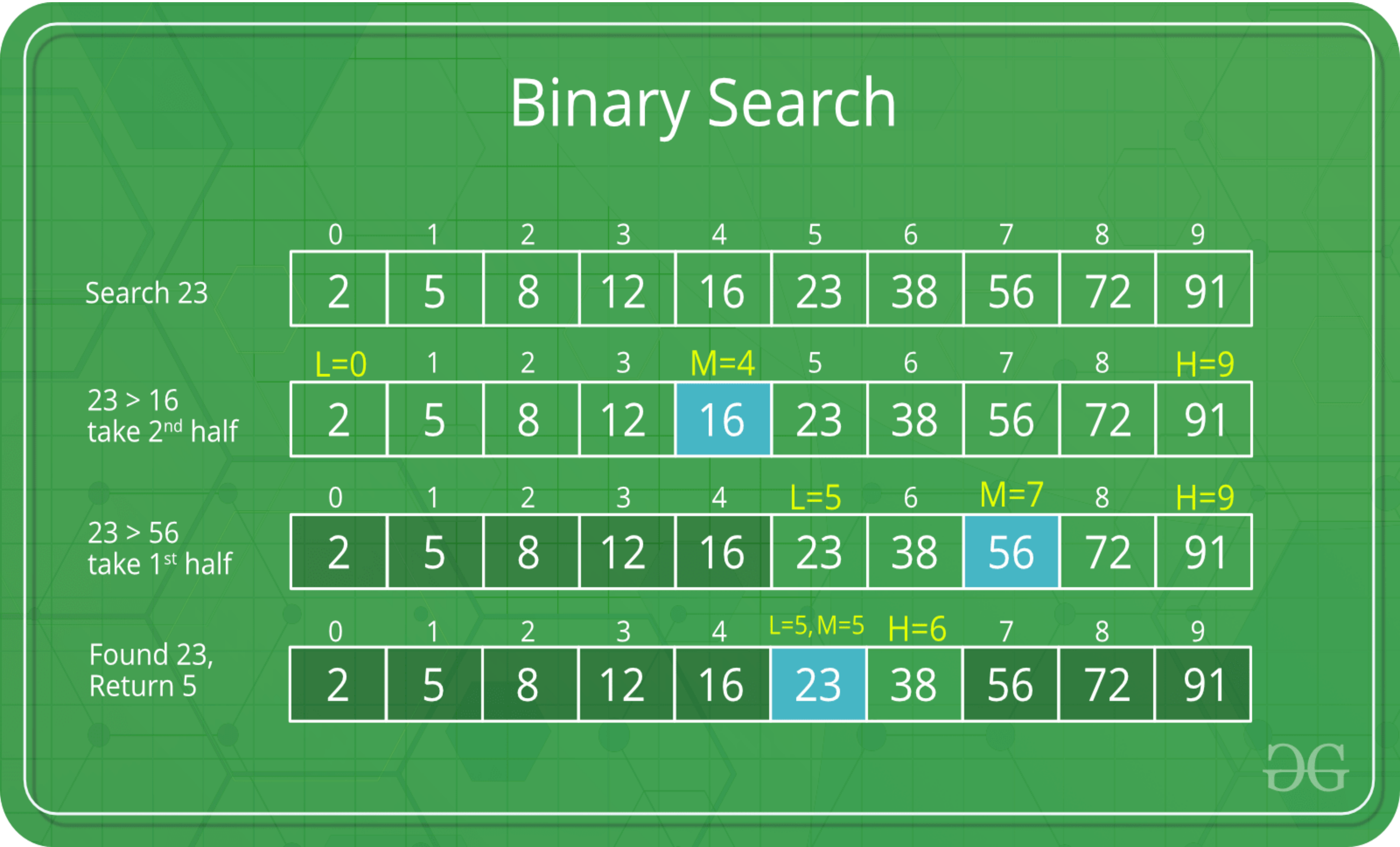
```
int binarySearchIterative(int A[i], int n, int data)
          int low=0;
          while (low<=high){
              mid=low + (high-low)/2; // To avoid overflow
              if(A[mid] == data)
                    return mid;
               else if (A[mid] < data)
                    low = mid + 1;
                else high = mid – 1;
          }
             return -1;
}
```

