Prasad V. Potluri Siddhartha Institute of Technology:: Vijayawada. Department of Computer Science and Engineering

I/II M.Tech. (CSE) (Second Semester)

17CSCS2T5A

ADVANCED LINEAR ALGEBRA Elective - III Credits: 4

Lecture: 4 Periods/week

Internal Assessment: 40 Marks Semester end examination: 60 Marks

Course Description:

This course highlights the basic properties of linear transformations between real vector spaces. The course also establishes structural properties of linear operators, structural properties of bilinear forms, and Hilbert Spaces.

Course Outcome:

At the end of the course, the graduate will be able to:

- **CO1:** Determine, with proof, whether a subset of a vector space is a subspace.
- **CO2:** Find the Jordan canonical form, and give a basis that attains it.
- **CO3:** Identify the Eigen spaces and other invariant spaces associated with each Eigen value.
- CO4: Represent a bilinear form on a vector space as a square matrix.
- **CO5:** Explain the Notion of Hilbert Spaces on a vector space.

UNIT - I

Basic Linear Algebra

Vector Spaces: Subspaces, Direct Sums, Spanning Sets and Linear Independence, The Dimension of a Vector Space, Ordered Bases and Coordinate Matrices, The Row and Column Spaces of a Matrix, The Complexification of a Real Vector Space.

Linear Transformations: Linear Transformations, the Kernel and Image of a Linear Transformation, Isomorphisms, the Rank plus Nullity Theorem, Linear Transformations from F^m to F^n , Change of Basis Matrices, The Matrix of a Linear Transformation, Change of Bases for Linear Transformations, Equivalence of Matrices, Similarity of Matrices, Similarity of Operators, Invariant Subspaces and Reducing Pairs, Projection Operators, Topological Vector Spaces, Linear Operators on V^c.

UNIT - II

The Structure of a Linear Operator: The Module Associated with a Linear Operator, The Primary Cyclic Decomposition of $V\tau$, The Characteristic Polynomial, Cyclic and Indecomposable Modules, The Big Picture, The Rational Canonical Form, Eigenvalues and Eigenvectors, Eigenvalues and Eigenvectors, Geometric and Algebraic Multiplicities, the Jordan Canonical Form, Triangularizability and Schur's Theorem.

Unit-III

Metric Vector Spaces: The Theory of Bilinear Forms: Symmetric, Skew-Symmetric and Alternate Forms, the Matrix of a Bilinear Form Quadratic Forms, Orthogonally, Linear Functionals, Orthogonal Complements and Orthogonal Direct Sums, Isometries, Hyperbolic Spaces, Nonsingular Completions of a Subspace.

Unit-IV

Metric Spaces: the Definition, Open and Closed Sets, Convergence in a Metric Space, The Closure of a Set, Dense Subsets, Continuity, Completeness, Isometries, The Completion of a Metric Space, Exercises, Hilbert Spaces, a Brief Review, Hilbert Spaces, Infinite Series, an Approximation Problem, Hilbert Bases, Fourier Expansions, Characterization of Hilbert Bases, Hilbert Dimension, A Characterization of Hilbert Spaces, The Riesz Representation Theorem.

Text book:

1. Steven Roman, "Advanced Linear Algebra", 3rd Edition, Springer Publications.

Reference Books:

- Yisong yang, "A Concise Text on Advanced Linear Algebra", Cambridge University Press
- Steven H. Weintraub, "A Guide to Advanced Linear Algebra", Publications Mathematical Association of America