Algorithm Method 2

- **//Recursive Binary Search Algorithm**

```
int binarySearchRecursive(int A[], low,int igh, int data)
        int mid = low+(high-low)/2 // To avoid overflow
        if((low>high)
                return -1;
        if(A[mid] == data)
                return mid;
        else if(A[mid] < data)
                return BinarySearchRecursive(A, mid+1,
                                                    high,data);
        else    return BinarySearchRecursive(A,low, mid-1, data);
        return -1;
}
```

Example



Binary Search

	0	1	2	3	4	5	6	7	8	9
Search 23	2	5	8	12	16	23	38	56	72	91
	L=0	1	2	3	M=4	5	6	7	8	H=9
23 > 16 take 2nd half	2	5	8	12	16	23	38	56	72	91
	0	1	2	3	4	L=5	6	M=7	8	H=9
23 > 56 take 1st half	2	5	8	12	16	23	38	56	72	91
	0	1	2	3	4	L=5, M=5	H=6	7	8	9
Found 23, Return 5	2	5	8	12	16	23	38	56	72	91

Advantages & Disadvantages

- Advantages:
  - Binary search is ***much* faster** than linear search
  - It eliminates half of the list from further searching by using the result of each comparison.
  - **Time Complexity of Binary Search Algorithm is O($\log_2 n$).**
    - Here, n is the number of elements in the sorted linear array.
  - **Linear** search takes, on **average N/2** comparisons (where N is the number of elements in the array), and **worst** case **N** comparisons.
  - It indicates whether the element being searched is before or after the current position in the list.

- **Disadvantages**
  - It works only on lists that are sorted and kept sorted.
  - It works only on element types for which there exists a less-than(<) relationship.
  - It employs recursive approach which requires more stack space.

Selection Sort

- Selection sort is an **in-place** sorting algorithm. Selection sort works well for **small files**.
- It is for sorting the files with **very large values used and small keys**.
- This is because **selection is made based on keys and swaps are made only when required**.

Advantages

- Easy to implement
- In-place sort (**requires no additional storage space**)

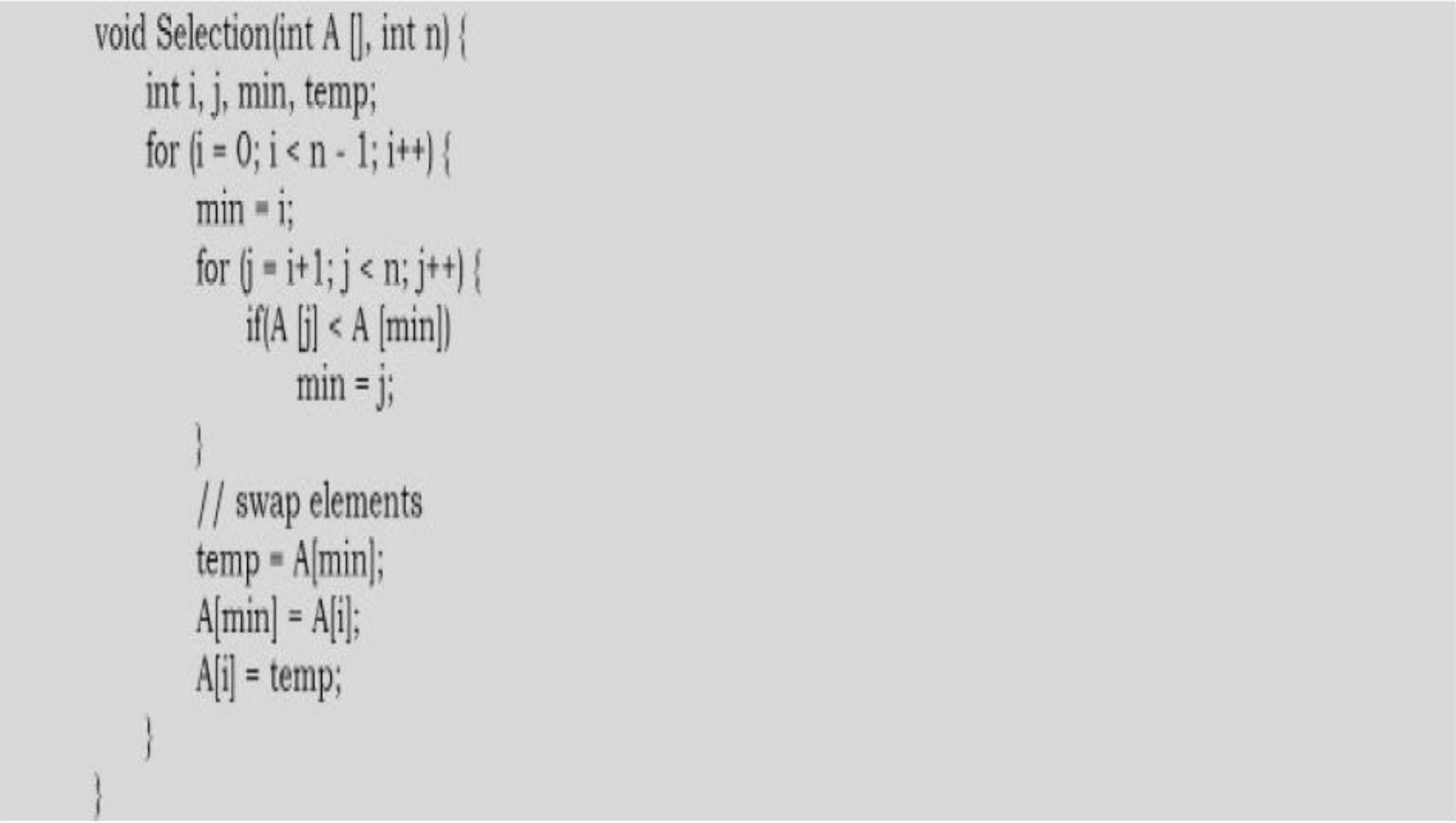
Disadvantages

- Doesn’t scale well: $O(n^2)$

Algorithm

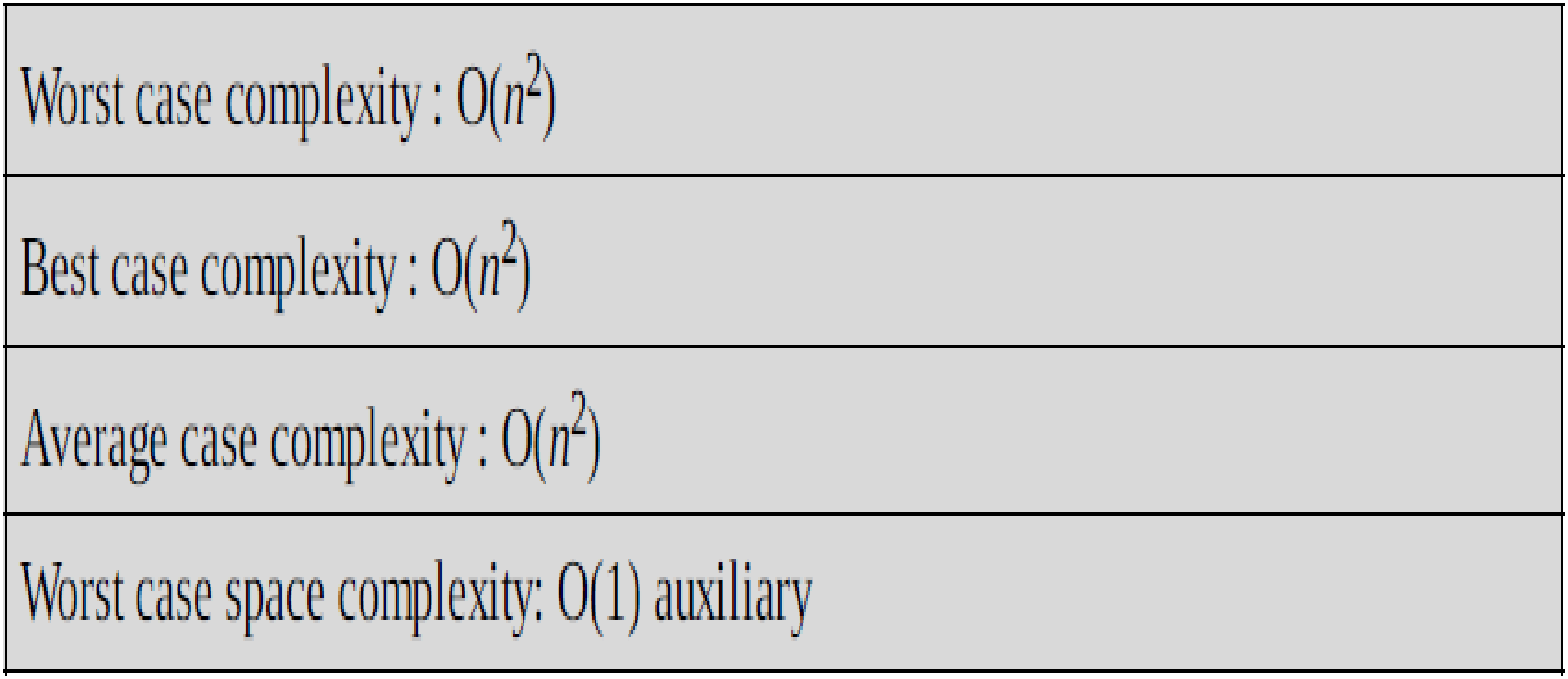
- 1. **Find** the **minimum value** in the list
- 2. **Swap it with the value** in the **current position**
- 3. **Repeat** this process for **all the elements** until the **entire array is sorted**
- This algorithm is called *selection sort* since it **repeatedly *selects* the smallest element.**

Implementation



```
void Selection(int A [], int n) {
    int i, j, min, temp;
    for (i = 0; i < n - 1; i++) {
        min = i;
        for (j = i+1; j < n; j++) {
            if(A [j] < A [min])
                min = j;
        }
        // swap elements
        temp = A[min];
        A[min] = A[i];
        A[i] = temp;
    }
}
```

Performance



Worst case complexity : $O(n^2)$

Best case complexity : $O(n^2)$

Average case complexity : $O(n^2)$

Worst case space complexity: O(1) auxiliary

Sorting

Definition

- *Sorting* is an algorithm that **arranges the elements of a list in a certain order** [either *ascending* or *descending*].
- The **output** is a **permutation or reordering of the input.**

Why is Sorting Necessary?

- Sorting can significantly **reduce the complexity** of a problem.
- Used for **database algorithms and searches**.

Classifications

- sorting algorithms are classified into
    - Internal Sort
    - External Sort

Internal Sort

- Sort algorithms use **main memory** exclusively during the sort are called ***internal*** sorting algorithms.
- This kind of algorithm assumes high-speed random access to all memory.
- **Bubble Sort**.
- **Insertion Sort**.
- Quick Sort.
- Heap Sort.
- Radix Sort.
- **Selection sort**.

External Sort

- Sorting algorithms that use **external memory**, such as **tape or disk**, during the sort come under this category.
- Distribution sorting,
  - which resembles quicksort,
- external merge sort,
  - which resembles merge sort.

Classification of Sorting Algorithms

- Sorting algorithms are generally categorized based on the following parameters.
- **By Number of Comparisons**
- **By Number of Swaps**
- **By Memory Usage**
- **By Recursion**
- **By Stability**
- **By Adaptability**

Bubble Sort

- Bubble sort is the simplest sorting algorithm.

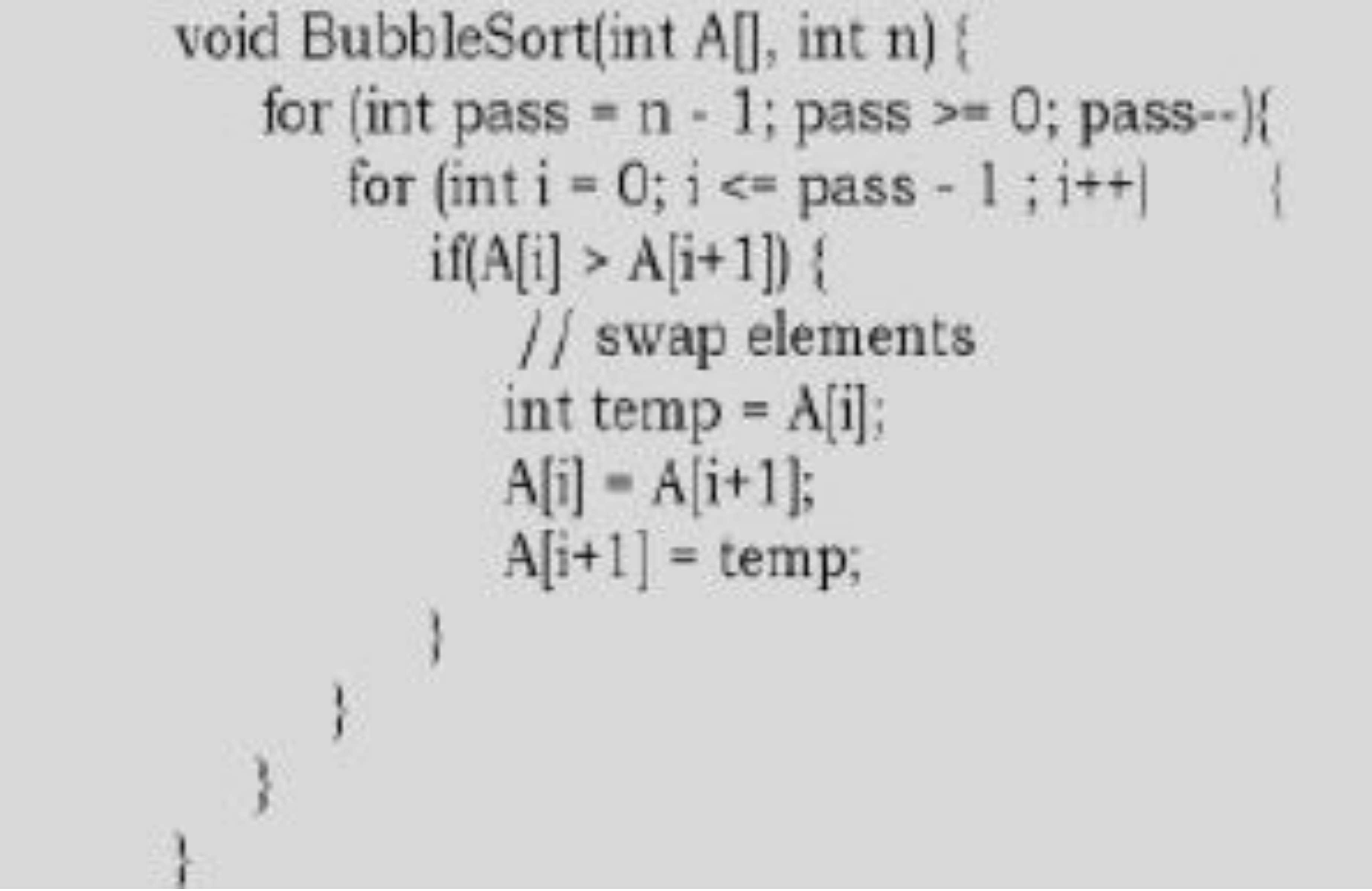
- It works by iterating the input array from the first element to the last, comparing each pair of elements and swapping them if needed.

- Bubble sort continues its iterations **until no more swaps are needed.**

- The algorithm gets its name from the way **smaller elements “bubble”** to the top of the list.

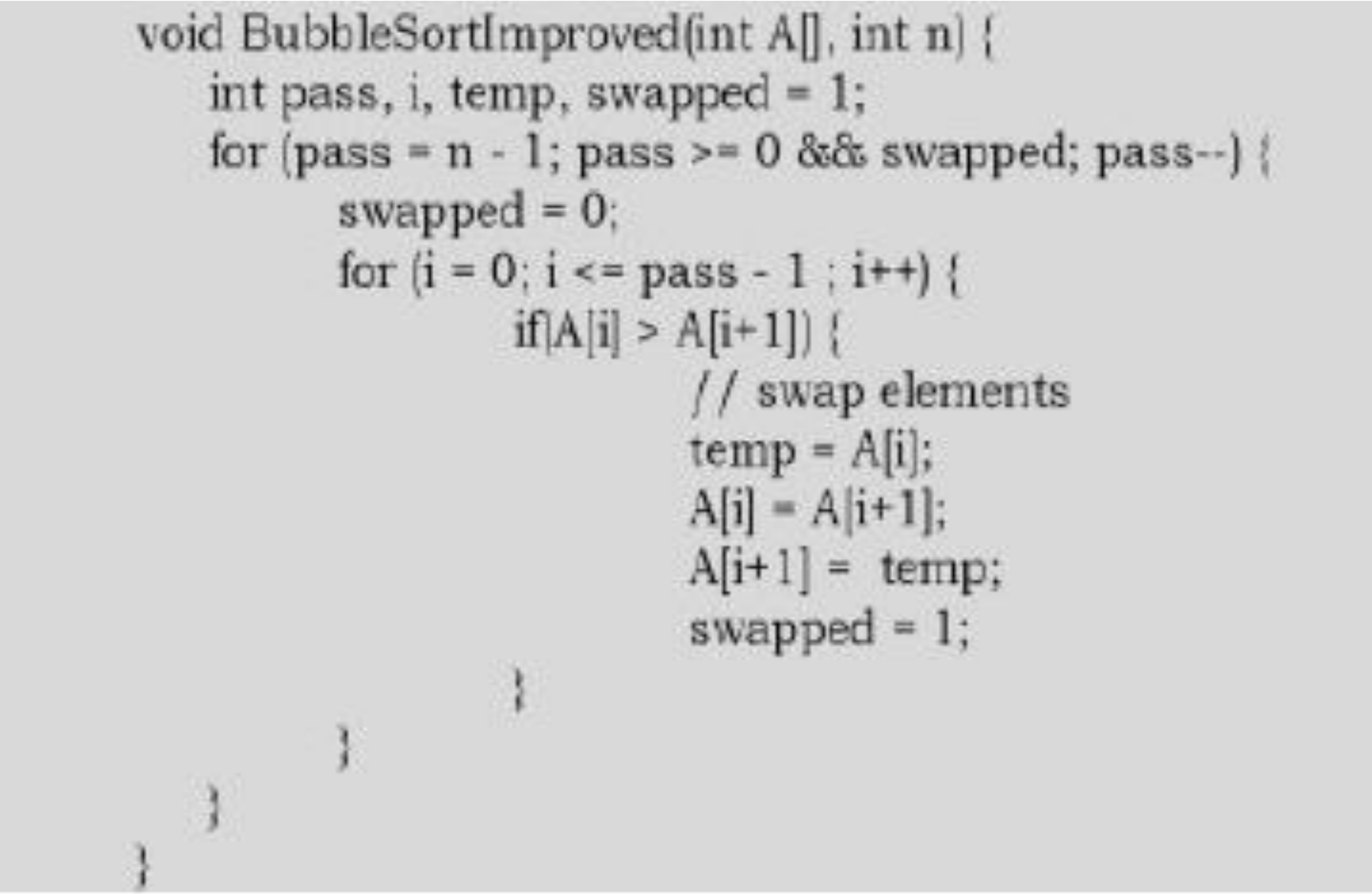
- The only significant advantage is that **it can detect whether the input list is already sorted or not.**

Implementation



```
void BubbleSort(int A[], int n) {
    for (int pass = n - 1; pass >= 0; pass--){
        for (int i = 0; i <= pass - 1 ; i++)      {
            if(A[i] > A[i+1]) {
                // swap elements
                int temp = A[i];
                A[i] = A[i+1];
                A[i+1] = temp;
            }
        }
    }
}
```

- Algorithm takes O(n^2) (even in best case).
- **We can improve it by using one extra flag.**
- No more swaps indicate the completion of sorting. If the list is already sorted, we can use this flag to skip the remaining passes.



```
void BubbleSortImproved(int A[], int n) {
    int pass, i, temp, swapped = 1;
    for (pass = n - 1; pass >= 0 && swapped; pass--) {
            swapped = 0;
            for (i = 0; i <= pass - 1 ; i++) {
                    if(A[i] > A[i+1]) {
                            // swap elements
                            temp = A[i];
                            A[i] = A[i+1];
                            A[i+1] =  temp;
                            swapped = 1;
                    }
            }
    }
}
```

Performance

- This modified version improves the best case of bubble sort to O(n).
- Worst case complexity : O(n2)
- Best case complexity (Improved version) : O(n)
- Average case complexity (Basic version) : O(n2)
- Worst case space complexity : O(1) auxiliary